Tachyonlike Excitations in Inverted Two-Level Media

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Electromagnetic fields dressed by *inverted* two-level atoms become tachyonlike excitations with group velocities which are faster than c, infinite, or negative. Such excitations describe the *stable modes* of the medium when it is weakly probed off resonance. The launching of these tachyonlike excitations is discussed, along with a proposed experiment to observe them. Their existence does not violate Einstein causality. [S0031-9007(96)00891-5]

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Special relativity prohibits a particle of positive or zero mass from ever possessing a speed exceeding that of light in the vacuum. However, there is nothing in the theory to prevent a particle from having an *imaginary* rest mass, and, correspondingly, a speed that is *always larger* than c [1]. Yet the very existence of such particles, called "tachyons" [2], has been doubted or even deemed unphysical, on the grounds that signals carried superluminally by tachyons would violate causality [3]. Furthermore, experimental searches for tachyons have so far yielded nothing [4,5]. Nevertheless, certain models have been suggested for tachyons. In one of these models, a chain of spring-coupled, inverted pendula near their unstable equilibria can give rise to collective modes whose dispersion relations correspond to an *effectively* imaginary mass and faster-than-c (i.e., superluminal) group velocities [6]. In another closely related model, the Frenkel-Kontorova equation for dislocations also leads to tachyonic behavior [7]. These mechanical models leave open the basic question of whether such collective modes are observable *even in principle*, let alone in a realistic experiment.

In this Letter we point out that tachyonic collective modes should indeed be observable, without implying causality violation. We propose here an experiment in which they can be launched and detected by conventional optical techniques in one of the most thoroughly studied systems of nonlinear optics, namely, a metastable system of inverted two-level atoms. The same cooperativity regime required for superfluorescence (SF) or self-induced transparency (SIT) in this system would, under certain circumstances, allow the stable propagation of a tachyonic wave packet with a superluminal group velocity. We show here that the characteristic signature of a tachyonic excitation is its unique dispersion relation.

Consider a mirrorless system of N two-level atoms per unit volume uniformly distributed over a length L along the z axis, in their inverted states at $t = 0$. The decay of such a system will be initiated by random acts of spontaneous emission, but will gradually become cooperative and form a SF pulse. Unlike previous treatments [8], we focus here on the transient linear response of the system to a weak probe, well before the onset of the SF pulse. This response will be shown to be stable, if the spectrum of the probe is restricted to an appropriate spectral range, in contrast to broadband probes implicitly considered in [9]. Despite certain similarities, there is a notable difference between tachyonic propagation and superluminal propagation in *steady-state* inverted media [10]. In the latter case, constant pumping of the medium produces a steady population inversion, which leads to the elimination of all *transient* coherences. By contrast, here the inversion is suddenly produced, and all damping is neglected, so that one must take full account of all transient coherences which produce undamped collective modes of propagation.

The semiclassical Maxwell-Bloch equations describing propagation in this medium with an arbitrary initial inversion, under the near-resonance, no-damping, rotating-wave, slowly-varying-envelope, and forward-propagation approximations, yield the well-known McCall-Hahn equation, which arose in connection with SIT [11–13], for the pulse area θ of the electric field envelope $\mathcal{E}$

$$\frac{\partial^2\theta}{\partial Z\partial T} = w \sin\theta\,. \tag{1}$$

Here the pulse area θ, which is also the tipping angle of the Bloch vector on the Bloch sphere, is defined by

$$\theta(z,t) \equiv (\mu/\hbar)\int_{-\infty}^{t} \mathcal{E}(z,t')\,dt'\,, \tag{2}$$

μ being the transition dipole moment, and

$$\begin{aligned} Z &\equiv \frac{1}{2}\,\omega_p z/c = z/c\tau_c\,, \\ T &\equiv \frac{1}{2}\,\omega_p(t - z/c) = (t - z/c)/\tau_c \end{aligned} \tag{3}$$

are, respectively, the dimensionless propagation distance and retarded time scaled by an effective plasma

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frequency [11]

$$\omega_p = \frac{2}{\tau_c} = \left(\frac{4\pi N e^2}{m} f\right)^{1/2}, \tag{4}$$

which is the inverse of τ_c, the Arrechi-Courtens cooperation time [12], where $f = 2m\omega_0|\mu|^2/\hbar e^2$ is the oscillator strength of the two-level transition. In the McCall-Hahn equation, w represents the fractional initial population inversion. In particular, $w = +1$ or $w = -1$ corresponds to a system with completely inverted or completely uninverted atoms, respectively.

