Non-Gaussian Fixed Point in Four-Dimensional Pure Compact U(1) Gauge Theory on the Lattice

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The line of phase transitions separating the confinement phase from the Coulomb phase in the four-dimensional pure compact U(1) gauge theory with extended Wilson action is reconsidered. By means of a high precision simulation on spherical lattices and a finite-size scaling analysis we find that along a part of this line, including the Wilson action the critical scaling behavior is determined by one fixed point with non-Gaussian critical exponent $\nu = 0.365(8)$. This indicates the existence of a nontrivial and nonasymptotically free continuum limit of this theory, as well as of its dual equivalent. [S0031-9007(96)01067-8]

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In contradistinction to lower dimensions, the only firmly established quantum field theories in four dimensions (4D) are either asymptotically free or so-called trivial theories. Both are defined in the vicinity of Gaussian fixed points, i.e., of noninteracting limit cases. In spite of numerous suggestions and circumstantial evidence, until now no candidate for a non-Gaussian fixed point in 4D has been established.

The suitability of numerical simulations on the lattice for a confirmation of the existence of non-Gaussian fixed points, and for an investigation of their properties, has been demonstrated in dimensions lower than four in numerous applications. For example, non-Gaussian values of critical exponents can be determined by means of finite-size scaling (FSS) or renormalization group (RG) analysis. Several attempts to use a similar approach in 4D have encountered severe problems, however.

In this Letter we reconsider the oldest candidate for a non-Gaussian fixed point in the 4D lattice field theory, the phase transition between the confinement and the Coulomb phases in the pure compact U(1) gauge theory [1,2] with Wilson action and extended Wilson action. After various pioneering studies, e.g., [3–5], the more detailed investigations were hindered mainly by a weak two-state signal [6,7]. This obscures the order of the phase transition and makes it difficult to determine critical exponents reliably. We demonstrate that the problems encountered, when considering the continuum limit at this phase transition, can be surmounted. The clues are the observation that the two-state signal disappears on lattices with spherelike topology, the construction of homogeneous spherical lattices, the use of modern FSS analysis techniques, and larger computer resources.

We find that at the confinement—Coulomb phase transition at strong bare gauge coupling, $g = O(1)$, the model exhibits a second order scaling behavior well described by the values of the correlation length critical exponent ν in the range $\nu = 0.35-0.40$. The measurements have been performed at various couplings and by different methods. The most reliable determination gives

$$\nu = 0.365(8)\,. \tag{1}$$

These results are quite different from $\nu = 0.25$, expected at a first order transition, as well as from $\nu = 0.5$, obtained in a Gaussian theory or in the mean field approximation. This strongly suggests the existence of a continuum pure U(1) gauge theory with properties different from theories governed by Gaussian fixed points with or without logarithmic corrections. It can be obtained from the lattice theory by the RG techniques.

Detailed numerical evidence for these claims will be presented elsewhere [8]. Some preliminary results have been published in Refs. [9,10].

Since the pure U(1) lattice gauge theory with the Villain (periodic Gaussian) action presumably belongs to the same universality class [11], rigorous dual relationships imply that also the following 4D models possess a continuum limit described by the same fixed point: the Coulomb gas of monopole loops [12], the noncompact U(1) Higgs model at large negative squared bare mass (frozen 4D superconductor) [13,14], and an effective string theory equivalent to this Higgs model [15].

These findings raise once again the question, whether in strongly interacting 4D gauge field theories further non-Gaussian fixed points exist that might be of use for theories beyond the standard model.

The pure compact U(1) gauge theory on the 4D cubic lattice with periodic boundary conditions (4D torus) can be described by the extended Wilson action [5]

$$S = -\sum_P w_P[\beta \cos(\Theta_P) + \gamma \cos(2\Theta_P)], \quad (2)$$

with $w_P = 1$. Here $\Theta_P \in [0, 2\pi)$ is the plaquette angle, i.e., the argument of the product of U(1) link variables along a plaquette P. Taking $\Theta_P = a^2 g F_{\mu\nu}$, where a is the lattice spacing, and $\beta + 4\gamma = 1/g^2$, one obtains for weak coupling g the usual continuum action $S = \frac{1}{4}\int d^4x\, F_{\mu\nu}^2$.

