Horizontal Symmetry for Quark and Squark Masses in Supersymmetric SU(5)

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Recent interest in horizontal symmetry model building has been driven mainly by the large top mass and hence strong hierarchy in quark masses, and the possibility of appropriately constrained soft squark mass matrices, in place of an assumed universality condition, for satisfying the relevant flavor changing neutral current constraints. Here we present the first successful supersymmetric SU(5) model that has such a feature. The horizontal symmetry is a gauged $[Q_{12} \times \mathrm{U}(1)]_H$ ($\subset [\mathrm{SU}(2) \times \mathrm{U}(1)]_H$). All nonrenormalizable terms compatible with the symmetry are allowed in the mass matrix constructions. Charged lepton masses can also be accommodated. [S0031-9007(96)00829-0]

Despite the success of the standard model (SM) and the very encouraging indication of its plausible supersymmetric unification (SUSY-GUT), we still lack a real understanding of flavor physics. In this perspective, the idea of a horizontal (flavor or family) symmetry has been resurrected as the most popular candidate theory to supplement the vertical (unified) gauge theory of particle physics. Various authors have illustrated the interesting model-building possibilities in using spontaneously broken horizontal symmetry to constrain the Yukawa sector of the SM with the aim of obtaining phenomenologically viable texture patterns for the quark mass matrices [1–7]. We have concentrated on the more restrictive scenario of a gauged non-Abelian horizontal symmetry, SU(2) and its discrete dicyclic subgroups Q_{2N} [5], which is compatible with vertical unification [6–8].

While there is quite a list of interesting extended applications of a horizontal symmetry, the most interesting one is, no doubt, its use in constraining squark mediated flavor changing neutral current (FCNC) in a SM supplemented with softly broken supersymmetry, which is favored by the unification picture. Any horizontal symmetry on the low energy fermions naturally constrains (soft) couplings among their SUSY partners. In fact, the use of a horizontal symmetry in place of an imposed degeneracy among squark masses is one of the major motivations in the recent resurrection of the theory [9,10]. A SU(2) [or U(2)] horizontal symmetry with the lighter two families forming a doublet has then been advocated by some authors [3,5,7,10,11]. In this Letter, we will present the first successful model, with a $[Q_{12} \otimes \mathrm{U}(1)]_H$ ($\subset [\mathrm{SU}(2) \otimes \mathrm{U}(1)_H]$) horizontal symmetry compatible with a vertical SUSY SU(5) unification.

The FCNC constraints.—Before going into the model-building specifics, we summarize below the relevant background concerning the squark mediated FCNC in neutral meson mixings [12].

The 6×6 squark mass-squared matrices $\tilde{M}^{u2}$ and $\tilde{M}^{d2}$ are each divided into four 3×3 submatrices. The lead-ing contributions to the off-diagonal blocks, $\tilde{M}^{u2}_{LR}$ and $\tilde{M}^{d2}_{LR}$, arise from the trilinear A terms, while the leading contributions to the diagonal blocks, $\tilde{M}^{u2}_{LL}$, $\tilde{M}^{u2}_{RR}$, $\tilde{M}^{d2}_{LL}$, and $\tilde{M}^{d2}_{RR}$, arise from the soft mass terms. The latter dominate over the former, and can generally lead to an unacceptably large FCNC effect in neutral meson mixing when universality of soft masses is not imposed. The flavor changing quark-squark-gluino couplings are the result of the fact that a generic squark mass-squared matrix cannot be simultaneously diagonalized with the corresponding quark mass matrix. For instance, constraints from K-$\bar{K}$ and B-$\bar{B}$ mixing on $\tilde{M}^{d2}_{LL}$ can be expressed by an upper bound on

$$(\delta^d_{LL})_{12} = \frac{1}{\tilde{m}^2}(\tilde{m}_1^2 K_{11}K_{12}^\dagger + \tilde{m}_2^2 K_{12}K_{22}^\dagger + \tilde{m}_3^2 K_{13}K_{32}^\dagger) \tag{1}$$

