# Shell Model Studies of the Double Beta Decays of $^{76}$Ge, $^{82}$Se, and $^{136}$Xe

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The double beta decays of $^{76}$Ge, $^{82}$Se, and $^{136}$Xe are studied in very large shell model spaces. The dimensions of the shell model spaces used in these calculations reach $O(10^8)$. The two neutrino matrix elements obtained are in good agreement with the available experimental data. Assuming that the mass of the neutrino is smaller than 1 eV, we get the following upper bounds to the half-lives for the neutrinoless decays: $T_{1/2}^{(0\nu)}(\mathrm{Ge}) > 1.85 \times 10^{25}$ yr, $T_{1/2}^{(0\nu)}(\mathrm{Se}) > 2.36 \times 10^{24}$ yr, and $T_{1/2}^{(0\nu)}(\mathrm{Xe}) > 1.21 \times 10^{25}$ yr. [S0031-9007(96)00984-2]

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The importance of the nuclear double beta decay ($\beta\beta$) is well established. The two neutrino mode ($\beta\beta_{(2\nu)}$), very sensitive to the nuclear correlations, provides a severe test of the nuclear wave functions. The neutrinoless mode ($\beta\beta_{(0\nu)}$) is one of the best probes for the physics beyond the standard model. It is particularly suitable to explore intrinsic properties of the neutrino such as its mass and the existence of right-handed weak currents [1–4]. Reasonable values of these lepton-number-violating parameters can be extracted from the experiment provided that accurate nuclear matrix elements are used.

Before this work, large shell model (SM) calculations of the $\beta\beta$ decay had been only possible in $^{48}$Ca [1,5]. In order to describe heavier nuclei different approximations had to be devised. Haxton, Stephenson, and Strottman implemented a weak coupling approximation in order to calculate the decays of $^{76}$Ge, $^{82}$Se, and $^{128,130}$Te [1]. They also used the closure approximation to circumvent the calculation of the $1^+$ states in the intermediate nuclei. The uncertainty in their results is difficult to estimate because the choice of the energy denominator in the two neutrino matrix element is not unique. Using a statistical method to determine the energy denominators and setting $g_a = 1$, they gave reasonable values for the lifetimes in all cases, except for the tellurium isotopes.

The quasiparticle random-phase approximation (QRPA) has also been used to study the decay of medium and heavy nuclei [6]. A shortcoming of this approach [7,8] is that the matrix elements of the Gamow-Teller and double Gamow-Teller operators are very sensitive to the particle-particle interaction. The strength of this interaction ($g_{pp}$) is treated as a parameter and fitted to the available $\beta_\pm$ data. The introduction of $g_{pp}$ as a phenomenological factor makes it possible to reproduce the observed half-lives. However, the predictive power of the QRPA approach is limited because of the large variation of the relevant matrix element $M_{\mathrm{GT}}^{(2\nu)}$ in the physical window for $g_{pp}$. The zero neutrino mode is less sensitive to the correlations; still, different predictions for $M_{\mathrm{GT}}^{(0\nu)}$ may differ by a factor of 3 [9–13].

Very recently the shell model Monte Carlo method (SMMC) [14] has been applied to the description of the two neutrino channel. For the decay $^{48}\mathrm{Ca} \rightarrow {}^{48}\mathrm{Ti}$ the SMMC matrix element $0.15 \pm 0.07$ MeV$^{-1}$ can be compared to the result of the exact diagonalization using the same interaction, 0.08 MeV$^{-1}$. For the decay of $^{76}$Ge a matrix element $0.13 \pm 0.05$ MeV$^{-1}$ is obtained.

In this Letter we study the $\beta\beta$ decays of $^{76}$Ge, $^{82}$Se, and $^{136}$Xe in the shell model framework. For $^{76}$Ge and $^{82}$Se the Schrödinger equation is solved in the valence space spanned by the orbits $p_{3/2}$, $f_{5/2}$, $p_{1/2}$, and $g_{9/2}$. For $^{136}$Xe the valence space contains the $d_{5/2}$, $s_{1/2}$, $g_{7/2}$, $d_{3/2}$, and $h_{11/2}$ shells. For each valence space we define a truncation scheme labeled by $t$: the number of particles that are allowed to jump from the $pf$ subshells to the $g_{9/2}$ orbital ($A = 76, 82$) or the number of particles that can be excited from the lower $g_{7/2}$ and $d_{5/2}$ subshells to the remaining three orbitals ($A = 136$). Later on we shall analyze the convergence of the relevant matrix elements as a function of the truncation. We can perform full calculations for the $A = 82$ (maximum dimension 70 757 366) and $A = 136$ nuclei. In $A = 76$ we are limited to $t = 4$ (Ge and Se) and $t = 5$ (As). These calculations have been made possible by the SM code ANTOINE [15].

