Comment on "Quantum Phase of Induced Dipoles Moving in a Magnetic Field"

It has recently been suggested [1] that an Aharonov-Bohm phase should be capable of detection using beams of neutral polarizable particles. A more careful analysis of the proposed experiment suffices to show, however, that it cannot be performed regardless of the strength of the external electric and magnetic fields.

To demonstrate this result one begins with the Lagrangian of Ref. [1],

$$L = \frac{1}{2}M\mathbf{V}^2 + \frac{1}{2}\alpha(\mathbf{E} + \mathbf{V} \times \mathbf{B})^2, \tag{1}$$

where α is the (intrinsically positive) polarizability of the particle and M is its mass. The electric field $\mathbf{E}$ is taken to have a magnitude k/r in a radial direction in a plane perpendicular to the uniform magnetic field $\mathbf{B}$. Upon applying the canonical formalism, one readily obtains from Eq. (1) the relevant Hamiltonian in the form

$$H = \frac{1}{2}(M + \alpha B^2)^{-1}(\mathbf{p} + \alpha B\overline{\mathbf{E}})^2 - \frac{1}{2}\alpha\mathbf{E}^2,$$

where (as in Ref. [1]) motion has been restricted to the plane perpendicular to the magnetic field. The two-dimensional vector $\overline{\mathbf{E}}$ is the dual of $\mathbf{E}$ [i.e., $(\overline{\mathbf{E}})_i = \epsilon_{ij}E_j$].

One readily finds that the relevant Schrödinger equation for a particle of energy $\mathcal{E}$ is

$$[\tfrac{1}{2}(M + \alpha B^2)^{-1}(-ih\nabla + \alpha B\overline{\mathbf{E}})^2 - \tfrac{1}{2}\alpha\mathbf{E}^2]\psi = \mathcal{E}\psi\,.$$

Standard separation of variables then yields for the radial wave function $f_m(r)$ the result

$$\left[\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{2(M + \alpha B^2)\mathcal{E}}{\hbar^2} - \frac{m^2 + 2m\alpha kB/\hbar - M\alpha k^2/\hbar^2}{r^2}\right]f_m(r) = 0\,, \tag{2}$$

where the angular momentum quantum number m takes the values $0, \pm 1, \pm 2, \ldots$.

The system described by Eq. (2) is one which allows quantum mechanically well-defined solutions only when the numerator of the $1/r^2$ term is non-negative for all m. This condition is readily seen to be violated in the case of $m = 0$ whenever there is a nonvanishing electric field present [2]. Moreover, the case $\mathbf{E} = 0$ is clearly trivial in that it implies the absence of a quantum phase with the only effect of the interaction being a mass renormalization (i.e., $M \to M + \alpha B^2$).

One could, of course, attempt to avoid the difficulties associated with the $1/r^2$ term by adding an infinitely large potential for r less than some fixed radius R. This, however, would require that an entirely different quantum mechanical problem be solved, one which would not have a physically acceptable limit for R going to zero.

In summary, the experiment in the form proposed cannot be carried out because its assumptions are in basic conflict with quantum mechanics. Whether a cutoff modified version could yield observable interference effects is, at present, no more than a matter of conjecture.

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[1] H. Wei, R. Han, and X. Wei, Phys. Rev. Lett. **75**, 2071 (1995).
[2] It may be worth emphasizing that this problem cannot arise in Aharonov-Bohm scattering since, in that case, the coefficient of the $1/r^2$ term can never be negative for any m value.

