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Atom de Broglie Wave Deflection by a Single Cavity Mode in the Few-Photon Limit: Quantum Prism

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It is shown that the deflection of an atom de Broglie wave at a nonresonant weak cavity field mode can yield a pure entangled quantum state in which discernable atomic beams are entangled to photon number states of the field and to internal states of the atom. The proposed experimental scheme is shown to be applicable for quantum nondemolition measurement of the photon statistics, and for quantum state engineering and reconstruction experiments. [S0031-9007(96)00939-8]

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Matter-wave interferometry has been significantly progressed due to the recent development of devices manipulating atomic beams. Several techniques for coherent splitting and recombination of atomic beams have been implemented so far [1–5]. One class of interferometers includes a beam splitting mechanism which produces the atoms' de Broglie wave in a superposition of different paths in real space. This kind of amplitude division can be achieved by diffraction at a material double slit [2,3] or at a material absorption grating [4]. Another beam splitting mechanism, diffraction at intense standing light wave, has been exploited in recently constructed atom de Broglie wave interferometers [5].

Atom wave deflection at electromagnetic fields relies on the mechanical effects of radiation. In these effects, however, the quantum nature of radiation can also manifest itself [6]. For example, in the absence of spontaneous decay, the absorbed momentum by an atom must be an integral multiple of the photon's momentum [7]. The momentum distribution of the atom after the interaction with a single cavity field mode is then sensitive to the quantum statistics of photons [8–10]. This fact makes it possible to measure the photon number observable in quantum demolition [11], and in quantum nondemolition (QND) measurements [9]. The correlation between the external motion of the atom and the field can also be revealed in the atomic wave pattern. Under certain conditions, the deflected atomic wave function can have a multifocal structure in which individual foci corresponding to neighboring Fock states can be resolved [12].

But can one construct a de Broglie wave beam splitter governed by microscopic degrees of freedom of the atom-field system? In this Letter we show that an atomic wave diffraction process can yield an entangled quantum state in which discernable, partial atomic waves propagating into different directions are entangled to direct products of the photon number states and the internal states of the atom. The proposed experimental scheme can be implemented at presently available experimental parameters. This atom-optical device enables us to manipulate atomic trajectories by controlling the field statistics or the internal atomic state. Once the distinct partial atomic waves are created, experiments can be developed for generating, as well as reconstructing, an arbitrary quantum state of the field.

Let us consider a two-level atom with states $|g\rangle$ and $|e\rangle$ crossing an opened high-Q cavity. Inside the cavity the atom is strongly coupled to one Gaussian mode having a frequency close to that of the atomic transition but sufficiently detuned by the frequency mismatch δ so as to inhibit photon exchange between the interacting subsystems. In the framework of the Jaynes-Cummings model the dipole coupling strength is characterized by the position-dependent Rabi frequency $\Omega f(\mathbf{r})$ where $f(\mathbf{r})$ is the dimensionless rms vacuum field amplitude. Suppose that the atom's longitudinal kinetic energy is large compared to the coupling and can be considered constant in the course of the passage. The transverse kinetic energy absorbed by the atom is assumed to be small so that the atom moves parallelly to the axis x inside the cavity. According to the nonresonant feature of the interaction, ensured by the inequality $\delta \gg \sqrt{n+1}\,\Omega$, the field energy and the atom's internal state emerges unchanged when the atom exits the cavity. If the atom moves slowly enough, the "dressed atom" system follows adiabatically a position-dependent level which means a potential for the atomic center-of-mass motion. Within the adiabatic approximation, the unitary operator connecting the input state of the atom-field system to the state at the cavity exit reads as

$$\hat{U}(z) = |g\rangle\langle g| \otimes e^{i\epsilon \hat{a}^\dagger \hat{a} \cos^2 kz} + |e\rangle\langle e| \otimes e^{-i\epsilon(\hat{a}^\dagger \hat{a}+1)\cos^2 kz}, \qquad (1)$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the creation and annihilation operators of the field mode, k is its wave vector, z specifies where the atom traverses the cavity mode, and ϵ characterizes the experimental parameters

$$\epsilon = \frac{\Omega^2 l_{\rm cav}}{\delta v_{\rm at}}. \qquad (2)$$

Here $l_{\rm cav}$ is the effective cavity length and $v_{\rm at}$ is the atomic velocity. The dependence of the effective cavity length on z can be considered negligible. Note that the spatial dependence of the evolution operator is due to the inhomogeneous coupling described by the mode function which is not sensitive to the quantum statistical features of the system.