The effective interactions used in the calculations are realistic $G$ matrices whose monopole part has been phenomenologically adjusted [16]. For $A = 76, 82$ we use a $G$ matrix calculated by Kuo [17]. To fix the interaction the monopole parameters are fitted to 60 energy levels of the Ni isotopes and $N = 50$ isotones [18]. The main monopole changes the amount to weaken the interaction among the $pf$ orbits and the $g_{9/2}$ shell. For $^{136}$Xe, the interaction is the $G$ matrix obtained from the Bonn potential [19] with monopole modifications in order to reproduce the excitation energies of 150 states of the $N = 82$ isotones [18]. In both cases the $\chi^2$ value is about 250 keV. Table I shows our predictions for the $2^+$ energies around $A = 82$. The electromagnetic properties are also well reproduced by our calculation.

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TABLE I. Selected $E_x(2^+)$.

| | | $^{80}$Ge | $^{82}$Ge | $^{82}$Se | $^{82}$Kr | $^{84}$Kr | $^{84}$Se |
|---|---|---|---|---|---|---|---|
| $E_x(2^+)$ | Th | 0.620 | 1.578 | 0.612 | 0.685 | 0.773 | 1.814 |
| | Exp | 0.659 | 1.348 | 0.655 | 0.776 | 0.882 | 1.455 |

The weak processes in the nucleus are described by the effective Hamiltonian proposed by Doi *et al.* [2] that consists of $V$ and $A$ currents and that is compatible with $SU(2)_L \otimes SU(2)_R \otimes U(1)$ grand unification models:

$$H_w = \frac{G}{\sqrt{2}}\left[j_L^{\mu}(J_{L\mu} + \chi J_{R\mu})^+ + j_R^{\mu}(\eta J_{L\mu} + \lambda J_{R\mu})^+\right] + \text{H.c.} \tag{1}$$

In addition to the right-left and right-right coupling constants, the neutrino mass (implicit in the leptonic currents) also breaks the maximal parity violation of the standard theory. This Hamiltonian leads us to a scenario without maximal parity violation and without lepton number conservation. The neutrinoless double beta decay is mediated by (virtual) massive Majorana neutrinos. Moreover, the description of the two neutrino mode is essentially the same as in the standard model. This is so because the relevant contributions come from the standard left-left coupling. In the hadronic currents, all the terms up to order $v/c$ are included [20].

*Results for the $(\beta\beta)_{2\nu}$ decays.*—The partial $2\nu$ half-life can be approximated as

$$[T_{1/2}^{(2\nu)}(0^+ \to 0^+)]^{-1} = G|M_{\rm GT}^{(2\nu)}|^2, \tag{2}$$

where $G$ is an integral kinematical factor and $M_{\rm GT}^{(2\nu)}$ is the energy weighted double Gamow-Teller matrix element. We adopt the effective value $g_A = 1$ that takes into account the reduction of the Gamow-Teller strength at low energies due to correlations outside the valence space. The operator in the definition of the matrix element is just $\boldsymbol{\sigma} \cdot \boldsymbol{\tau}$; the constants $g$ are included in the integral kinematic factor. The precise definition of this matrix element as well as the algorithm that we use to calculate it can be found in Ref. [5]. By means of this algorithm we can get a reliable approximation to the Gamow-Teller strength in the intermediate nucleus.

Partial $(\beta\beta)_{2\nu}$ lifetimes have been measured for several emitters. We summarize our results in Table II. Two sets of matrix elements are listed. The first one is obtained using the theoretical energies of the $1^+$ states. In the second, the spectrum of these states is globally shifted in order to place the first $1^+$ at its experimental energy. As it can be seen in Table II the matrix elements increase by 20% when the experimental energies are used. The agreement with the experimental data is, in both cases, reasonably good. In the $A = 76, 82$ region the matrix element increases very slowly as the valence space is enlarged. Consequently, the half-life decreases and there is a factor of 2 between the $t = 2$ and the final predictions. This behavior is different from that found in the decay of $^{136}$Xe, where the matrix element is almost constant.

