Nonlinear Spin Dynamics and Magnetic Field Distortion of the Superfluid ^{3}He-B Order Parameter

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We have measured magnetic field distortion of the order parameter and the longitudinal resonance frequency of superfluid ^{3}He-B using pulsed nuclear magnetic resonance. The experiments were guided by numerical solutions of Leggett's equations which we have generalized to include gap distortion. We have found stable regions for the nonlinear spin dynamics and taken advantage of this to obtain the first measurements of the temperature dependence of the distortion of the energy gap in the B phase. At low temperatures and pressures the results agree with theoretical predictions of Fishman and Sauls. [S0031-9007(96)00881-2]

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At temperatures of order 2 mK ^{3}He condenses into a highly ordered p-wave spin-triplet superfluid state [1] that exhibits spontaneously broken spin-orbit symmetry. As a consequence, the nuclear dipole interaction plays a significant role in the spin dynamics [2] creating a molecular field that shifts the nuclear magnetic resonance (NMR) precession frequency [3,4] from that of the normal Fermi liquid [5,6]. Shortly after the discovery of superfluid ^{3}He, the measurements of these frequency shifts in the linear spin-dynamic regime led to the identification of the microscopic structure of the order parameter [1,7,8]. In the B phase of superfluid ^{3}He, the order parameter is significantly distorted by large magnetic fields. The use of NMR frequency shifts to measure this structure requires large excitations where the spin dynamics become nonlinear. Our measurements and numerical simulations have uncovered a regime of quasistable response permitting us to determine experimentally, for the first time, the distortion of the energy gap over a wide range of temperature and to measure the temperature dependence of the longitudinal resonance frequency Ω in superfluid ^{3}He-B.

One effect of an external magnetic field is to produce depairing of the $m = 0$ spin state of the triplet manifold resulting in distortion of the otherwise isotropic gap structure of the B phase [9,10]. The distorted gap can be parametrized theoretically by a longitudinal gap Δ_2 in the direction of the magnetic field and a transverse gap Δ_1, with $\Delta_2 < \Delta_0 < \Delta_1$, where Δ_0 is the zero field value. Gap distortion affects many properties of the superfluid. The nonlinear magnetic field dependence of both the magnetization [11] and the frequencies of order parameter collective modes [12] are examples. It is also responsible for the small shift in NMR frequency from the normal fluid resonance frequency [13–15] which can be observed for small radio frequency (RF) excitation as in the case of continuous wave NMR. Although many experiments have been performed showing these various effects, it has not been possible to determine distortion of the order parameter from them. The NMR frequency shifts found for small excitations, known as g shifts, were first noted in the early high pressure experiments of Osheroff [13]. More recently, the g shift has been the subject of extensive study [14,15] and has been interpreted near T_c with the theory of Greaves [16]. In the current work we extend our previous NMR experiments [14] to the nonlinear dynamic regime accessible with large RF excitation. From the combination of these experiments we can self-consistently determine the full temperature dependence of gap distortion and the longitudinal resonance frequency. As a first step, we generalized the Leggett equations [8] for spin dynamics to the case of gap distorted order parameter. Secondly, we performed numerical simulations of the actual experiments based on these equations and identified specific conditions of RF excitation for which the NMR precession frequency is given by stationary solutions of the Leggett equations [10]. Our numerical work shows that the precession frequency under these special conditions provides a direct measure of Ω and Δ_2/Δ_1. Finally, our experiments, guided by the simulations, give results consistent with earlier work where a comparison is possible.

In a typical pulsed NMR experiment, a 10–100 μs excitation pulse exerts a torque on the nuclear magnetization, tipping it away from its equilibrium position parallel to the external field. The resulting tip angle is $\phi \approx \gamma H_1 t$, with H_1 the amplitude of the RF field, γ the gyromagnetic ratio, and t the duration of the pulse. From the earliest observations of NMR in superfluid ^{3}He-B [3,13], it has been known that the precession frequency of the nuclear magnetization is a function of tip angle as a consequence of the dipole torque, in contrast to more usual

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NMR. There exist two distinct regimes [3,4]. For tip angles less than the Leggett angle $\phi_L = 104°$, the ^{3}He nuclear spins precess in the laboratory frame at a frequency ω, which is approximately the Larmor frequency $\omega_0 = \gamma H_0$, but is given more precisely as $(1 + g)\gamma H_0$. The g shift, $g = \Delta\omega/\omega_0$, depends on both Ω and Δ_2/Δ_1. For tip angles greater than ϕ_L there is a strong dependence of the frequency shift on tip angle. This behavior was first explained by Brinkman and Smith [17] in terms of low field stationary solutions of the Leggett equations. They showed that for the case $\phi > 104°$ there should be a frequency shift from the Larmor precession $\Delta\omega = -(4\Omega^2/15\gamma H_0)(1 + 4\cos\phi)$.

