The Gravitational Constant, the Chandrasekhar Limit, and Neutron Star Masses

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The Chandrasekhar mass limit sets the scale for the late evolutionary stages of massive stars, including the formation of neutron stars in core collapse supernovae. Because its value depends on the gravitational constant G, the masses of these neutron stars retain a record of past values of G. Using Bayesian statistical techniques, I show that measurements of the masses of young and old neutron stars in pulsar binaries limit $\dot{G}/G = (-0.6 \pm 2.0) \times 10^{-12}$ yr^{-1} (68% confidence limit) or $\dot{G}/G = (-0.6 \pm 4.2) \times 10^{-12}$ yr^{-1} (95% confidence limit). [S0031-9007(96)00910-6]

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Whether or not the fundamental "constants" of nature vary with time has been a question of considerable interest since Dirac suggested that the gravitational force may be weakening with the expansion of the universe [1]. Although general relativity predicts $\dot{G}$ is identically zero, a variable G is expected in theories such as the Brans-Dicke scalar-tensor theory and its extensions [2–5], and has recently received renewed attention in the context of extended inflationary cosmology [6].

The most direct experimental limits on $\dot{G}/G$ come from monitoring the separations of orbiting bodies, since from Kepler's laws it is easily shown that $\dot{G}/G = -\dot{a}/a$, where a is the orbital semimajor axis. Early studies based on ancient occultation and eclipse observations had relatively low precision [7–9], but strong limits are now available from direct measurements. For convenience, I define $\zeta_{-12} \equiv (\dot{G}/G)/10^{-12}$ yr^{-1}. Lunar laser ranging experiments yield $\zeta_{-12} = 0 \pm 11$ [10], while radar ranging to *Viking* gives $\zeta_{-12} = 2 \pm 4$ [11] or -2 ± 10 [12], depending on assumptions about solar system mass uncertainties and correlations between model parameters. Similarly, observations of the pulsar–white-dwarf binary PSR B1855+09 yield $\zeta_{-12} = -9 \pm 18$ [13]. (Observations of the double neutron-star binary PSR B1913+16 give $\zeta_{-12} = 4 \pm 5$ [14], but this limit is greatly weakened when the $\dot{G}$-driven variation in the gravitational self-energy of the stars is considered [15].)

Indirect evidence about past values of G can be obtained from comparison of big-bang nucleosynthesis models with the observed ^{4}He abundance [16]. A recent reanalysis argues that $0.7G_0 < G_{\rm BBN} < 1.4G_0$ [17], corresponding to $|\zeta_{-12}| \lesssim 0.9$ for a power law variation of G, or $|\zeta_{-12}| \lesssim 40$ for a linear variation of G. Limits are also obtained from considerations of the long term stability of clusters of galaxies and globular clusters [18] ($|\zeta_{-12}| \lesssim 40$–$60$), or from evolution of the Sun or other stars, since the luminosity of a star $L \propto G^7$ [19,20]. Unfortunately, the Earth preserves only a crude memory of the early luminosity of the Sun, so paleontological evidence gives only a weak limit $|\zeta_{-12}| < 100$ [21].

Fortunately, a much more precise record of early stellar evolution can be found in the galactic population of neutron stars, whose masses are set at their time of formation by the balance between the Fermi degeneracy pressure of a cold electron gas and the gravitational force, through the Chandrasekhar limit [2,22]:

$$M_{\rm ch} \sim \left(\frac{\hbar^{3/2}c^{3/2}}{G^{3/2}m_N^2}\right), \tag{1}$$

where m_N is the mass of the neutron. Because $M_{\rm ch}$ sets the mass scale in the late stages of stellar evolution [23], we expect the average neutron star mass $\mu \sim M_{\rm ch}$, which implies $\dot{G}/G = -2\dot{\mu}/3\mu$. In this Letter, I show how observations of neutron star masses and ages can be used to set tight limits on $\dot{\mu}$ and hence $\dot{G}$.