Our closure matrix elements $[(M_{\rm GT}^{2\nu})_c]$ are very different from those of Ref. [1]; 0.68 compared to 2.56 ($^{76}$Ge) and 0.74 compared to 1.876 ($^{82}$Se). These discrepancies also lead to quite different effective $1^+$ centroids ($\overline{E}$). Table II lists also the total Gamow-Teller strengths $S_{+/-}$. For the $^{76}$Ge decay the SM result, 0.14, is very close to the SMMC extrapolation, $0.13 \pm 0.05$. Nevertheless, other relevant quantities ($(M_{\rm GT}^{2\nu})_c$, $S_+$, $\overline{E}$) are quite different. Notice, however, that, although the valence space is the same, the effective interaction is different.

There are two sources of uncertainty in our calculation: the first one is the absence in the valence spaces of some of the spin orbit partners, leading to total Gamow-Teller strengths that are smaller than the $3(N - Z)$ values ($S_-^{(\rm Ge)} = 17.14$, $S_-^{(\rm Se)} = 21.66$, and $S_-^{(\rm Xe)} = 52.30$

TABLE II. Two neutrino matrix elements and half-lives. $M_{\rm GT}^{(2\nu)}$ in MeV$^{-1}$ and $T_{1/2}$ in years. The experimental values are taken from Refs. [24–26].

| | | $M_{\rm GT}^{(2\nu)}$ | | | $T_{1/2}^{2\nu}$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Decay | $t$ | $\Delta E$(exp) | $\Delta E$(th) | $(M_{\rm GT}^{(2\nu)})_{\rm exp}$ | $\Delta E$(exp) | $\Delta E$(th) | $(T_{1/2}^{2\nu})_{\rm exp}$ | $(M_{\rm GT}^{(2\nu)})_c$ | $S_-$ | $S_+$ |
| | 0 | 0.000 | 0.000 | | | | | 0.000 | 16.73 | 0.000 |
| $^{76}$Ge → $^{76}$Se | 2 | 0.112 | 0.088 | | $5.678 \times 10^{21}$ | $9.197 \times 10^{21}$ | | 0.465 | 17.13 | 0.146 |
| | 4 | 0.180 | 0.140 | | $2.198 \times 10^{21}$ | $3.634 \times 10^{21}$ | | 0.676 | 17.14 | 0.258 |
| | Full | | | 0.22 | | | $1.80 \times 10^{21}$ | | | |
| | 0 | 0.000 | 0.000 | | | | | 0.000 | 21.66 | 0.000 |
| $^{82}$Se → $^{82}$Kr | 2 | 0.128 | 0.102 | | $1.312 \times 10^{20}$ | $2.065 \times 10^{20}$ | | 0.483 | 21.61 | 0.121 |
| | 4 | 0.198 | 0.155 | | $5.482 \times 10^{19}$ | $8.946 \times 10^{19}$ | | 0.745 | 21.56 | 0.209 |
| | Full | 0.208 | 0.164 | 0.14 | $4.968 \times 10^{19}$ | $7.991 \times 10^{19}$ | $1.08 \times 10^{20}$ | 0.799 | 21.55 | 0.226 |
| | 0 | 0.026 | 0.028 | | $2.487 \times 10^{21}$ | $2.455 \times 10^{21}$ | | 0.106 | 52.75 | 0.004 |
| $^{136}$Xe → $^{136}$Ba | 2 | 0.036 | 0.039 | | $1.485 \times 10^{21}$ | $1.265 \times 10^{21}$ | | 0.178 | 52.37 | 0.007 |
| | 4 | 0.032 | 0.035 | | $1.879 \times 10^{21}$ | $1.571 \times 10^{21}$ | | 0.146 | 52.30 | 0.008 |
| | Full | 0.031 | 0.034 | $<0.06$ | $2.003 \times 10^{21}$ | $1.665 \times 10^{21}$ | $5.50 \times 10^{20}$ | 0.143 | 52.30 | 0.008 |

