Superfluidity in β-Stable Neutron Star Matter

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(Received 26 April 1996)

In this work we present results for pairing gaps in β-stable neutron star matter with electrons and muons using a Dirac-Brueckner-Hartree-Fock approach, starting with modern meson-exchange models for the nucleon-nucleon interaction. Results are given for superconducting $^1S_0$ protons and superfluid $^3P_2$ and $^1D_2$ neutrons. A comparison is made with recent nonrelativistic calculations, and the implications for neutron star cooling are discussed. [S0031-9007(96)00970-2]

PACS numbers: 26.60.+c, 21.30.Fe

Superfluidity and superconductivity of matter in neutron stars are expected to have a number of consequences directly related to observation; see Refs. [1–5]. Among the processes that will be affected is the emission of neutrinos. Neutrino emission from, e.g., various URCA processes is expected to be the dominant cooling mechanism in neutron stars less than 10^5-10^6 yr old. Typically, proton superconductivity reduces considerably the energy losses in so-called modified URCA processes and may have important consequences for the cooling of young neutron stars. Another possible manifestation of superfluid phenomena in neutron stars is glitches in rotational frequencies observed in a number of pulsars. Moreover, the estimation of superfluid gaps and studies of pairing are not only important issues in neutron star matter, but also in the rapidly developing field of neutron-rich systems such as heavy nuclei close to the neutron drip line [6] or the study of light halo nuclei [7]. Therefore theoretical studies of pairing in neutron-rich assemblies form currently a central issue in nuclear physics and nuclear astrophysics.

The aim of this Letter is to present results from self-consistent calculations for neutron and proton pairing gaps in β-stable matter relevant for neutron star studies. Relativistic effects for pairing in the partial waves $^1S_0$, $^3P_2$, and $^1D_2$ will be studied using the Dirac-Brueckner-Hartree-Fock (DBHF) approach with modern meson-exchange potential models to describe the nucleon-nucleon (*NN*) potential [8]. A comparison with the corresponding nonrelativistic approach is also made. To our knowledge, this is the first estimate of pairing gaps within the framework of the DBHF approach. The only parameters which enter our approach are those which define the free *NN* potential [8].

Our computational scheme is as follows.

The first ingredient in our calculation is the self-consistent evaluation of single-particle energies in β-stable matter starting from the meson-exchange potential models of the Bonn group [8]. These single-particle energies are obtained within the framework of the DBHF scheme [9–11], using a medium renormalized *NN* potential G defined through the solution of the G-matrix equation

$$G(\omega) = V + VQ \frac{1}{\omega - QH_0Q} QG(\omega), \tag{1}$$

where ω is the unperturbed energy of the interacting nucleons, V is the free *NN* potential, H_0 is the unperturbed energy of the intermediate scattering states, and Q is the Pauli operator preventing scattering into occupied states. Only ladder diagrams with two-particle states are included in Eq. (1). In this work we solve Eq. (1) using the Bonn A potential defined in Table A.2 of Ref. [8]. This potential model employs the Thompson [10,12] reduction of the Bethe-Salpeter equation, and is tailored for relativistic nuclear structure calculations. For further details; see Refs. [8,10,11].

The DBHF is a variational procedure where the single-particle energies are obtained through an iterative self-consistency scheme. To obtain the relativistic single-particle energies we solve the Dirac equation for a nucleon in the nuclear medium, with $c = \hbar = 1$,

$$[\not p - m + \Sigma(p)]\tilde{u}(p,s) = 0, \tag{2}$$

where m is the free nucleon mass and $\tilde{u}(p,s)$ is the Dirac spinor for positive energy solutions, $p = (p^0, \mathbf{p})$ being a four momentum and s the spin projection. The self-energy $\Sigma(p)$ for nucleons can be written as

$$\Sigma(p) = \Sigma_S(p) - \gamma_0\Sigma^0(p) + \boldsymbol{\gamma} \cdot \mathbf{p}\Sigma^V(p). \tag{3}$$

Since $\Sigma^V \ll 1$ [10,13], we approximate the self-energy by

$$\Sigma \approx \Sigma_S - \gamma_0\Sigma^0 = U_S + U_V, \tag{4}$$

where U_S is an attractive scalar field and U_V is the timelike component of a repulsive vector field. The Dirac spinor reads then

$$\tilde{u}(p,s) = \sqrt{\frac{\tilde{E}_p + \tilde{m}}{2\tilde{m}}} \begin{pmatrix} \chi_s \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{\tilde{E}_p + \tilde{m}}\chi_s \end{pmatrix}, \tag{5}$$

where χ_s is the Pauli spinor and terms with tilde like $\tilde{E}_p = \sqrt{\mathbf{p}^2 + \tilde{m}^2}$ represent medium modified quantities.