and

$$(\delta^d_{LL})_{13} = \frac{1}{\tilde{m}^2}(\tilde{m}_1^2 K_{11}K_{13}^\dagger + \tilde{m}_2^2 K_{12}K_{23}^\dagger + \tilde{m}_3^2 K_{13}K_{33}^\dagger), \tag{2}$$

respectively, where $\tilde{m}_i^2$ are the three eigenvalues and $\tilde{m}^2$ their average, and K is actually $K^d_L = V^d_L \tilde{V}^{d\dagger}_L$ with $\tilde{V}^d_L$ being the unitary matrix that diagonalizes $\tilde{M}^{d2}_{LL}$, and V^d_L the usual notation for the matrix involved in diagonalizing quark masses. There are also constraints on the respective elements of δ^d_{RR}, and mixed product of the form $\langle \delta^d_{ij} \rangle = [(\delta^d_{LL})_{ij}(\delta^d_{RR})_{ij}]^{1/2}$. There are similar constraints from D-$\bar{D}$ mixing on the corresponding up-sector quantities. While the actual numerical bounds depend on the details of the SUSY spectrum, an illustrative set of numbers are listed in Table I.

In principle, there are other very important flavor changing processes, such as $b \rightarrow s\gamma$ [13], that constrain the off-diagonal blocks, $\tilde{M}^{d2}_{LR}$. However, while universality of squark masses is not a natural consequence of horizontal symmetry, proportionality of the trilinear soft A terms to the quark Yukawa couplings could be, provided that the horizontal symmetry is not an R symmetry. This

TABLE I. Constraints from neutral meson mixings and results of our model.

K-$\overline{K}$ mixing	$(\delta_{LL}^d)_{12}$	$(\delta_{RR}^d)_{12}$	$\langle\delta_{12}^d\rangle$
Upper bound	0.05	0.05	0.006
Our model	$\sim\lambda^5$	$\sim\lambda^5$	$\sim\lambda^5$
B-$\overline{B}$ mixing	$(\delta_{LL}^d)_{13}$	$(\delta_{RR}^d)_{13}$	$\langle\delta_{13}^d\rangle$
Upper bound	0.1	0.1	0.04
Our model	$\sim\lambda^3$	$\sim\lambda^4$	$\sim\lambda^{3.5}$
D-$\overline{D}$ mixing	$(\delta_{LL}^u)_{12}$	$(\delta_{RR}^u)_{12}$	$\langle\delta_{12}^u\rangle$
Upper bound	0.1	0.1	0.04
Our model	$\sim\lambda^6$	$\sim\lambda^6$	$\sim\lambda^6$

then would take care of the necessary FCNC suppression arising from $\tilde{M}_{LR}^{u2}$ and $\tilde{M}_{LR}^{d2}$. Hence, we are not going to discuss the off-diagonal blocks any further.

Another important question involved is the scale where any structure on the squark masses is imposed. On the one hand, it is possible to have universality among the soft SUSY breaking terms imposed at the Planck scale yet significantly corrected at the GUT scale [14–16] leading to interesting lepton-flavor-violating and CP-violating signal [16]. On the other hand, there is the scenario where nonuniversal squark masses are rendered sufficiently degenerate by large common contributions from renormalization group (RG) evolution due to particularly heavy gauginos [17]. Scenarios of this second type are also possible in some string-inspired supergravity models [18].

For our model-building consideration, we are interested only in constraints on nonuniversal squark masses which result from a horizontal symmetry spontaneously broken at some high energy scale. A recent analysis by Choudhury et al. [19] in the minimal supersymmetric standard model (MSSM) framework is most relevant. The result can be summarized by three points: (1) large gaugino masses enhance the diagonal squark masses; (2) nonuniversal A terms decrease the off-diagonal mass-squared matrix elements; (3) this A-term suppression effect decreases as the top Yukawa gets large and approaches zero at its IR (quasi)fixed point. We then conclude that, for a horizontal symmetry model with a hierarchical quark mass texture, it is sufficient for the FCNC constraints to be satisfied naively by the high energy texture of the squark mass-squared matrices ($\tilde{M}_{LL}^{u2}$, $\tilde{M}_{RR}^{u2}$, $\tilde{M}_{LL}^{d2}$, and $\tilde{M}_{RR}^{d2}$); and in the absence of very massive gauginos, the necessary FCNC bounds are not going to be very much weakened at the high scale [20]. We aim at providing such a model with the FCNC constraints satisfied by the squark mass-squared texture from a broken horizontal symmetry, the energy scale of which to be specified later.