Let us transform the McCall-Hahn equation into the sine-Gordon equation

$$\frac{\partial^2\theta}{\partial X^2} - \frac{\partial^2\theta}{\partial Y^2} = 4w\sin\theta \tag{5}$$

by introducing the variables X and Y, which are linearly related to Z and T by

$$X \equiv \frac{1}{2}(Z + T) = \frac{1}{4}\,\omega_p t\,,$$

$$Y \equiv \frac{1}{2}(Z - T) = \frac{1}{4}\,\omega_p\left(\frac{2z}{c} - t\right). \tag{6}$$

Thus X is proportional to the time variable t, and Y represents the average of the local position z and the retarded position $z - t/c$.

For small tipping angles such that $\sin\theta \approx \theta$, this equation becomes linear, coinciding with either the tardyonic ($w < 0$) or the tachyonic ($w > 0$) Klein-Gordon equation

$$\frac{\partial^2\theta}{\partial X^2} - \frac{\partial^2\theta}{\partial Y^2} - 4w\theta = 0\,, \tag{7}$$

depending on the sign of the inversion w.

The dispersion relation for weak excitations of the medium now possesses a diagonalized form. Let us introduce the plane-wave ansatz $\theta = \exp(iqY - i\nu X)$, which, after substitution into Eq. (7), yields the relation between the dimensionless wave number q and the dimensionless frequency ν

$$\nu^2 = q^2 - 4w\,. \tag{8}$$

Apart from constants of proportionality, this equation has the form of the relativistic energy-momentum relation $E^2 = p^2c^2 + m^2c^4$, except that for positive inversions ($w > 0$), the invariant rest mass m is imaginary, which is the signature of tachyons [14].

Using Eq. (6) to transform the plane-wave ansatz to the laboratory space and time variables z and t

$$\theta = \exp(iKz - i\Omega t)\,, \tag{9}$$

we obtain from Eq. (8) the quadratic equation

$$\Omega^2 - Kc\Omega + \frac{1}{4}\,w\omega_p^2 = 0\,, \tag{10}$$

whose solution gives the dispersion relation

$$\Omega = \frac{1}{2}\,Kc \pm \frac{1}{2}(K^2c^2 - w\omega_p^2)^{1/2}. \tag{11}$$

The same dispersion relations are obtained from the small-θ limit of the McCall-Hahn equation in the laboratory coordinates z and t,

$$\frac{\partial^2\theta}{\partial t^2} + c\,\frac{\partial^2\theta}{\partial z\partial t} - \left(\frac{1}{4}\,w\omega_p^2\right)\theta = 0\,, \tag{12}$$

upon substitution of Eq. (9) into Eq. (12). Here $K \equiv k - k_0$ and $\Omega \equiv \omega - \omega_0$ are defined with respect to the bare resonance at (k_0, ω_0). In what follows we take advantage of the fact that the detuning Ω can vary, under the near-resonance and slowly-varying-envelope approximations, over a range much broader than ω_p, since it must satisfy only $|\Omega| \ll \omega_0$ [11–13].

The branches of Eq. (11) with $w = \pm 1$ are plotted in Fig. 1. For the uninverted case $w = -1$, we recover the well-known polaritonic dispersion relations, but for the inverted case $w = +1$, we obtain new tachyonic dispersion relations, which are the central results of this Letter. Just as the polaritonic solutions are physical, we believe that the tachyonic ones should also be physical, since the inversion w is a continuous parameter which can vary smoothly from -1 to $+1$, with nothing physically discontinuous happening at $w = 0$. These relations have the form of hyperbolas with asymptotes $\Omega = 0$ and $\Omega = Kc$, the latter corresponding to modes with a limiting group velocity of c. In the tachyonic case, a gap opens up between $-K_c$ and $+K_c$, at the points where the discriminant in Eq. (11) vanishes. These points (Fig. 1 for $w = 1$) with coordinates $A_\pm = (\pm K_c, \pm\Omega_c)$, where

$$K_c = w^{1/2}\omega_p/c\,, \qquad \Omega_c = \frac{1}{2}\,w^{1/2}\omega_p\,, \tag{13}$$