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Here we have defined [10,13] $\tilde{m} = m + U_S$. In all equations below, a momentum p refers to the three-momentum $\mathbf{p}$. The single-particle energies $\tilde{\varepsilon}_p$ can then be written as

$$\tilde{\varepsilon}_p = \langle p|\boldsymbol{\gamma} \cdot \mathbf{p} + m|p\rangle + u_p = \tilde{E}_p + U_V\,, \quad (6)$$

where the single-particle potential u_p is given by $u_p = U_S\tilde{m}/\tilde{E}_p + U_V$ and can in turn be defined in terms of the G matrix

$$u_p = \sum_{h \le k_F} \frac{\tilde{m}^2}{\tilde{E}_h \tilde{E}_p} \langle ph|G(\omega = \tilde{\varepsilon}_p + \tilde{\varepsilon}_h)|ph\rangle\,, \quad (7)$$

where p, h represent quantum numbers like momentum, spin, isospin projection, etc. of the different single-particle states and k_F is the Fermi momentum. Eqs. (6) and (7) are solved self-consistently starting with adequate values for the scalar and vector components U_S and U_V. The proton fraction in β equilibrium is determined by imposing the relevant equilibrium conditions on the processes $e^- + p \to n + \nu_e$ and $e^- \to \mu^- + \overline{\nu}_\mu + \nu_e$. The conditions for β equilibrium require that $\mu_n = \mu_p + \mu_e$, where μ_i is the chemical potential of particle species i, and that charge is conserved $n_p = n_e$, where n_i is the particle number density for particle species i. We also include muons and the condition for charge conservation becomes $n_p = n_e + n_\mu$, and chemical equilibrium gives $\mu_e = \mu_\mu$. Throughout we have assumed that neutrinos escape freely from the neutron star. The proton and neutron chemical potentials are determined from the energy per baryon, calculated self-consistently in the above DBHF approach.

The next step in our calculations is to evaluate the pairing gaps for various partial waves. To evaluate the pairing gap we follow the scheme of Baldo *et al.* [14], originally proposed by Anderson and Morel [15]. These authors introduced an effective interaction $\tilde{V}_{k,k'}$. This effective interaction sums up all two-particle excitations above a cutoff momentum k_M, $k_M = 3\ \mathrm{fm}^{-1}$ in this work. It is defined according to

$$\tilde{V}_{k,k'} = V_{k,k'} - \sum_{k'' > k_M} V_{k,k''} \frac{1}{2\mathcal{E}_{k''}} \tilde{V}_{k'',k'}\,, \quad (8)$$

where the energy $\mathcal{E}_k$ is given by $\mathcal{E}_k = \sqrt{(\tilde{\varepsilon}_k - \tilde{\varepsilon}_F)^2 + \Delta_k^2}$, $\tilde{\varepsilon}_F$ being the single-particle energy at the Fermi surface, $V_{k,k'}$ is the free nucleon-nucleon potential in momentum space, defined by the three-momenta k, k'. The renormalized potential $\tilde{V}_{k,k'}$ and the free *NN* potential $V_{k,k'}$ carry a factor $\tilde{m}^2/\tilde{E}_k\tilde{E}_{k'}$, due to the normalization chosen for the Dirac spinors in nuclear matter. These constants are also included in the evaluation of the G matrix, as discussed in [10,11]. For the $^1S_0$ channel, the pairing gap Δ_k is [14–16]

$$\Delta_k = -\sum_{k' \le k_M} \tilde{V}_{k,k'} \frac{\Delta_{k'}}{2\mathcal{E}_{k'}}\,. \quad (9)$$

For the $^3P_2$ partial wave, we employ the expressions given in Ref. [17], modified as well by the above normalization constants. For further details; see, e.g., Refs. [14,17,18]. In summary, first we obtain the self-consistent DBHF single-particle spectrum $\tilde{\varepsilon}_k$ for protons and neutrons in β-stable matter using the method detailed in Ref. [18]. Thereafter, we solve self-consistently Eqs. (8) and (9) in order to obtain the pairing gap Δ for protons and neutrons for different partial waves.