compared to 36, 42, and 84, respectively). However, the influence of the excluded orbits in $M_{\rm GT}^{(2\nu)}$ is expected to be small because (i) the missing one-particle–one-hole (1p-1h) intermediate states are outside the energy window relevant for the matrix element ($\sim$6 MeV) and (ii) the 2p-2h correlations in the ground states are hindered by the large effective energy denominators ($\simeq$20 MeV) involved. We have calculated the $M_{\rm GT}^{(2\nu)}$ for the decay of $^{60}$Ni in a model space that includes the $1f7/2$ and $1g7/2$ orbits and the effect amounts to less than 30%. The second one is related to possible defects in the effective interaction. A good test of the combined effect of truncation and interaction is provided by the comparison of the predicted $\beta_\pm$ strength functions with the experimental ones, measured in $(p,n)$ and $(n,p)$ reactions. Unfortunately, only the former is available in some cases. Our predictions for the Gamow-Teller strength functions compare fairly well with the $(p,n)$ results of Ref. [21], in the energy range relevant for the $2\nu$ decay. As an example, the calculated $B$(GT)'s to the first $1^+$ states in $^{76}$As and $^{82}$Br are 0.15 and 0.28, compared to the measured 0.15 and 0.34. These two states contribute more than 50% to the $2\nu$ matrix element. Another estimation of the quality of the interaction is given by its predictions for the binding energies. Recent SMMC calculations show that they are in reasonable agreement with the experimental results [22]. All put together, it would be safe to affect our $M_{\rm GT}^{(2\nu)}$ values of a "theoretical" error of 50%.

*Results for the $(\beta\beta)_{0\nu}$ decays.*—In the closure approximation the half-life of the $0^+ \to 0^+$ decay can be written as

$$[T_{1/2}^{(0\nu)}(0^+ \to 0^+)]^{-1} = |M_{\rm GT}^{(0\nu)}|^2 \left\{ C_{mm}\left(\frac{\langle m_\nu\rangle}{m_e}\right)^2 + C_{\lambda\lambda}\langle\lambda\rangle^2 + C_{\eta\eta}\langle\eta\rangle^2 + C_{\lambda m}\langle\lambda\rangle\frac{\langle m_\nu\rangle}{m_e}\cos\psi_1 + +C_{\eta m}\langle\eta\rangle\frac{\langle m_\nu\rangle}{m_e}\cos\psi_2 + C_{\lambda\eta}\langle\lambda\rangle\langle\eta\rangle\cos(\psi_1-\psi_2)\right\}, \quad (3)$$

where $\langle m_\nu\rangle$, $\langle\lambda\rangle$, and $\langle\eta\rangle$ are the effective lepton violating parameters, $\psi_{1(2)}$ are the *CP* phases, and the $C_{xy}$ coefficients are linear combinations of the nine matrix elements and nine integral factors. A clear and comprehensive definition of them all can be found in Ref. [2].

The nine matrix elements shown in Table III are calculated in the light neutrino approximation (see Doi [2]) and using $g_A/g_V = 1.25$. Since this mode has not been observed yet, we assume a half-life of $10^{25}$ yr, which is very close to the expected experimental limits [23], to obtain the upper bounds to the three lepton violating parameters $\langle m_\nu\rangle$, $\langle\lambda\rangle$ and $\langle\eta\rangle$. The results are compiled in Table IV. For $\langle m_\nu\rangle < 1$ eV we find the following lower bounds: $T_{1/2}^{(0\nu)}({\rm Ge}) > 1.85 \times 10^{25}$ yr, $T_{1/2}^{(0\nu)}({\rm Se}) > 2.36 \times 10^{24}$ yr, and $T_{1/2}^{(0\nu)}({\rm Xe}) > 1.21 \times 10^{25}$ yr. Although the result for $^{76}$Ge has been obtained in a $t = 4$ calculation, we expect the matrix element and therefore the half-life to be close to convergence, as it is the case in $^{82}$Se. Among the three nuclei studied, $^{82}$Se is the best candidate for the detection of the zero neutrino mode as it has the smallest half-life and the largest ratio $T_{1/2}^{(2\nu)}/T_{1/2}^{(0\nu)}$.

Table V compares the $M_{\rm GT}^{(0\nu)}$ values predicted by several authors. Notice that there is at least a factor of 2 among the different predictions. Our matrix elements are similar to those of Refs. [11,13].