Actual NMR experiments do not excite stationary solutions and are very sensitive to the strength of the RF pulse H_1 [18,19], as is shown in Fig. 1(a). This effect has been investigated numerically by Gould [20]. Consequently, large tip angle experiments cannot be easily interpreted as a quantitative measure of the order parameter structure. However, our numerical work demonstrates that quasistationary modes can be excited at one specific large tip angle $\phi \approx 125°$ and can be used to determine Ω and Δ_2/Δ_1. This provides a new experimental probe of the ^{3}He order parameter.

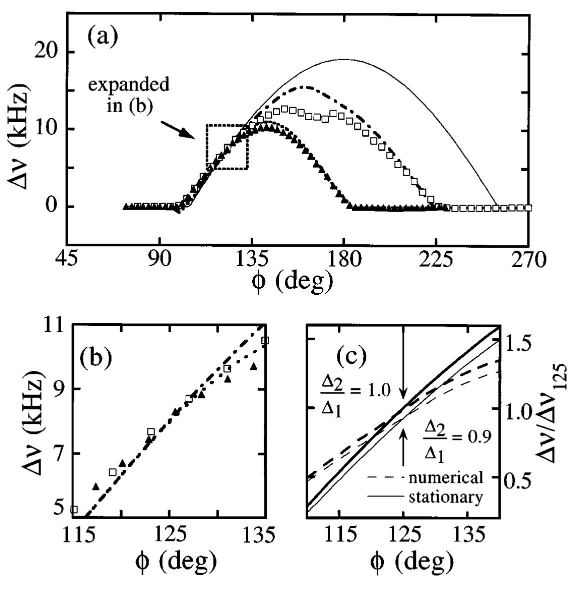

FIG. 1. (a) NMR frequency shifts $\Delta\nu$ plotted versus tip angle ϕ with different excitations H_1 (1.2 G, triangles; 3.4 G, squares) at $P = 28.7$ bars, $T = 0.4T_c$. The dotted and dot-dashed lines are the corresponding numerical simulations with $\Delta_2/\Delta_1 = 0.996$. The solid line is the gap-distorted stationary solution [10]. (b) The expanded region has the same notation as (a) and shows that numerical solutions and the data concur near $\phi = 125°$ independent of H_1. Here the stationary solutions are omitted for clarity but are indistinguishable from the dash-dotted simulation. (c) Numerical and stationary solutions also agree near $\phi = 125°$ (see arrows), independent of gap distortion; $\Delta_2/\Delta_1 = 1.0$ (heavy curves); $\Delta_2/\Delta_1 = 0.9$ (light curves).

We performed pulsed NMR experiments at two tip angles, $\phi = 20°$ and $125°$, at 3.62 MHz (0.112 T) on superfluid ^{3}He-B, with $H_1 = 1.2$ G. The tip angles were calibrated in the normal fluid to within $\pm 1°$. In the superfluid, the angle of the spin with respect to the external field following an RF pulse differs from the expected $\phi = \gamma H_1 t$. However, for our experimental work the discrepancy is less than $1°$ for $\phi \leq 125°$. The experimental resolution of the precession rate for tip angle $\phi < 104°$ was 0.2 ppm and was determined from a fit to the power spectrum. In the nonlinear spin-dynamic regime, $\phi > 104°$, frequency shifts, $\Delta\omega = \omega - \omega_0$, were determined as a function of time from the phase velocity of the magnetization precession. We report only the initial frequencies obtained immediately after the excitation pulse to avoid complication from Leggett-Takagi [21] relaxation. Further experimental details are given elsewhere [6,15,22].