The Masses and Ages of Neutron Stars—There are now five double neutron star binaries known. In each case, five Keplerian parameters can be very precisely measured by pulse timing techniques [24]: the binary period P_b, the projection of the orbital semimajor axis on the line of sight $x \equiv a_1 \sin i$, the eccentricity e, and the time and longitude of periastron, T_0 and ω_0. These parameters are related to the pulsar and companion masses, m_1 and m_2, through the mass function

$$f = \frac{(m_2 \sin i)^3}{(m_1 + m_2)^2} = \frac{4\pi^2 x^3}{GP_b^2}. \tag{2}$$

In each case, the relativistic advance of the angle of periastron, $\dot{\omega}$, has also been measured, which yields an estimate of the total system mass $m_t = m_1 + m_2$. For three systems (PSRs B1534+12, B1913+16, and B2127+11C) the measurement of the combined effects of the transverse Doppler shift and the gravitational redshift allow the individual determination of the pulsar and companion masses. In Table I, I collect the measurements of f, m_1, m_2, and m_t, and their uncertainties (the uncertainty in f can be neglected). To a good approximation, the measurements are independent and normally distributed, so if $\hat{m}_i$ is the true value of one of m_1, m_2, or m_t, then the probability that we will measure m_i is

$$P(m_i|\hat{m}_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} e^{-(m_i-\hat{m}_i)^2/2\sigma_i^2}. \tag{3}$$

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TABLE I. Ages and masses of binary pulsars.

	Mass $M_\odot$	Pulsar mass $M_\odot$	Mass function $M_\odot$	Spindown age t_c (Gyr)	Ref.
J1518+4904	2.65(7)	⋯	0.1159876	> 16	[36]
B1534+12	2.67835(16)	1.338(12)	⋯	0.25	[37]
B1913+16	2.82843(2)	1.442(3)	⋯	0.11	[38]
B2127+11C	2.7121(6)	1.349(40)	⋯	0.10	[39]
B2303+46	2.60(6)	⋯	0.2462832	0.03	[37]

An upper limit to the age of a pulsar can be determined from the rate at which the pulsar period P is increasing because of the loss of rotational energy to radiation, $t_c = P/2\dot{P}$. The progenitors of the double neutron star systems were binary systems consisting of two massive stars that underwent successive supernova explosions. Because the lifetime of a neutron star progenitor is only about 10^7 yr, the age difference between the pulsar and companion can be neglected. In the case of PSR J1518+4904, for which t_c is at least 16 Gyr, I assume instead an upper age limit of 10 Gyr, about the age of the Galactic disk.

PSR B2127+11C is a special case, because it is in the globular cluster NGC 7078 (M15), where interactions can result in new companions being exchanged into binaries. Indeed, in the standard scenario, neutron star formation is completed within the first $\sim 10^7$ yr of the cluster lifetime, so the small characteristic age of the pulsar is understood as the time since the pulsar was (last) spun-up by a companion. In this case, the relevant age for both neutron stars is the cluster age, which can be found by comparing stellar structure models with observations of the color-magnitude distribution of cluster members. The most recent calculations, using the latest nuclear equation of state and opacity data, find a cluster age of 12–13 Gyr [25], somewhat younger than previous estimates [26,27].

Variability of the Average Neutron Star Mass.—In Fig. 1, I display the available information on the ages and masses of neutron stars in double neutron star binaries. There is certainly no evidence that the average mass has changed by more than a few tenths of a solar mass in the last 12 Gyr. I introduce a model in which the average mass varies as $\mu = \mu_0 - \dot{\mu} t$, where t is the neutron star age, with the goal of estimating the posterior density $P(\dot{\mu}|\{x_i\},\{t_i\})$, where $\{x_i\}$ and $\{t_i\}$ are the observations of neutron star masses and ages.