2 + 1 *family structure in SUSY-GUT.*—Consider in the SUSY SU(5) framework a general 2 + 1 family structure. We label the chiral supermultiplets that contain the low energy chiral fermions as, for the doublets containing the first and second families, 10_2 and $\bar{5}_2$, and

for the third family singlets, 10_1 and $\bar{5}_1$; and 5 and $\bar{5}$ represent the Higgs bosons. We want only the top quark to have a mass term invariant under the horizontal symmetry. We can take both the 10_1 and the 5 to be in representation $(1, 0)$ of the $[Q_{2N} \otimes U(1)]_H$, where the zero U(1) charge is taken for simplicity. Denote the representation of the 10_2 by $(2_k, A)$, where 2_k is a general doublet of a Q_{2N} and A the U(1) charge. The nontrivial U(1) charge is what forbids an invariant mass for the doublet. Now, if we take a SU(5) singlet ϕ in $(2_k, -A)$, with a horizontal symmetry breaking vacuum expectation value (VEV) in the direction $[1, 1]$ [21] of the doublet, we have from the terms

$$10_2 10_1 \langle 5\rangle \langle\phi\rangle_{\text{sym}}/M_{\text{Pl}}, \qquad 10_2 10_2 \langle 5\rangle \langle\phi\rangle_{\text{sym}}^2/M_{\text{Pl}}^2,$$

($M_{\text{Pl}} \sim 2.4 \times 10^{18}$ GeV) an up-quark mass matrix of the form

$$M_u \sim \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^4 & \lambda^2 \\ \lambda^2 & \lambda^2 & 1 \end{pmatrix}, \qquad (3)$$

where $\lambda \sim 0.22$, coefficients of order one are neglected, and we set

$$\langle\phi\rangle_{\text{sym}}/M_{\text{Pl}} \sim \lambda^2, \qquad (4)$$

The VEV $\langle\phi\rangle_{\text{sym}}$ together with its conjugate also give us off-diagonal terms in $\tilde{M}_{10}^2$, through similar higher dimensional terms, as

$$\tilde{M}_{10}^2 \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^2 \\ \lambda^4 & 1 & \lambda^2 \\ \lambda^2 & \lambda^2 & 1 \end{pmatrix}. \qquad (5)$$

If we put in another VEV for ϕ (denoted by $\langle\phi\rangle_{\text{antisym}}$) in the $[1, -1]$ direction of the doublet, with

$$\langle\phi\rangle_{\text{antisym}}/M_{\text{Pl}} \sim \lambda^4, \qquad (6)$$

this gives nonzero mass to the up. We have then a mass matrix of the form

$$M_u = \begin{pmatrix} a + x & a & (c + y) \\ a & a - x & (c - y) \\ (c + y) & (c - y) & 1 \end{pmatrix}, \qquad (7)$$

where

$$a \sim \lambda^4, \qquad x \sim \lambda^6, \qquad c \sim \lambda^2, \qquad y \sim \lambda^4. \qquad (8)$$

There are also extra contributions to $\tilde{M}_{10}^2$ of higher order in λ that we neglect.

The choice of scales for the VEVs of ϕ are consistent. The two VEVs correspond to two linear independent states of the 2_k doublet. If Q_{2N} breaks to a Z_2 remnant at $\lambda^2 M_{\text{Pl}}$, with the $[1, 1]$ state from the doublet transforming trivially under Z_2 and the $[1, -1]$ state transforming nontrivially, the latter VEV would be further suppressed until the breaking of the Z_2 remnant. So, in the hierarchical basis, the Z_2 symmetry protects the first family, the

u quark, from getting a mass; in the horizontal symmetry basis considered here, it enforces the degeneracy between the two lighter families.