Our results for the pairing gaps, scalar and vector potentials for neutrons and protons, proton and neutron fractions, and the chemical potential for electrons (and muons for total baryonic densities greater than $\rho = 0.15\ \mathrm{fm}^{-3}$) are displayed in Tables I and II as functions of the total baryonic density. The results of these tables can in turn be used in relativistic equations for various modified URCA processes, in a similar way as done in the nonrelativistic approach of Friman and Maxwell [19]. In Fig. 1 we plot as function of the total baryonic density the pairing gap for protons in the $^1S_0$ state, together with the results from the nonrelativistic approach discussed in Refs. [18,20]. The results in the latter references were also obtained with the Bonn A potential of Ref. [8]. These results are all for matter in β equilibrium. In Fig. 2 we plot the corresponding relativistic results for the neutron energy gap in the $^3P_2$ channel. For the $^1D_2$ channel we found both the nonrelativistic and the relativistic energy gaps to vanish. The nonrelativistic results for the Bonn A potential are taken from Ref. [17]. We have omitted a discussion on neutron pairing gaps in the $^1S_0$ channel, since these appear at densities corresponding to the crust of the neutron star. The gap in the crustal material is unlikely to have any significant effect on cooling processes [2], though it is expected to be important in the explanation of glitch phenomena.

As can be seen from Fig. 1, there are only small differences (except for higher densities) between the nonrelativistic and relativistic proton gaps in the $^1S_0$

TABLE I. Proton fractions χ_p, scalar and vector single-particle potentials U_S^p and U_V^p, respectively, for protons, the proton pairing gap Δ_p for protons in the $^1S_0$ state and the electron (and muon) chemical potential μ_e as functions of total baryonic density ρ. Densities are in units of fm^{-3}, U_S^p, U_V^p, Δ_p, and μ_e in units of MeV.

ρ	χ_p	U_S^p	U_V^p	Δ_p	μ_e
0.0013	0.0032	−7.8479	3.2471	0.0121	11.7231
0.0068	0.0050	−77.7002	61.7252	0.0483	20.3904
0.0281	0.0096	−172.0541	135.3744	0.2024	38.9884
0.0583	0.0156	−236.5725	181.5207	0.4386	58.5459
0.0944	0.0229	−285.0128	213.1141	0.7036	78.1881
0.1377	0.0307	−329.1642	242.7944	0.9107	98.3550
0.1811	0.0403	−365.8355	270.4411	1.0160	115.8907
0.2007	0.0462	−381.3338	283.5829	1.0173	123.0215
0.2212	0.0524	−396.7707	297.3635	0.9742	130.1985
0.2627	0.0658	−424.5634	325.2710	0.7712	143.9456
0.3072	0.0801	−451.9637	357.1098	0.4490	158.2441
0.3304	0.0877	−464.7640	373.9551	0.2638	165.5386
0.3544	0.0953	−476.8407	391.2967	0.1826	172.9228
0.3594	0.0968	−479.2122	394.8924	0.0856	174.4599

TABLE II. Proton fraction χ_p, neutron scalar and vector single-particle potentials U_S^n and U_V^n, respectively, and the neutron pairing gap $\Delta(^3P_2)$ as functions of total baryonic density ρ. Densities are in units of fm^{-3}, U_S^n, U_V^n, and Δ in units of MeV.

ρ	χ_p	U_S^n	U_V^n	$\Delta(^3P_2)$
0.0756	0.0191	−118.4076	90.1259	0.009
0.0811	0.0202	−127.8562	97.9057	0.013
0.0849	0.0210	−134.2159	103.1913	0.014
0.0949	0.0230	−150.7538	116.9925	0.017
0.1012	0.0243	−161.1272	125.6867	0.017
0.1056	0.0252	−167.9521	131.2468	0.017
0.1125	0.0266	−179.0345	140.6626	0.015
0.1172	0.0275	−186.6448	147.1867	0.013
0.1196	0.0279	−190.5106	150.5173	0.011

wave. (Even smaller differences are obtained for neutrons in the $^1S_0$ channel.) This is expected since the proton fractions (and their respective Fermi momenta) are rather small; see Table I.