TABLE III. Nuclear matrix elements (ME) for the $(\beta\beta)_{(0\nu)}$ mode. $X_{\rm GT}^\omega = 2 - X'_{\rm GT}$, $X_F^\omega = 2X_F - X'_F$.

| ME | | $t=0$ | $t=2$ | $t=4$ | Full |
|---|---|---|---|---|---|
| $M_{\rm GT}^{(0\nu)}$ | $^{76}$Ge | 0.721 | 1.294 | 1.568 | |
| | $^{82}$Se | 0.505 | 1.340 | 1.846 | 1.970 |
| | $^{136}$Xe | 0.484 | 0.630 | 0.649 | 0.651 |
| $\chi_F$ | $^{76}$Ge | −0.068 | −0.098 | −0.106 | |
| | $^{82}$Se | −0.101 | −0.107 | −0.107 | −0.108 |
| | $^{136}$Xe | −0.172 | −0.156 | −0.157 | −0.158 |
| $\chi'_{\rm GT}$ | $^{76}$Ge | 1.074 | 1.107 | 1.115 | |
| | $^{82}$Se | 1.069 | 1.114 | 1.119 | 1.120 |
| | $^{136}$Xe | 1.103 | 1.106 | 1.099 | 1.097 |
| $\chi_{F'}$ | $^{76}$Ge | −0.060 | −0.102 | −0.109 | |
| | $^{82}$Se | −0.104 | −0.112 | −0.112 | −0.112 |
| | $^{136}$Xe | −0.184 | −0.166 | −0.167 | −0.167 |
| $\chi_T$ | $^{76}$Ge | 0.186 | 0.043 | 0.017 | |
| | $^{82}$Se | 0.156 | 0.049 | 0.031 | 0.028 |
| | $^{136}$Xe | 0.039 | −0.006 | −0.031 | −0.031 |
| $\chi_P$ | $^{76}$Ge | −1.435 | −0.832 | −0.544 | |
| | $^{82}$Se | 1.710 | 0.852 | 0.574 | 0.494 |
| | $^{136}$Xe | 0.898 | 0.411 | 0.280 | 0.256 |
| $\chi_R$ | $^{76}$Ge | 0.761 | 0.707 | 0.684 | |
| | $^{82}$Se | 0.873 | 0.706 | 0.683 | 0.680 |
| | $^{136}$Xe | 0.780 | 0.872 | 0.942 | 0.955 |

TABLE IV. Shell model predictions for the upper bounds to the three lepton violating parameters (LPV): the effective mass $\langle m_\nu\rangle$ and the coupling constants $\langle\lambda\rangle$ and $\langle\eta\rangle$ for $T_{1/2} > 10^{25}$ yr.

| | | $t=0$ | $t=2$ | $t=4$ | Full |
|---|---|---|---|---|---|
| $^{76}$Ge | $\langle m_\nu\rangle$ (eV) | 2.98 | 1.60 | 1.32 | |
| | $\langle\eta\rangle\ 10^8$ | 2.78 | 1.83 | 1.65 | |
| | $\langle\lambda\rangle\ 10^6$ | 4.31 | 2.65 | 2.24 | |
| $^{82}$Se | $\langle m_\nu\rangle$ (eV) | 1.93 | 0.72 | 0.52 | 0.49 |
| | $\langle\eta\rangle\ 10^8$ | 3.03 | 1.22 | 0.86 | 0.79 |
| | $\langle\lambda\rangle\ 10^6$ | 1.93 | 0.82 | 0.61 | 0.57 |
| $^{136}$Xe | $\langle m_\nu\rangle$ (eV) | 1.47 | 1.14 | 1.10 | 1.10 |
| | $\langle\eta\rangle\ 10^8$ | 3.10 | 1.77 | 1.53 | 1.49 |
| | $\langle\lambda\rangle\ 10^6$ | 1.88 | 1.53 | 1.50 | 1.49 |

TABLE V. SM versus QRPA $M_{\mathrm{GT}}^{(0\nu)}$ matrix elements.

| | This work | Haxton [1] | Tomoda [8] | Muto [10] | Engel [11] | Staut [12] | Pantis [13] |
|---|---|---|---|---|---|---|---|
| $^{76}$Ge | 1.57 | 4.18 | 3.36 | 3.01 | 1.75 | 10.91 | 1.85 |
| $^{82}$Se | 1.97 | 3.45 | 3.06 | 2.85 | 1.29 | | 1.15 |
| $^{136}$Xe | 0.65 | | 1.12 | | 1.25 | | 1.35 |

In summary, we have presented the first large scale SM calculations of the $\beta\beta$ decays of $^{76}$Ge, $^{82}$Se, and $^{136}$Xe. The reasonable agreement between theory and experiment for the spectroscopy of the regions around these $\beta\beta$ emitters, as well as for the $\beta\beta_{(2\nu)}$ half-lives, strongly support our predictions for the zero neutrino mode.

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