Comment on "Fixed-Point Structure of Scalar Fields"

In a recent Letter [1], certain properties of the local potential approximation (LPA) to the Wilson renormalization group were uncovered, which led the authors to conclude that $D > 2$-dimensional O(N) scalar field theories endowed with *nonpolynomial* interactions allow for a continuum of renormalization group fixed points (FP's), and that around the Gaussian FP, asymptotically free (actually relevant) interactions exist. If true, this could herald very important new physics, particularly for the Higgs sector of the standard model ($N = D = 4$) [1]. Continuing work [2] in support of these ideas has motivated us to point out that we previously studied the same properties and showed that they lead to very different conclusions [3]. Indeed, in as much as the statements in Ref. [1] are correct, they point to some deep and beautiful facts about the LPA and its generalizations [4], but however, no new physics.

The LPA, which has a long history [5], approximates the renormalization group by allowing interactions only in the form of a general effective potential $V(\phi, t)$. [Here $t = \ln(\Lambda_0/\Lambda)$, where Λ_0 (Λ) is the overall (effective) cutoff.] The sharp cutoff form [6] can be written as

$$\dot{V} + d\phi V' - DV = (N - 1)\ln(1 + V'/\phi) + \ln(1 + V''). \quad (1)$$

Here $d = D/2 - 1$, $\dot{V} \equiv \partial V/\partial t$, and $' \equiv \partial/\partial\phi$. Reference [1] computed the Taylor expansion coefficients u_{2n} : $V = \sum_{n=0}^{\infty} \zeta^{2-2n} u_{2n} \phi^{2n}$, where $\zeta = (4\pi)^{D/4}\sqrt{\Gamma(D/2)}$. For the FPs, i.e., $\dot{V} = 0$, these coefficients can all be solved in terms of the mass coefficient $\sigma = u_2 > -1$. The fact that σ appears otherwise to be arbitrary led the authors to conclude that a continuum of FP's exist. If this were correct, it would be difficult to understand why universality is seen experimentally (including in simulations) in the continuous phase transitions of many different systems, where it is certainly not the case that the (bare) effective potential is always polynomial. Indeed if such a continuum really did exist for (1) (with $D > 2$ [4]), we would have to conclude that the LPA was too severe an approximation. Fortunately some magic occurs [3,4]: all but a few FP solutions to (1) are unacceptable since they have singular derivatives at some critical value of the field $\phi = \phi_c$, and fail to exist for $\phi > \phi_c$. We plot ϕ_c in Fig. 1, for $N = D = 4$. We see that only the Gaussian FP potential $V \equiv 0$ exists for all $\phi \geq 0$, supporting the standard lore on triviality. Furthermore, for values of σ around the minimum in Fig. 1, ϕ_c is the singularity closest to the origin in the ϕ complex plane, implying that $u_{2n} \sim \phi_c(\sigma)^{-2n}$ for large enough n. Thus, these $u_{2n}(\sigma)$ have a local maximum at $\sigma = -0.64$. Halpern *et al.* found this maximum and attributed it to a maximum density of FP's; however, we see that it actually corresponds to the most singular possible putative FP potential.

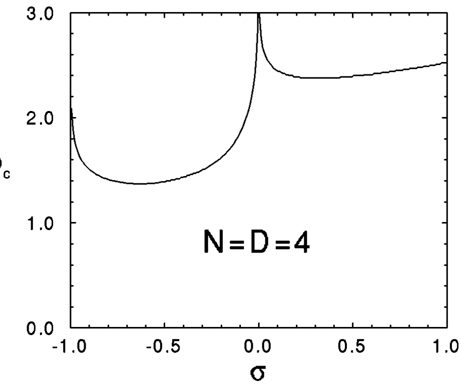

FIG. 1. Numerical results for $\phi_c(\sigma)$.

Around any FP, for finite ϕ, the form of the allowed interactions may be studied by linearization: $\delta V(\phi, t) = \epsilon v(\phi) e^{\lambda t}$. From this we tentatively deduce the existence of a small (renormalized) coupling $g(t) = \epsilon e^{\lambda t}$ of (scaling) dimension λ. However, once again, experimentally we know that λ is quantized, and it is crucial that the LPA reproduce this. Studying the limit $\phi \to \infty$, we find that the $v(\phi)$ divide into two classes: quantized-λ perturbations (QP's) that behave as $\sim\phi^{(D-\lambda)/d}$, and nonquantized-λ perturbations (NQP's) that behave as $\sim \phi^q \exp(c\phi^p)$ with c and p positive. It is $\lambda > 0$ choices of the latter that Ref. [1] argues give asymptotically free interactions. We have argued [3] that the NQP's lead to singular potentials at some $t > 0$ [7], but irrespective of this, we can see that NQP's do not scale as required, since for any *finite* ϵ the right-hand side of (1) contributes negligibly as $\phi \to \infty$ and mean field evolution takes over: $\delta V(\phi, t) \sim \epsilon e^{Dt} v(\phi e^{-dt})$. While this t evolution can still be absorbed into $g(t)$ in the case of the QP's, it cannot for the NQP's. Indeed, at the Gaussian FP, where the QP's correspond to Laguerre polynomials, it follows that as soon as $t > 0$ the NQP's become integrable with the Laguerre weight and may be reexpanded in terms of the QP's. Thus, clearly the QP's already span all the continuum physics.

Tim Morris
Physics Department
University of Southampton
Southampton, SO17 1BJ, United Kingdom

Received 24 January 1996 [S0031-9007(96)00927-1]
PACS numbers: 11.10.Hi, 11.10.Kk, 11.10.Lm

[1] K. Halpern and K. Huang, Phys. Rev. Lett. **74**, 3526 (1995).
[2] K. Halpern and K. Huang, Phys. Rev. D **53**, 3252 (1996); V. Periwal, Princeton Report No. PUPT-1567 (1995).
[3] T. R. Morris, Phys. Lett. B **334**, 355 (1994).
[4] T. R. Morris, Nucl. Phys. **B(Proc. Suppl.)42**, 811 (1995).
[5] T. R. Morris, Nucl. Phys. **B458[FS]**, 477 (1996).
[6] J. F. Nicoll *et al.*, Phys. Rev. Lett. **33**, 540 (1974).
[7] They are already singular at $t = 0$, when $N = \infty$.

 0031-9007/96/77(8)/1658(1)$10.00