For neutrons, however, see Table II, the Fermi momenta are larger, and we would expect relativistic effects to be important. At Fermi momenta which correspond to the saturation point of nuclear matter, $k_F = 1.36$ fm^{-1}, the lowest relativistic correction to the kinetic energy per particle is of the order of 2 MeV. At densities higher than the saturation point, relativistic effects should be even more important, as can clearly be seen in the calculations of Ref. [10]. Since we are dealing with very small proton fractions in Table II, a Fermi momentum of $k_F = 1.36$ fm^{-1} would correspond to a total baryonic density ~0.09 fm^{-3}. Thus at larger densities relativistic effects for neutrons should be important. This is also reflected in Fig. 2 for the pairing gap in the $^3P_2$ channel. The relativistic $^3P_2$ gap is less than half the corresponding nonrelativistic one, and the density region is also much smaller. This is mainly due to the inclusion of relativistic single-particle energies in the energy denominator of Eq. (9) and the normalization factors for the Dirac spinors in the *NN* potential. As an example, at a neutron Fermi momentum $k_F = 1.5$ fm^{-1}, the gap has a value of 0.17 MeV when one uses free single-particle energies and a bare *NN* potential. Including the normalization factors in the *NN* potential, but employing free single-particle energies, reduces the gap to 0.08 MeV. If we employ only DBHF single-particle energies and the bare *NN* potential, the gap drops from 0.17 to 0.04 MeV. Thus the largest effect stems from the change in the single-particle energies, although the combined action of both mechanisms reduce the gap from 0.17 MeV to 0.015 at $k_F = 1.5$ fm^{-1}. The *NN* potential in the $^3P_2$ channel depends also strongly on the spin-orbit force, see, e.g., Fig. 3.3 in Ref. [8], and relativistic effects tend to make the *NN* spin-orbit interaction from the ω meson in *P* waves more repulsive [8]. This leads to a less attractive *NN* potential in the $^3P_2$ channel and a smaller pairing gap.

Even the nonrelativistic energy gaps for neutron star matter in β equilibrium are small compared with the

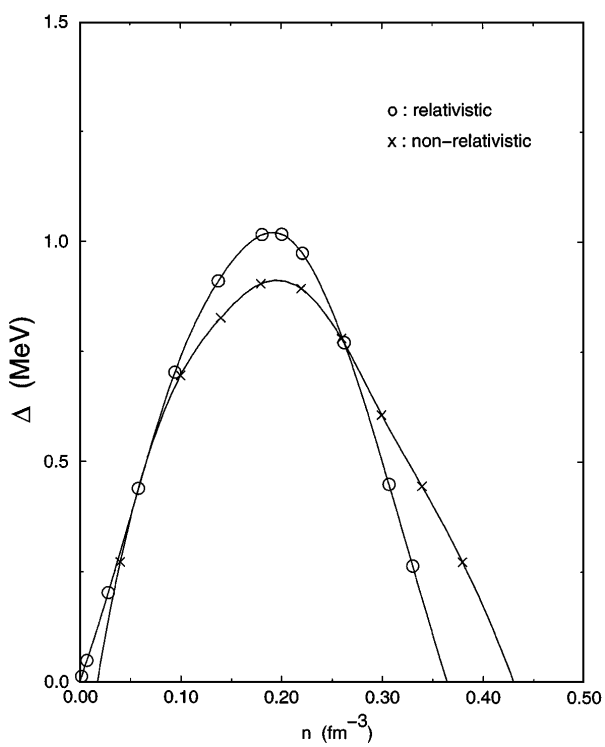

FIG. 1. Proton pairing in β-stable matter for the $^1S_0$ partial wave. The nonrelativistic results are taken from Ref. [18].