In one of the very first studies [4], restricted to the $\gamma = 0$ case (Wilson action [1]) and small lattices, a behavior consistent with a second order phase transition at $\beta \simeq 1$ was observed, and $\nu \simeq 1/3$ was found. However, any inference about the continuum limit has been hindered by the subsequent observation of a two-state signal on larger, but finite lattices [6]. This either could be a finite-size effect or it could imply that the phase transition at $\gamma = 0$ is actually of first order, preventing a continuum limit at $\gamma = 0$.

In the model with the extended Wilson action (2), it was found that the confinement Coulomb phase transition is clearly of first order for $\gamma \geq 0.2$, and weakens with decreasing γ [5,7]. Various studies suggested that the transition becomes second order at slightly negative γ [7], or around $\gamma = 0$ [16,17].

The order of the transition at $\gamma = 0$ has remained a controversial subject [18–21]. Though the values $\nu \simeq 0.3$–0.4 have been obtained consistently by various methods [4,5,16,17,22,23], the continuum limit has not appeared to be possible.

Also the hope that the continuum limit might be taken at least at negative γ was spoiled by the observation of a weaker, but still significant, two-state signal on finite lattices even there [7]. Though this signal is probably only a finite-size effect, and the transition in the infinite volume limit is genuinely of second order, it impedes a precise FSS and RG analysis.

It was known that monopole loops winding around the toroidal lattice occur [16,24] and cause difficulties in simulations with local update algorithms. Suspecting that this might be a reason for the two-state signal, two of the present authors performed simulations at $\gamma = 0$, using the 4D surface of a 5D cubic lattice instead of the torus. They observed that on such lattices with spherelike topology the two-state signal vanishes [18]. This suggests that the two-state signal at $\gamma \leq 0$ is related to the nontrivial topology of the toroidal lattice.

Related observations have been made for the Schwinger model [25]. On the other hand, it has been checked in spin models that weak two-state signals are not washed out on lattices with spherelike topology, if they are due to a genuine first order transition [10].

However, the lattice on the surface of a cube is rather inhomogeneous and causes complex finite-size effects, preventing a reliable FSS analysis.

For our present study at $\gamma \leq 0$, we have chosen again lattices with spherelike topology. To alleviate the problem of inhomogeneity, we have used lattices obtained by projecting the 4D surface $SH[N]$ of a 5D cubic lattice N^5 onto a concentric 4D sphere. On such a spherical lattice $S[N]$, the curvatures concentrated on the corners, edges, etc., of the original lattice $SH[N]$ are approximately homogenized over the whole sphere by the weight factors

$$w_P = A'_P/A_P \quad (3)$$

in the action (2). A_P and A'_P are the areas of the plaquette P on $S[N]$, and of its dual, as on any irregular, e.g., random lattice [26]. These areas are determined by means of a two-triangle approximation of the plaquettes.

It has been checked in some spin and gauge models with second order transitions that universality for spherical lattices holds, and that the FSS analysis works very well if $V^{1/D}$ is used as a linear size parameter, $V \equiv \frac{1}{6}\sum_P w_P$ being the volume of the sphere $S[N]$ [10,27].

Another new feature in our study of the theory is the FSS analysis of the first zero z_0 of the partition function in the complex plane of the coupling β (Fisher zero). Applying the multihistogram reweighting method [28] for its determination, the expected FSS behavior

$$\mathrm{Im}\, z_0 \propto V^{-1/D\nu} \quad (4)$$

has been used for measuring ν. This has turned out to be superior to—though consistent with—the more common FSS analysis of specific-heat and cumulant extrema.

Finally, we have performed the FSS study of the confinement Coulomb transition not only at $\gamma = 0$, but also at $\gamma = -0.2$ and -0.5. This allows us to compare the behavior of the system at $\gamma = 0$, where the order of the transition is disputed, with the commonly expected second order behavior at negative γ, and to test the universality of the critical properties.