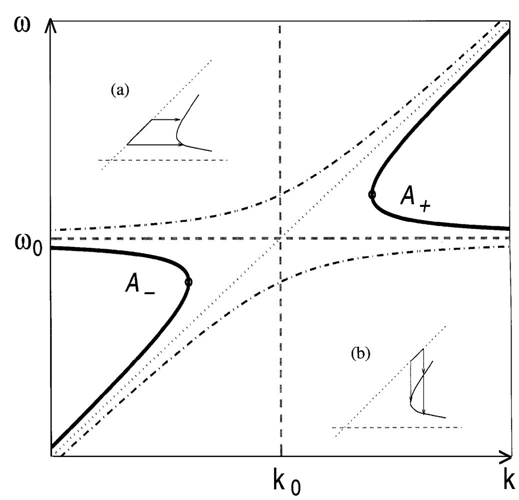

FIG. 1. Polaritonic dispersion relations (dash-dotted curves; $w = -1$) and tachyonic ones (thick solid curves; $w = 1$) from Eq. (11). Inset (a): The probe pulse spectrum is projected horizontally onto the tachyonic curve. Inset (b): Vertical projection for the alternative method [18].

demarcate the boundaries of the gap of instability, and correspond to points of infinite slopes or group velocities. The complex solutions inside the gap correspond to unstable modes. By contrast, the tachyonic solutions outside the gap are purely real, corresponding to *stable* propagating modes of the system. Physically, these modes represent conditions for equality of stimulated emission and absorption, namely, gain-free (and loss-free) propagation. They are the optical analogs of zero-force solutions in the mechanical model [6], which correspond to balancing the gravity force on inverted pendula by the restoring spring force.

Does the existence of the above tachyonlike solutions violate Einstein causality? They represent in fact a *causal* pulse-reshaping process, in which the early tail of a pulse undergoes amplification due to virtual stimulated emission by the inverted atoms, followed by the virtual stimulated absorption of the peak of the pulse by these atoms. Consequently, the pulse is reshaped in such a way that its peak is displaced forwards in time, i.e., the pulse envelope is *advanced,* rather than *retarded,* and thus the group velocity becomes faster than c. The causality of this pulse-reshaping process is revealed when we consider a pulse with a sharp front, corresponding to the first appearance of a nonzero response of the medium to an impulsive excitation. This front, as shown below, always moves *exactly* at the vacuum speed of light, and the rest of the field produced by the inverted medium can never outrace it.

In a more formal analysis of this causality problem, we consider the response to a weak delta-function impulse moving at c through the inverted medium. The resulting electric field envelope is given by

$$\mathcal{E}(z,t) \propto z^{1/2}(t - z/c)^{-1/2} \times I_1(w^{1/2}\omega_p[(z/c)(t - z/c)]^{1/2})\Theta(t - z/c)\,, \tag{14}$$

where I_1 is the modified first-order Bessel function. Einstein causality is not violated due to the presence of the Θ step function. However, instead of oscillatory solutions, as in the uninverted medium [11], we now have exponentially growing solutions, which indicate the presence of instability in the system. Such an instability arises because this impulsive excitation contains Fourier components inside the gap, where there exists exponential gain from the inverted atoms. Thus we see that a superposition of stable and unstable modes conspires in just such a way as to ensure that causality is not violated, as was noted earlier in [6].

When the tipping angle θ becomes large, this linear analysis breaks down, and one must use the nonlinear, large-angle oscillatory solutions of the full sine-Gordon equation [11–13,15,16]. Einstein causality is still preserved on account of the fact that the characteristics of the McCall-Hahn equation,

$$Z \propto z = \text{const}\,, \qquad T \propto t - z/c = \text{const}\,, \tag{15}$$

which determine the propagation of delta-function source terms, are independent of the nonlinear $w \sin\theta$ term. In the nonlinear case as well, superluminal solutions are possible, particularly π solitons [17]. Yet such nonlinear superluminal effects are again due to pulse reshaping, i.e., early tail amplification. The front velocity is still c, as evidenced by SF pulse propagation [15,16].

We now propose an experiment that could verify the existence of these tachyonlike excitations. For concreteness, we consider a situation in which a strong pump pulse, directed *perpendicular* to the medium axis z, causes sudden and uniform population inversion of the medium at $t = 0$. This inversion is followed by the injection along the z axis of a weak probe pulse whose spectrum is detuned from resonance by around $\Omega_c = \frac{1}{2}\,\omega_p$. The resulting tachyonic wave packet is determined by the *horizontal* (constant-frequency) projection of the pulse spectrum onto the tachyonic dispersion curve [Fig. 1, inset (a)] centered near A_+, where the group velocity is practically infinite ($\gg c$) [18].