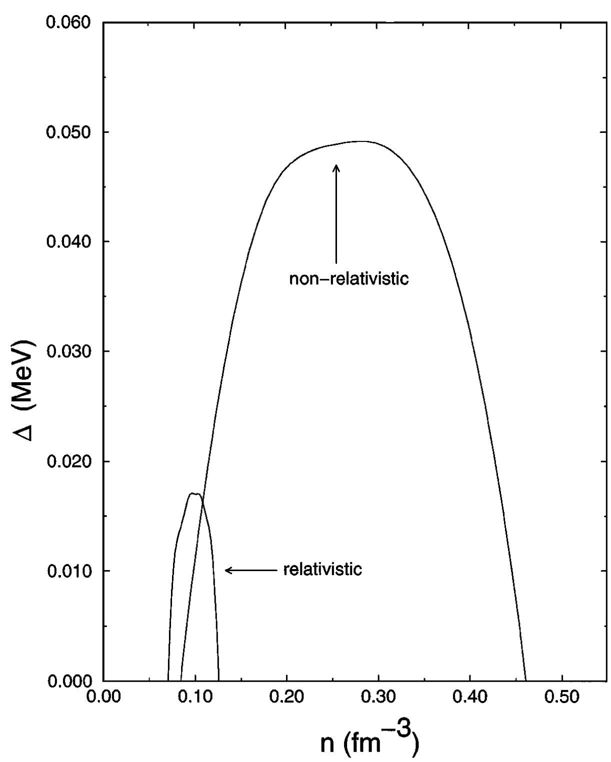

FIG. 2. Neutron pairing in β-stable matter for the $^3P_2$ partial wave. The nonrelativistic results are taken from Ref. [17].

results for pure neutron matter, where the $^3P_2$ energy gap has a maximum around $\sim$0.12$-$0.13 MeV; see Refs. [17,21,22]. The consequences for cooling rates and the interior composition of a neutron star are significant. A recent investigation of various cooling mechanisms by Schaab *et al.* [23] found that an agreement with observed surface temperatures was reached if the $^3P_2$ energy gaps were of the order $\sim$0.05 MeV. Our nonrelativistic results for β-stable matter are of this size, while the relativistic energy gaps result in an almost negligible suppression of, e.g., various modified URCA processes in the interior of a neutron star. These results, and those of Schaab *et al.* [23] as well, differ from those of Page [4], where, in order to explain the observed temperature of Geminga, baryon pairing has to be present in most, if not all, of the core of the star.

In summary, in this work we have calculated in a self-consistent way single-particle energies and energy gaps using a relativistic DBHF approach. Although we have focused on pairing in dense matter, our many-body approach allows also for a consistent treatment of other neutron star properties. The same *NN* force used here has also been used in Ref. [24] to calculate the equation of state and the total mass and radius for a neutron star. Combining the results from this work and those of Refs. [20,24], the following picture emerges: Within the DBHF approach, the direct URCA processes are only allowed for densities larger than 0.52 fm^{-3}; see Ref. [20]. A neutron star with total mass $1.6M_\odot$ would have a central density of $\rho_c = 0.4\ \mathrm{fm}^{-3}$ in β-stable matter [24]. For such a central density, various modified URCA processes are possible mechanisms for neutrino production in a neutron star. The main suppression of these processes would then come from protons in the $^1S_0$ state. The reader should note that there are other possible cooling mechanisms than those discussed here, such as neutrino-pair bremstrahlung [25], direct URCA with hyperons or neutrino emissions from more exotic states, such as pion and kaon condensates or quark matter; see, e.g., Refs. [4,5,23] for recent reviews. However, for a star with central density $\rho_c = 0.4\ \mathrm{fm}^{-3}$, many of these more exotic neutrino emissivities are less likely. Hyperons appear at densities $\rho \sim 0.3$ or greater [26]. Similar densities are expected for kaons and quark matter [23,26]. In addition, neutrino-pair bremstrahlung was recently found [25] to be much less important than previously estimated. Thus for a $1.6M_\odot$ neutron star with central density of 0.4 fm^{-3} obtained with our DBHF approach [24], the most likely cooling scenario is through modified URCA processes, and the main suppression comes from superconducting protons in the $^1S_0$ state. Finally, it ought to be noted that we have not included effects from medium polarization effects, as discussed in Ref. [27]. These effects are expected to be quite large at densities above nuclear matter saturation density and may further change the size of the energy gaps and the neutrino emissivities (the $^1S_0$ gap should decrease, while the $^3P_2$ gap is expected to increase). However, the aim here was to get an indication of the importance of relativistic effects using an approximate relativistic scheme like the DBHF approach and compare these results with a nonrelativistic calculation.

This work has received support from The Research Council of Norway (NFR) (Programme for Supercomputing) through a grant of computing time. M. H. J. thanks the Istituto Trentino di Cultura, Italy, and the NFR for financial support.

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