Note that $\tilde{M}_{10}^2$ contains $\tilde{M}_{LL}^{u2}$, $\tilde{M}_{RR}^{u2}$, and $\tilde{M}_{LL}^{d2}$ which share the same texture pattern of the parent $\tilde{M}_{10}^2$. We have, then, through introducing the two ϕ VEVs, a M_u, which corresponds to an acceptable symmetric texture pattern, and a $\tilde{M}_{10}^2$ that satisfies all the correspondent constraints, when a compatible down-quark mass matrix is assumed. The great economy of the scheme is self-evident.

We leave the details concerning the admissible texture patterns for quark and squark masses in the half-democratic half-hierarchical form given above to a separate publication [22].

Gauge anomaly cancellation.—Before presenting our complete model, we comment on the gauge anomaly cancellations. We have a gauged $SU(5) \otimes [Q_{2N} \otimes U(1)]_H$ symmetry ($N = 6$ in particular), with $U(1)$ being replaceable by a Z_N subgroup. The first thing to notice is that all chiral supermultiplets have to be embeddable into complete $SU(5) \otimes SU(2)$ representations to be free from any anomalies involving only $SU(5)$ and $SU(2)$. This is a nontrivial condition, making the situation different from gauging Abelian discrete symmetries [23]. In our model, for example, we take a 10 and a $\bar{10}$ from a 4 and 2 of $SU(2)$, respectively, assuming conjugate $U(1)$ charges. Breaking the $SU(2)$ to the discrete Q_{12} (or any Q_{2N} with $N \geq 4$) subgroup, we have the splitting

$$4 \rightarrow 2_3 + 2_1, \qquad 2 \rightarrow 2_1 .$$

A Q_{12} invariant Dirac mass term can develop for the 2_1 doublet, leaving behind a chiral $(10, 2_3)$, to be identified as our 10_2.

We assume that the supermultiplets containing the quarks and leptons are the only chiral content, with all other multiplets in matching vectorlike pairs. The latter are naturally heavy, except the electroweak- (EW-) breaking Higgs doublets. Cancellation of the $[SU(5)]^2U(1)$ anomaly has to be enforced. The situation for the $[SU(2)]^2U(1)$ and $U(1)$ anomalies is, however, more like the Abelian scenario. It is possible, for example, to introduce extra $SU(5)$ singlet supermultiplets that can develop Dirac or Majorana masses invariant under $Q_{12} \otimes Z_N$.

The full $[Q_{12} \otimes U(1)]_H$ model.—Along the lines considered above, it is possible to build a full model which has a gauged horizontal symmetry that accounts for both the quark and squark mass matrix textures and fits all the phenomenological constraints. Here we present the example which we believe to be the most economic. It remains to be seen whether the assumed sequence of horizontal symmetry breaking can be naturally obtained from a scalar potential.

We have 10_1 and the Higgs multiplets 5 and $\bar{5}$ in $(1, 0)$, and the 10_2 horizontal doublet in a $(2_3, 1)$ of $[Q_{12} \otimes U(1)]_H$ as mentioned above. We further put the $\bar{5}_1 + \bar{5}_2$ in a $SU(2)$triplet, which then splits into a $1' + 2_2$

at the Q_{12} level. The full representation assignments of the chiral supermultiplets are shown in Table II. Now we can simply take the above results on M_u and $\tilde{M}_{10}^2$, setting $k = 3$ and $A = 1$. To complete the model for M_d and $\tilde{M}_{\bar{5}}^2$, we need a few extra heavy VEVs, as given in Table II. Tracking down all the lower order coupling (up to λ^6), we obtain

$$M_d \sim \lambda^2 \begin{pmatrix} a' + x' & a' & c' + y' \\ a' & a' - x' & c' - y' \\ z' & z' & 1 \end{pmatrix}, \qquad (9)$$

where

$$a' \sim \lambda^3, \qquad x' \sim \lambda^4, \qquad c' \sim \lambda^2,$$
$$y' \sim \lambda^4, \qquad z' \sim \lambda^3; \qquad (10)$$

and for the squark mass matrix $\tilde{M}_{RR}^{d2}$ from

$$\tilde{M}_{\bar{5}}^2 \sim \begin{pmatrix} 1 & \lambda^4 & \lambda^3 \\ \lambda^4 & 1 & \lambda^3 \\ \lambda^3 & \lambda^3 & 1 \end{pmatrix}. \qquad (11)$$