The underlying distribution of neutron star masses is unknown, but the tight clustering of masses of young stars, for which $\dot{G}$ is unimportant, suggest that a normal distribution with variance s about μ is reasonable. Hence the probability that a neutron star of age $\hat{t}$ will have mass $\hat{m}$ is

$$P(\hat{m}|\hat{t}, \mu_0, \dot{\mu}, s) = \frac{1}{\sqrt{2\pi s}} e^{-(\hat{m}-\mu_0-\dot{\mu}\hat{t})^2/2s}. \qquad (4)$$

I consider a normal distribution because the present data set is too small to justify a more complex model; other distributions (e.g., uniform between an upper and lower bound [28]) are possible, but the final results are fairly insensitive to the form of Eq. (4).

Assuming a uniform prior density for $\dot{\mu}$, I use Bayes' Theorem to write $P(\dot{\mu}|\{x_i\},\{t_i\}) \propto P(\{x_i\}|\dot{\mu},\{t_i\})$, where the proportionality constant is set by the normalization condition $\int P(\dot{\mu}|\{x_i\},\{t_i\})\, d\dot{\mu} = 1$. Then

$$P(\{x_i\}|\dot{\mu},\{t_i\}) = \int \int P(\{x_i\}|\dot{\mu}, \mu_0, s, \{t_i\}) \times \pi(\mu_0)\pi(s) d\mu_0\, ds\,, \qquad (5)$$

where I take the prior density $\pi(\mu_0)$ to be uniform for positive μ_0, and $\pi(s)$ as uniform in $\log s$ [29].

To evaluate Eq. (5), note that the independence of the measurements of the five binary systems allows the

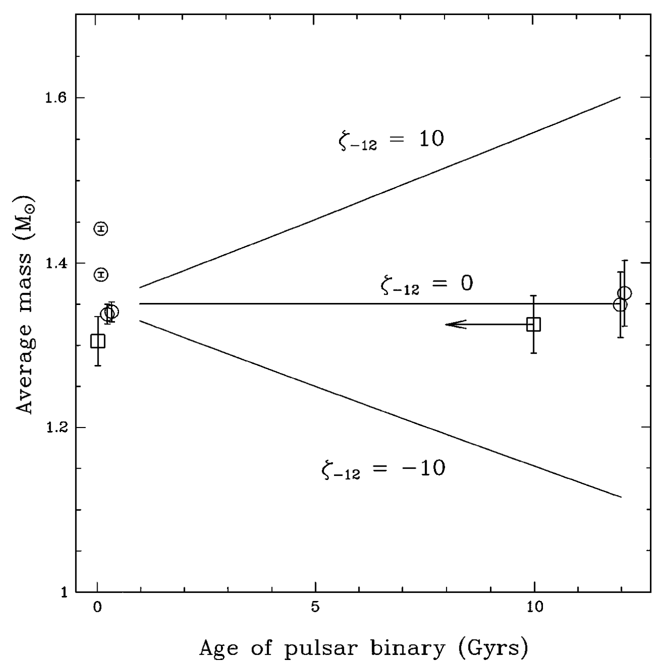

FIG. 1. Masses of the neutron stars in the five binaries of Table I. Circles indicate individual stars; squares are the average mass in cases where the individual masses cannot be determined. Ages shown are upper limits, except for PSR B2127+11C, as described in text. Ages of the components of a binary are offset slightly for clarity; uncertainties in the mass of a pulsar and its companion are not independent. The variation in the average neutron star mass corresponding to $\zeta_{-12} = \pm 10$ is shown.

factorization

$$P(\{x_i\}|\dot{\mu}, \mu_0, s, \{t_i\}) = \prod_i P(x_i|\dot{\mu}, \mu_0, s, t_i) = \prod_i \int P(x_i|\dot{\mu}, \mu_0, s, \hat{t}_i) P(\hat{t}_i|t_i) d\hat{t}_i\,, \tag{6}$$