If a transversely extended atom de Broglie wave impinges on the cavity mode, the dispersive interaction described by $\hat{U}(z)$ yields a z dependent phase shift of the wave function. The initial wave front then experiences a transformation determined by the shape of the field mode. There is a certain regime between a nodal and an adjacent antinodal of the standing light wave where the cosine-square function appearing in Eq. (1) can be approximated by the linear function $1/2 + kz$. In this regime the interaction with the field gives rise to a linear phase shift in function of z, hence the cavity mode acts as a *prism* for atomic wave. In the presence of exactly n photons in the mode, a plane wave with a wave vector $k_{\rm at}$, $\psi_{\rm in}(x,z) = \mathcal{N}\ \exp(ik_{\rm at}x)$, is tilted to result in the plane wave $\psi_{\rm out}(x,z) = \mathcal{N}\ \exp(ik_{\rm at}x + in\epsilon[1/2 + kz])$ that propagates in the direction

$$\alpha(n,g) = n\epsilon \frac{k}{k_{\rm at}}, \qquad (3)$$

when the atom was prepared in the state $|g\rangle$. For an atom initially in the state $|e\rangle$, the same argument leads to an angle

$$\alpha(n,e) = -(n+1)\epsilon \frac{k}{k_{\rm at}}. \qquad (4)$$

The "refractive angle" depends on the excitation number of the cavity mode and the internal state of the atom, i.e., on microscopic, substantially quantum features of the system. This fact can be interpreted as if a *"quantum prism"* operated on the atom de Broglie wave. The underlying physical effect is a virtual photon exchange, mediated by the atom, between the two counterpropagating waves with momenta $\hbar\vec{k}$ and $-\hbar\vec{k}$, composing the standing-wave mode in the cavity. The probability of this process increases with the photon number that results in different absorbed transversal momentum. The internal state of the atom affects the direction of the deflection.

Let us analyze in more detail the propagation of an atomic wave when an atom in the state $|g\rangle$ is sent across the cavity containing a field in the number state $|n\rangle$. Suppose the atomic wave is prepared to approximate a plane wave in front of the cavity. The transverse extension of the atomic wave is confined to the linear regime by placing an aperture of width A in front of the cavity. Without losing any important physical effect, we can limit the treatment to two dimensions, i.e., the system in the y direction is assumed to be uniform. Behind the cavity, the atom de Broglie wave follows a free evolution. Provided the deflection angle is small, the time of flight is simply determined by the coordinate x as $t = x/v_{\rm at}$. Using Eq. (1) and the unitary operator of the free evolution, in the far field limit the state of the total atom-field system in the coordinate representation for the atomic wave function can be expressed as

$$\Psi_{\rm far}(x,z) = \mathcal{N} e^{ik_{\rm at}(x+z^2/2x)} \frac{k_{\rm at}}{2\pi i x} e^{in\epsilon/2} \times \int_{-A/2}^{A/2} e^{i(n\epsilon k - k_{\rm at}z/x)\zeta} d\zeta. \qquad (5)$$

On carrying out the integral with respect to ζ for a fixed photon number n, one can obtain the position distribution of the atom

$$|\psi_{\rm far}(x,z)|^2 \propto \frac{\sin^2\eta}{\eta^2}, \qquad \eta = \frac{k_{\rm at}A}{2}\left[\alpha(n,g) - \frac{z}{x}\right]. \qquad (6)$$

This profile well approximates a peak in the position z that corresponds to the direction $z/x = n\epsilon k/k_{\rm at} = \alpha(n,g)$, in agreement with Eq. (3). The half-width of the peak, found to be $\lambda_{\rm at}x/A$, describes the divergence of the beam which is attributed to the effect of the aperture's finite size. For other z values the probability distribution oscillates in a small vicinity of zero. The distance between two peaks associated with adjacent photon numbers determines the splitting angle: $\Delta\alpha \equiv \alpha(n+1,g) - \alpha(n,g) = \epsilon k/k_{\rm at}$. The resolution requires that the splitting dominates the divergence of the beams, which leads to the following condition:

$$\epsilon kA \geq 2\pi . \tag{7}$$

This condition expresses the possibility of distinguishing the atomic waves corresponding to different photon numbers. Note that the overlap between distributions associated with adjacent photon number states is minimized when the main peak of one distribution of type Eq. (6) coincides with a zero point of the other. This imposes a supplementary, periodic condition on the size of the aperture