For example,

$$a'\lambda^2 \sim \langle \bar{5} \rangle \langle (2_2, -1) \rangle_{\rm sym} \langle (2_3, 1) \rangle_{\rm sym} \langle (1, 1) \rangle_{\rm sym} / M_{\rm Pl}^3 ,$$
$$(12)$$

$$x'\lambda^2 \sim \langle \bar{5} \rangle \langle (2_5, -1) \rangle_{\rm antisym} \langle (1', 2) \rangle / M_{\rm Pl}^2 . \qquad (13)$$

Note that M_d is (slightly) not symmetric. This is a general feature of $SU(5)$ unification. The asymmetric mass matrix can be put into a symmetric form by a rotation of the right-handed down-quark field, raising the $31-$ and $32-$ entries to the same order as the $13-$ and $23-$ ones, as noted previously [7]. The extra rotation makes V_R^d different from V_L^d, and is relevant to the constraints on $\tilde{M}_{RR}^{d2}$. Detail analysis shows that this actually leads to further slight suppression of the $13-$ and $23-$ entries in K_R^d. The results concerning the FCNC constraints are shown in Table I.

TABLE II. Supermultiplet and heavy VEV content of our model. The $SU(5)$ VEVs should correspond to scalar states of complete supermultiplets in vectorlike pairs and with heavy masses, for instance, Planck scale masses. Note that for the horizontal doublets VEVs, $\langle \phi_i \rangle_{\rm sym}$ are in the $[1, 1]$ direction while $\langle \phi_i \rangle_{\rm antisym}$ are in the $[1, -1]$ direction. $\langle \phi_i \rangle$ corresponds to a VEV for a Q_{12} singlet. Notice that there are two different singlets for any Q_{2N} group, a $1'$ and a 1. Only the latter is truly Q_{2N} invariant. Hence the $(1, 1)$ VEV breaks only the $U(1)_H$ but not the $(Q_{12})_H$ symmetry.

$SU(5)$ multiplet	10_2	10_1	$\bar{5}_2$	$\bar{5}_1$	$5 + \bar{5}$
$[Q_{12} \otimes U(1)]_H$ rep.	$(2_3, 1)$	$(1, 0)$	$(2_2, -2)$	$(1', -2)$	$(1, 0)$
$SU(5)$ singlet heavy VEVs—their $[Q_{12} \otimes U(1)]_H$ rep.					
$-\langle \phi_i \rangle_{\rm sym} \sim \lambda^2 M_{Pl}$			$(2_3, 1)\ (2_2, -1)\ (2_3, 1)$		
$-\langle \phi_i \rangle_{\rm antisym} \sim \lambda^4 M_{Pl}$			$(2_3, -1)\ (2_5, -1)$		
$-\langle \phi_i \rangle$			$(1', 2) \sim \lambda^2 M_{Pl}\ (1, 1) \sim \lambda M_{Pl}$		

To accommodate the charged lepton masses, either the Georgi-Jarlskog [24] or the Ellis-Gaillard [25] mechanism can be used. While there may be potential complications and interesting phenomenology involved [16], we will leave the detailed features of the leptonic sector for future investigation.

We also note that there is the possibility of obtaining the gravitationally induced nonrenormalizable terms through a Froggatt-Nielsen [26] mechanism, thereby reducing the horizontal symmetry breaking scale.

Finally, we want to point out that the model has not addressed the doublet-triplet splitting problem. One can assume the simple fine-tuning solution. Apart from its being "unnatural," there is also an extra recent objection from the perspective of precise gauge coupling unification. The latter problem can, however, be corrected by some other strategy [27]. "Missing doublet" models provide a very interesting alternative that is free from both problems [28], as well as giving less unnatural mass constraints for an acceptable proton decay rate [29]. Extensions or modifications of the model to incorporate a missing doublet structure and a suppression of squark-mediated proton decay without R parity are under investigation.

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