The following conditions have to be satisfied in the experiment: (i) It is important to filter out of the probe pulse all frequencies detuned from resonance by much less than Ω_c, down to the level of the spontaneous emission noise; otherwise the probe can actually stimulate the instabilities. The spontaneous emission noise corresponds to pulse area $\theta_0 \propto \mathcal{N}^{-1/2}$, $\mathcal{N}$ being the total number of atoms (typically $\theta_0 \sim 10^{-8} - 10^{-3}$) [15]. Hence, the probe pulse area θ must be restricted to $\theta \lesssim \theta_0$ in the instability band $|\Omega| \ll \Omega_c$, whereas outside this band it must satisfy $1 \gg \theta \gg \theta_0$, to ensure a good signal-to-noise ratio. (ii) The probe-pulse duration τ_{probe} must not exceed $\tau_R = 4c/L\omega_p^2$, which is the SF radiation time for a pencil-shaped sample of length L [15]. Conditions (i) and (ii) guarantee that the spontaneous emission noise be kept much smaller than θ outside the instability band well after the probe has traversed the sample, since the delay time for SF amplification [15,19] is $\tau_D = (10-100) \times \tau_R$. (iii) We should try to fulfill Rayleigh's criterion for resolving the peak of a superluminally advanced tachyonic pulse relative to that of a "twin" pulse which has propagated through a length L in the vacuum, by cross-correlation measurements. (iv) On the other hand, we wish to avoid adverse dispersion effects on the superluminal pulse, which implies $\tau_{\text{probe}} \gtrsim \omega_p^{-1}$. To sum up, we must satisfy the inequalities [15]

$$\omega_p^{-1} \lesssim \tau_{\text{probe}} \sim L/c < \tau_R \ll \tau_D \lesssim T_2^*\,, \tag{16}$$

where T_2^*, the inhomogeneous lifetime, is typically shorter than the homogeneous (dephasing) lifetime T_2.

Specifically, we consider the hydrogen fluoride (HF) gas system used by Skribanowitz *et al.* [16] to observe SF on the 84 μm far-infrared $J = 3 \rightarrow 2$ rotational transition of the HF molecule. This transition was pumped by a pulsed HF laser operating on the 2.7 μm near-infrared $P_1(4)$ vibrational line. The gas was at a pressure of 4.5 mTorr in a sample cell of length $L = 1$ m. Doppler broadening introduced an inhomogeneous lifetime of $T_2^* = 330$ ns in this system. To observe tachyonlike excitations in this same system, we detune (e.g., by a corner-cube whisker-diode modulator) a probe pulse from a 84 μm HF laser to near the A_+ point. The probe pulse can be prevented from exciting instabilities by passing it first through an absorption cell [20] of HF gas, which absorbs down to spontaneous emission levels all near-resonant frequencies which could stimulate SF, and then passing it through the population-inverted HF gas cell to launch the stable A_+ tachyonic mode. We choose the probe pulse duration to be $\tau_{\text{probe}} \simeq L/c \simeq 3.3$ ns. All the inequalities in Eq. (16) are then indeed satisfied, since in this system [16] $\omega_p^{-1} = 1.9$ ns, $\tau_R = 4.7$ ns, and the observed $\tau_D = 740$ ns. It is then clearly possible to transmit a tachyonic probe through the population-inverted system and resolve its superluminal peak advancement (by Rayleigh's criterion relative to its "twin"), long before the system can produce appreciable spontaneous emission noise [19]. Repeated cross-correlation measurements at different probe detunings Ω can reproduce both tachyonic branches of Fig. 1.

To conclude, our analysis predicts a novel propagation regime in inverted media. Our estimates imply the possibility of performing an experiment in which collective modes with group velocities much larger than c and unique dispersion relations could be observed, thereby rendering physical reality to the hitherto elusive concept of the tachyon.

The present analysis is semiclassical, yet we anticipate that these dressed-state modes of the system would become upon quantization stable optical tachyons, i.e., tachyonic quasiparticles or elementary excitations which are just as physical as polaritons. Just as a polariton is viewed as a subluminal quasiparticle consisting of a photon dressed by uninverted atoms, an optical tachyon could then be thought of as a photon dressed by inverted two-level atoms to become a superluminal quasiparticle.

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