We have performed simulations [8] on $S[N]$ for N between 4 and 12. Note that $S[12]$ has about $(19.6)^4$ lattice points. The values of β were chosen in the immediate neighborhood of its critical values for $\gamma = 0, -0.2$, and -0.5. For each lattice size at each γ value, we have accumulated typically 10^6 updates distributed over 8–12 β points.

We have found no indication of a two-state signal, neither in the individual nor in the multihistogram distributions of $e = [\sum_P w_P \cos(\Theta_P)]/(\sum_P w_P)$, at any of the three γ values. This is demonstrated in Fig. 1 for $\gamma = 0$. The values of the studied cumulants at their respective extrema are compatible with a second order transition. The critical behavior at all three γ values is very similar, except that the transition weakens with decreasing γ, which means that larger lattices are needed for the same height of the specific-heat peak.

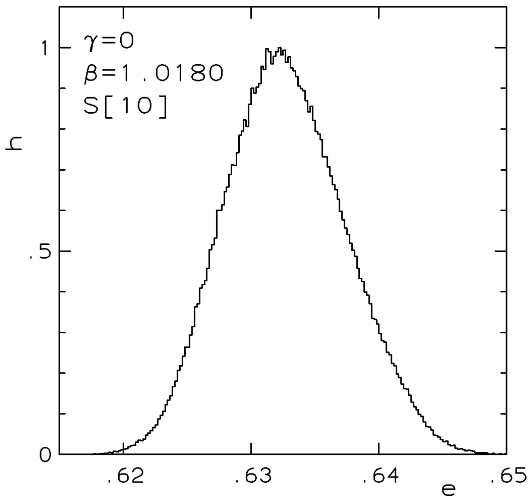

FIG. 1. Histogram of the energy density e on the $S[10]$ lattice (about 16^4 points) at $\gamma = 0$ and $\beta = 1.0180$, very close to the pseudocritical point $\beta_{\mathrm{pc}} = 1.018\,35(4)$ on that lattice. h is the relative occurrence of e values.

In test runs at $\gamma = +0.2$, a two-state signal has been clearly observed. This confirms that at sufficiently large positive γ the phase transition is of first order, and that the spherical lattice $S[N]$ does not wash out such a signal.

Furthermore, at all three investigated values $\gamma \leq 0$, the FSS analysis assuming a second order transition works remarkably well and leads to consistent results for all observables [8]. All our evidence thus points towards the conclusion that the phase transition is of second order for $\gamma \leq 0$.

In Table I we present results for the critical exponent ν obtained from all the data with $N \geq 4$. The most reliable ones come from the FSS analysis of the Fisher zero (first column). The approximate agreement between the obtained values of ν at all three γ values demonstrates that the confinement Coulomb critical line belongs to one universality class. The FSS behavior of $\mathrm{Im}\, z_0$ according to (4), and the consistency of this behavior at different γ are illustrated in Fig. 2. The value (1) has been obtained from a joint fit by means of (4) to the data for $\mathrm{Im}\, z_0$ with $N \geq 6$ (parallel straight lines in Fig. 2).

The next two columns in Table I show the results from the FSS analysis of the maximum of the specific-heat $c_{V,\max}$ and the minimum of the Challa-Landau-Binder cumulant $V_{\mathrm{CLB,min}}$. Here certain assumptions about nonleading terms have been necessary [8], and small systematic errors are therefore possible. Nevertheless,

TABLE I. Results for ν from fits to $\mathrm{Im}\, z_0$, $c_{V,\max}$, and $V_{\mathrm{CLB,min}}$ at various γ. The indicated errors are statistical.

γ	$\mathrm{Im}\, z_0$	$c_{V,\max}$	$V_{\mathrm{CLB,min}}$
0	0.345(3)	0.361(6)	0.361(6)
−0.2	0.378(7)	0.374(6)	0.365(6)
−0.5	0.368(8)	0.404(9)	0.396(9)

all the shown results are consistent with ν lying in the interval 0.35–0.40, thus supporting universality.