We have generalized the spin dynamic equations proposed by Leggett [8] to include gap distortion that enters through the dipole torque. Fifth order Runge-Kutta integration [23] was performed with inputs of H_1, Ω, and Δ_2/Δ_1. In general, the simulated spin and order parameter motion exhibits precession and nutation after RF excitation. Averaging the nutation over a time of approximately $1/\Omega$, we define an average shift in precession frequency $\overline{\Delta\omega}$ from the simulation. In Fig. 1(a), the dependence of $\overline{\Delta\omega}$ on tip angle is shown by the dotted curves for two different values of H_1. The numerical inputs Ω and Δ_2/Δ_1 were adjusted to fit the experimental data. A comparison is made in the figure with $\Delta\omega$ from two simultaneous NMR experiments shown as triangles ($H_1 = 1.2$ G) and squares ($H_1 = 3.4$ G). These RF field values are also used in the simulation. The numerical results show that the two tip angles which mark the region of large frequency shifts vary systematically with and depend strongly on H_1 in very good agreement with the experiment. This contrasts with the gap-distorted stationary solution of Hasegawa [10], shown as a solid line, for which RF excitation is not involved.

We have found that the frequency shifts determined experimentally, $\Delta\omega$, depend strongly on H_1 at tip angles greater than 125° and near 104°. However, at the tip angle $\phi \approx 125°$ and for $\phi < 60°$ the frequency shifts are insensitive to H_1 for all temperatures and pressures [15]. In Fig. 1(b) we show results emphasizing the region near 125°. For $\phi < 60°$ the dynamic response appears to be linear and the dependence on H_1 is too small to measure. Likewise, $\overline{\Delta\omega}$ predicted by our numerical solutions is also insensitive to H_1 at $\phi \approx 125°$ and for $\phi < 60°$ where it agrees with the stationary solution results as a function of Ω and Δ_2/Δ_1 [10], Fig. 1(c). Furthermore, the amplitude of the nutation has minima near tip angles of 125° and as ϕ approaches zero, suggesting a stability that is not found at other tip angles. On this basis we hypothesize that $\overline{\Delta\omega}$, taken from numerical simulations at $\phi = 125°$ and small tip angles, specifically 20°, accurately reflect the corresponding experimental shifts $\Delta\omega$ allowing us to infer their dependence on Ω and Δ_2/Δ_1. Surprisingly, both the observed and numerical frequency shifts at these stable tip angles are also very close to those given by the stationary solutions as we discuss next.

Hasegawa [10] has developed a hydrodynamic theory of the stationary solutions of the spin-dynamic equations including gap distortion following methods of analysis introduced by Fomin [18]. For small tip angles, $\phi < \phi_L \approx 104°$, he obtained

$$\Delta\omega = \frac{2}{15}\frac{\Omega^2}{\omega_0}\frac{\chi(0)}{\chi(H)}(\Delta_1^2 - \Delta_2^2), \tag{1}$$

and for large tip angles, $\phi > \phi_L \approx 104°$,

$$\Delta\omega = -\frac{16}{15}\frac{\Omega^2}{\omega_0}\frac{\chi(0)}{\chi(H)}\frac{\Delta_1 + \Delta_2}{2} \times \left\{\frac{\Delta_1 + \Delta_2}{2}\cos\phi + \frac{\Delta_1}{4}\right\}. \tag{2}$$

$\chi(H)$ is the field dependent susceptibility which we take from previous measurements [11]. The ratio of Eqs. (1) and (2) is independent of $\chi(0)/\chi(H)$ and gives Δ_2/Δ_1. Equation (1) can also be obtained by linearizing the generalized Leggett equations of motion for small tip angle. Our simulations agree with Eqs. (1) and (2) at the tip angles of $\phi = 20°$ and $125°$ within 2% for conditions corresponding to our experiments and for $0.1 < H_1 < 3.0$ G. Therefore we use the analytic, closed forms of Eqs. (1) and (2) to obtain Ω and Δ_2/Δ_1 from the experimental $\Delta\omega$ at the specified tip angles, rather than perform numerical simulations at each data point. With this prescription we are able to measure the temperature dependence of Δ_2/Δ_1 and Ω, as is given in Fig. 2, in a manner which is insensitive to H_1. We estimate the experimental uncertainty to be less than 10%.

The results for Δ_2/Δ_1 extrapolate to the acoustic pair-breaking measurements performed at low temperatures [24]; inset Fig. 2(a). The theoretical predictions of Fishman and Sauls [25] are shown as a dashed line in Fig. 2(a) for $P = 5.7$ bars. We have adapted the theory to include trivial strong-coupling corrections, which may affect Δ_2/Δ_1 by as much as 20% [26]. Their theory is expected to be applicable at low pressures and temperatures and in this regime it is consistent with our experiment, Figs. 2(a) and 3. At pressures above $P = 5.7$ bars we find 1-Δ_2/Δ_1 to be about 10% greater than theory, likely due to nontrivial strong-coupling effects which increase gap distortion and are estimated to be of this order [26]. At higher temperatures our data extrapolate to predictions of Ginzburg-Landau theory [26] with coefficients of the fourth order invariants taken from experiment [14]. Our results for the temperature dependence of the longitudinal resonance frequency at various pressures are shown in Fig. 2(b). These are in excellent agreement with a previous experiment [27] shown here interpolated to the pressures of our work. The consistency confirms that quantitative properties of the superfluid order parameter