$$\epsilon kA = j \times 2\pi , \qquad j = 1, 2, \ldots . \tag{8}$$

In a general case the atom, as well as the field, is prepared initially in superposition states $a_g|g\rangle + a_e|e\rangle$, and $\sum C_n|n\rangle$, respectively. Fulfillment of the conditions (7) and (8) ensures that each photon number and internal atomic state is entangled to a separate atomic beam. Using the linearity of quantum mechanics, we get the total state

$$\sum_n C_n a_g|n, g, \vec{\alpha}(n,g)\rangle + C_n a_e|n, e, \vec{\alpha}(n,e)\rangle , \tag{9}$$

where $\vec{\alpha}$ symbolizes a discernable atomic partial wave propagating into the given direction α. The diffraction pattern is sketched in Fig. 1 presenting how the microscopic degrees of freedom select the direction of the propagation which is clearly a macroscopic feature of the system. The complex coefficients $C_n a_g$, $C_n a_e$ are associated with partial atomic waves propagating into distinct directions. The probability of finding the atom in a certain direction is given by the absolute square of these coefficients. Note that we assumed a nonresonant interaction between the atom and the field that requires $\Omega\sqrt{n+1} \ll \delta$. This condition limits the maximum photon number at which transitions between internal states are avoided and hence determines the maximum number of trajectories represented in Fig. 1.

Far enough from the cavity, due to the highly entangled state of the system shown in Eq. (9), the detection in the position $z_{\rm det} = \alpha(m,g)x_{\rm det}$ has the field part jump into the corresponding number state $|m\rangle$. Furthermore, a QND readout of the photon number takes place this way since the energy stored in the cavity is a constant of motion by virtue of the nonresonant feature of the interaction. The collapse of the field state can be elucidated by inserting the relation $z_{\rm det} = \alpha(m,g)x_{\rm det}$ into Eq. (5). Substituting $\varphi = \epsilon kz$ and $\varphi_{\min} = -\epsilon kA/2 = -\varphi_{\max}$, if $\varphi_{\max} - \varphi_{\min} = 2\pi$ in accordance with Eq. (8), one can see that the output field state appears as the action of the projection operator

$$|m\rangle\langle m| = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-i(m-\hat{a}^\dagger\hat{a})\varphi}\, d\varphi \tag{10}$$

on the arbitrary initial one. Applying this operator, describing the collapse, on a cavity field initially in the coherent state $|\beta\rangle$, the well-known representation of the $|m\rangle$ number state [13] can be recognized:

$$|m\rangle = \text{const} \times \int_{-\pi}^{\pi} e^{-im\varphi}|\beta e^{i\varphi}\rangle d\varphi . \tag{11}$$

The expansion of the Fock state $|m\rangle$ in terms of coherent states having the same mean energy but different phases in phase space reflects that the back action of the energy measurement appears in increasing the uncertainty of the phase observable. The condition for the boundaries, that is $\varphi_{\max} - \varphi_{\min}$ be an integer multiple of 2π, is required for the complete erasure of the phase information stored in the initial field. It ensures the possibility of reading out a photon number in a single measurement. This condition naturally coincides with (8) that was deduced using different considerations.

The quantum prism offers several possibilities of application in various experimental setups. It can be viewed as a device producing separate atomic partial waves from a single input one, where these beams are numbered by the quantum states $|n\rangle$. Controlling the photon statistics, i.e., the coefficients C_n in the cavity, one can associate different amplitudes and phases with the trajectories. In the most elementary case, when the cavity mode is initially in vacuum state and the atom is prepared in the superposition $(1/\sqrt{2})(|g\rangle + |e\rangle)$, the beam entangled to $|g\rangle$ crosses the cavity without altering its direction, while in the state $|e\rangle$ the wave is subject to a deflection by an angle $\epsilon k/k_{\rm at}$. The vacuum limit of the optical Stern-Gerlach experiment [2,10] can be realized this way.