The physical content of the continuum limit of the pure compact U(1) gauge theory at the confinement Coulomb phase transition, discussed, e.g., in [13,29], depends on the phase from which the critical line is approached. In the confinement phase, a confining theory with monopole condensate is expected, as the string tension scales with a critical exponent consistent with the value (1) [22]. The physical spectrum consists of various gauge balls. In the Coulomb phase, massless photon and massive magnetic monopoles, both already observed in Monte Carlo simulations [30,31], should be present. The renormalized electric charge e_r is large but finite [22,32], and has presumably a universal value [22,32,33]. The numerical result $e_r^2/4\pi = 0.20(2)$ [22] agrees with the Lüscher bound [34], as explained in [31].

To our knowledge, the existence of such continuum quantum field theories in 4D is in no way indicated by the perturbation expansion. The non-Gaussian character of the fixed point might be rather understood as a consequence of the complex dynamics of the systems obtained in the dual representation of the theory with Villain action [12–15,31].

There are some questions deserving further discussion. For $\gamma < 0$, the studied theory does not satisfy reflection positivity, which is a sufficient, albeit not necessary, condition for unitarity. If the phase transition in the reflection positive case at $\gamma = 0$ is of second order, as strongly suggested by our results, then unitarity should hold also for $\gamma < 0$ by universality arguments. If it is of weak first order, unitarity at $\gamma < 0$ is made plausible by our finding that the scaling behavior at $\gamma = 0$ (on lattices of limited size) and at $\gamma < 0$ is the same, and that the regions with $\gamma < 0$ and $\gamma \geq 0$ are connected by the RG flows [17].

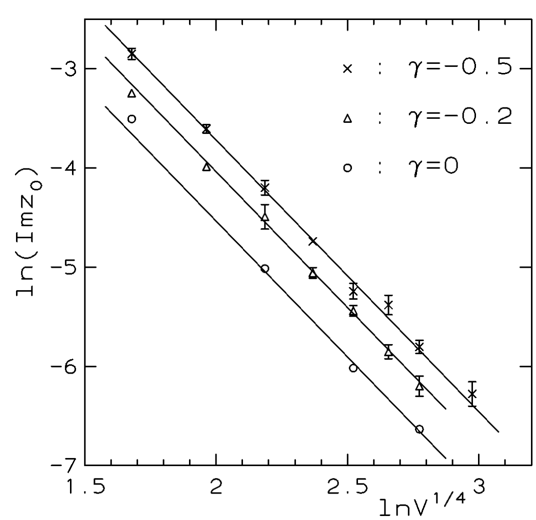

FIG. 2. Joint fit by $\mathrm{Im}\, z_0$ according to (4) for all three γ values and $N \geq 6$. The values of $L = V^{1/4}$ correspond to $N = 4 - 10, 12$.

Though improbable, it is not completely excluded that the phase transition we have studied is of first order for any $\gamma \leq 0$ [35], but so weak that the two-state signal cannot be detected by the currently available numerical means. Then the correlation length would remain finite for all finite γ. Presumably, the effective cutoff, arising in this way, may be made arbitrarily large by suitably decreasing γ. However, unitarity then might be questionable.

At small $\gamma > 0$, where the order of the transition probably changes, a tricritical point with special values of indices is expected [7]. As the domain of dominance of such a point is unknown, it could be that the measured value of ν, and the corresponding non-Gaussian fixed point are actually tricritical. Alternatively, the change of the order might be more complicated than in metamagnets with tricritical points, since in our case only nonlocal order parameters are available.

Though this may seem plausible, we cannot firmly conclude that the two-state signal observed on toroidal lattices is due to the monopole loops winding around them. The evidence for such an explanation is controversial [19–21,24], and the question, which configurations make the difference between the compact U(1) gauge theory on finite toroidal and spherical lattices, requires further study. The thermodynamic limit ought to be the same.

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