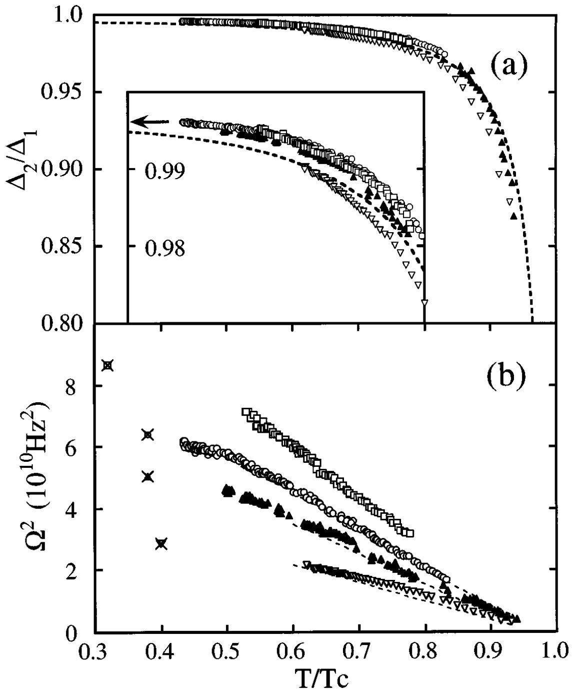

FIG. 2. (a) Gap distortion Δ_2/Δ_1 versus temperature for $P = 5.7$ (open triangles), 12.9 (solid triangles), 18.3 (circles), and 28.7 bars (squares). Theory [25] is shown as the dotted curve at $P = 5.7$ bars. Inset: An expanded view. The arrow gives a limit taken from Ref. [24] at 4.85 bars. (b) Longitudinal resonance frequency compared with Hakonen *et al.* (dotted lines) [27] and Candela *et al.* (crossed symbols) [27], interpolated to the pressures given above.

can be extracted from an analysis of nonlinear NMR response. The measured pressure dependence of gap distortion at $0.62T_c$ is shown as solid circles in Fig. 3, together with low temperature, $T \approx 0$, acoustic pair-breaking data [24], shown as triangles. Theory [25], including trivial strong-coupling effects, is shown as a solid line for $T = 0.62T_c$ and as a dotted line at $T \approx 0$. It is expected that it should agree best with experiment

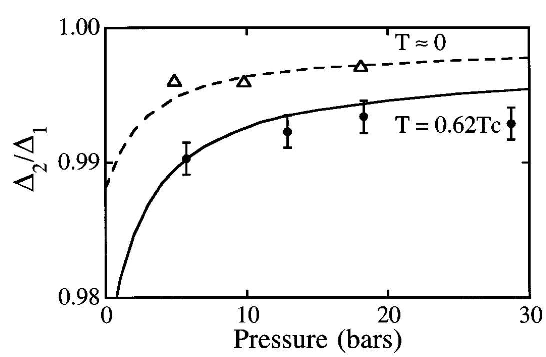

FIG. 3. Pressure dependence of gap distortion at $0.62T_c$ (circles). The quasiclassical theory [25], has been modified to include trivial strong coupling effects at zero temperature (dotted line) and $0.62T_c$ (solid line). The triangles are low temperature limits from Ref. [24].

at low pressures and low temperatures [26]. The trivial strong coupling corrections [12] are obtained from heat capacity measurements. From theory [28] the magnetic field dependence of $1\text{-}\Delta_2/\Delta_1$ is quadratic for the temperature range and magnetic field of our work with only small contributions at quartic order.

In conclusion, we have derived and numerically integrated the equations for spin and order parameter motion of superfluid ^{3}He-*B* in the high field limit where the order parameter is distorted. Our measurements and numerical simulations show that for certain tip angles, $\phi < 60°$ and $\phi = 125°$, the NMR precession rate is quasistable, insensitive to RF excitation strength, and in agreement with analytical stationary solutions. We have taken advantage of these facts to determine the longitudinal resonance frequency and the distortion of the energy gap in superfluid ^{3}He-*B*.

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