By the measurement of the atomic position, QND readout of the photon number distribution is possible, as previously discussed. But before detecting the atom, which results in a collapse of the highly entangled state (9), one could use an additional atom interferometer [5] to recombine different trajectories. For example, by the use of an intense, classical standing-wave field, the neighboring beams can be led to the same position at the observation plane. On detecting the atom in such a position, the field reduces to a superposition of the two corresponding number states. The probability of detecting

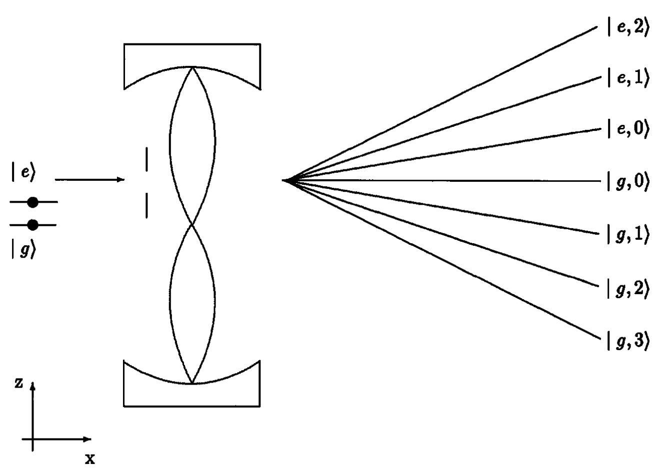

FIG. 1. The diffraction pattern: Each photon number and internal atomic state pair selects one deflection angle. An initial superposition results in a highly entangled state.

the atom in this position is proportional to the squared modulus of the *sum* of these number state coefficients. Hence the quantum interference in the Hilbert space appears in the probability distribution of the atom at the detection plane. The phase of the complex coefficients can also be extracted from the mapping of the atom's position distribution in repeated measurements. Thus, besides the measurement of the absolute values of the Fock coefficients, the quantum prism makes it possible to get complete information of the quantum state of the field mode. This type of experiment is referred to as quantum state reconstruction.

The discernable atomic beams, numbered by Fock states, can be the input of an atom "multiport" system. In such a multiport, the atomic beams can be interfered in a manner that includes an adjustment of their relative phases and amplitudes at will. At the output, the resulting beams are numbered by quite general superpositions of number states. Observation of the atom at a specific output would then result in a collapse into the corresponding superposition of Fock states. This technique is a promising variant of quantum state engineering.

Several conditions must be considered with respect to the feasibility of the quantum prism. Basically, condition (7) determines the resolution. Since the extent of the linear regime leads to $kA \leq 1$, one should seek a system where $\epsilon \geq 2\pi$. To enhance the effect of beam splitting, i.e., to increase the splitting angle $\Delta\alpha$, the maximum, realizable $\epsilon = \Omega l_{\mathrm{cav}}(\Omega/\delta)v_{\mathrm{at}}$ must be chosen. Taking into account the adiabatic condition for avoiding resonant transitions, the ratio Ω/δ is maximized by the photon number which one invents to operate the quantum prism. Thus ϵ is proportional to $1/v_{\mathrm{at}}$ and, as a consequence, $\Delta\alpha \propto 1/v_{\mathrm{at}}^2$. However, the atomic velocity is limited from below by the fact that, within the coherence time τ of the system, the atom should take a minimal distance $x_{\min}$ where it can be detected ($\tau v_{\mathrm{at}} > x_{\min}$). These simultaneous, controversial requirements can be accomplished by using a microwave cavity coupled to a Rydberg atom with two highly excited, circular levels [14]. Here $\Omega = 1.6 \times 10^5/\mathrm{s}$, $l_{\mathrm{cav}} = 7.4 \times 10^{-3}$ m, $k = 1\ \mathrm{mm}^{-1}$, and $\tau = 0.033$ s are the essential parameters describing the interaction. For a photon number $n = 3$ a good compromise for δ is 5Ω, and the optimum atomic velocity was then found to be 3 m/s, which results in $\epsilon = 66.6 \gg 2\pi$. An aperture $A = 90\ \mu$m derived from $\epsilon kA = 2\pi$ cuts out strictly the linear regime of the mode function. In this case, one gets a splitting angle $\Delta\alpha = 17\ \mu$rad. Note that this 17 μrad is the angle between the two beams, associated with the internal atomic states $|e\rangle$ and $|g\rangle$, and deflected by the vacuum state of the cavity field. The obtained minimum distance of the detection plane for the atom's position is $x_{\min} = v_{\mathrm{at}}\tau \approx 10$ cm, which can be achieved by the application of an atomic lens [15] with a focal length of 10 cm. This lens maps the far-field limit in its focal plane, where one can observe a spacing 1.7 μm between the images of partial waves corresponding to neighboring photon numbers. According to this analysis, the proposed experimental scheme seems to be feasible at the present state of art.

In conclusion, we have shown that, by deflecting an atom de Broglie wave at a quantum field under certain conditions, discernable beams can be produced which are entangled to the system's internal, microscopic degrees of freedom.

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