where now $x_i = \{m_2, m_t\}$ for PSRs B1534+12, B1913+16, and B2127+11C, and $x_i = \{f, m_t\}$ for PSRs J1518+4904 and B2303+46. I take $P(\hat{t}_i|t_i)$ uniform for $\hat{t}_i < t_i$ and zero otherwise, except for PSR J1518+4904, where I make the further (conservative) assumption that $P(\hat{t}_i|t_i) = 0$ for $\hat{t}_i > 10$ Gyr, and for PSR B2127+11C, for which I take $P(\hat{t}_i|t_i) = \delta(\hat{t}_i - 12 \text{ Gyr})$. We can further factor

$$P(m_2, m_t|\dot{\mu}, \mu_0, s, t_i) = \int\int P(m_2|\hat{m}_2) P(m_t|\hat{m}_t)\, P(\hat{m}_2|\dot{\mu}, \mu_0, s, \hat{t}) P(\hat{m}_t - \hat{m}_c|\dot{\mu}, \mu_0, s, \hat{t}) d\hat{m}_t\, d\hat{m}_2\,, \tag{7}$$

and a similar, more complex expression for $P(f, m_t|\dot{\mu}, \mu, s, t_i)$, using Eq. (2) and $P(\cos i) = 1/2$. Using Eqs. (3) and (4), we can then (numerically) evaluate $P(\dot{\mu}|\{x_i\}, \{t_i\})$. I find $\dot{\mu} = -1.2 \pm 4.0(\pm 8.5) \times 10^{-3} M_\odot\ \text{Gyr}^{-1}$ at the 68% (95%) confidence level, corresponding to $\dot{G}/G = -0.6 \pm 2.0(\pm 4.2) \times 10^{-12}$.

The measurement $\zeta_{-12} = -0.6 \pm 2.0$ is a factor five tighter than earlier limits. It is important to understand how sensitive this result is to our model assumptions, the most critical of which concern the neutron star ages. Independent evidence that pulsars with small spin-down rates are old comes from studies of pulsars in binaries with cooling white dwarf companions. For example, an upper limit on the optical luminosity of the companion of PSR B1855+09 yields a minimum system age of 4 Gyr, comparable to the pulsar timing age $t_c = 5$ Gyr [13,30].

Some postformation mass transfer is required to recycle the pulsars to their observed periods, however, a very small transfer ($\ll 0.1 M_\odot$) is sufficient in all cases (and the physics of spin-up is presumed independent of time). Of more concern is the age of PSR B2127+11C, which is based on the standard model of neutron star formation: core collapse in massive stars. It has been proposed that some neutron stars in globular clusters may result from accretion induced collapse (AIC) of O-Ne-Mg white dwarfs [31,32], though theoretical and observational uncertainties remain [32]. In the most unfavorable case, both PSR B2127+11C and its companion were (separately) formed by AIC before exchanging into the current binary. The pulsar may then be young, with $\hat{t}_i < t_c$, while the companion could have any age less than the cluster age. Using these assumptions, I find the weaker limit $\zeta_{-12} = 2.3 \pm 5.0$.

Mass determinations of pulsars such as PSR B1855+09, for which white dwarf cooling ages are available, are clearly of great interest. In fact, observations of the Shapiro time delay in this system yield $m_1 = 1.50^{+0.26}_{-0.14} M_\odot$ [13], but the uncertainties are still too large for this measurement to contribute significantly to our estimate of ζ_{-12}.

Time variability of the Chandrasekhar limit has other potentially observable implications, most notably for Type Ia supernovae, which are widely believed to be the thermonuclear disruptions of Chandrasekhar mass white dwarfs [33], and which are remarkable standard candles, with an intrinsic dispersion of only 0.12 magnitudes when multicolor light curve shape corrections are done [34]. The optical emission is due to decay of ^{56}Ni. In a naive model, ^{56}Ni production will be roughly proportional to mass, and hence to $G^{-3/2}$. If G is proportional to the expansion parameter R^σ, then $\dot{G}/G = \sigma H_0$. An ambitious program on existing telescopes could measure the average luminosity of supernovae at $z = 1$ to $\sim$0.05 magnitudes [35], corresponding (for $H_0 = 60$ km s^{-1} Mpc^{-1}) to the magnitude change produced by $\zeta_{-12} = 2.7$.

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