Spin Motive Force and Faraday Law for Electrons in Mesoscopic Rings

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(Received 2 May 1995)

The spin motive force and Faraday law for electrons due to the time-dependent Aharonov-Casher effect are studied by using the Goldhaber-Anandan gauge theory for a low energy spin particle. The spin motive force and Faraday law associated with the time-dependent magnetic field are also discussed based on the same gauge theory. The gauge theoretic approach provides a unified view for the various spin motive forces and spin Faraday laws.

PACS numbers: 72.10.Bg, 03.65.Bz, 71.70.Ej

The remarkable discovery of the Berry phase [1] has drawn considerable interest during the last decade [2]. The existence of a topological phase for a cyclic Hamiltonian system has brought new insight into quantum mechanics as well as into classical mechanics. The simplest example of the Berry phase is the Aharonov-Bohm (AB) phase [3] acquired by a charged particle encircling a magnetic flux. The localized magnetic flux in a solenoid can affect the quantum phase of the charged particle, although its classical effect is null. Since a solenoid can be viewed as a line of magnetic moments, from an opposite point of view, a magnetic moment encircling an electric charge can accumulate a quantum phase. Indeed, in 1984, Aharonov and Casher (AC) [4] noticed this reverse effect of the AB phase and discovered the AC effect for a neutral magnetic moment encircling a charged line. The AC effect, which may be considered dual to the AB effect, has been experimentally verified by Cimmino *et al.* [5] for thermal neutrons, and by Sangster *et al.* [6] for an atomic system.

The AC effect is very similar to the AB effect in many respects, but essentially differs from the latter in that it involves the spin, which requires an extra degree of freedom. Hence the similarities and differences between the AB effect and the AC effect have become the subject of intense discussion [7] since the discovery of the AC effect. Goldhaber [8] made an important step in this regard by pointing out that when the spin is a quantum operator the interaction of a spin with the electromagnetic field becomes isomorphic to that of isospin with the Yang-Mills field. Anandan expounded this concept and demonstrated the SU(2) gauge structure in the AC effect for a low energy spin particle [9]. According to Anandan's analysis, the interaction of a spin with the electromagnetic field behaves as if the spin is a gauge charge and the interaction is due to the SU(2) gauge field. And, as a result, the AC effect is found to contain much richer physics than the AB effect, [9,10] because of the non-Abelian nature of the SU(2) gauge structure. For instance, Anandan found that the spin magnetic moment in the classical limit is subject to a new nonlinear force arising from the non-Abelian property of the interaction between the spin and the electromagnetic field. This force was soon shown by Casella and Werner not to be measurable when the electromagnetic field is constant or slowly varying [11]. The understanding of the physics of the AC effect is not yet complete, and discovery of interesting physics is still underway. The deeper meanings of the physics associated with the AC effect may be more clearly understood from the gauge theoretical point of view [9,10,12].

In the other direction, an interesting connection between the Berry phase and the solid state transport problem has been discovered. Loss *et al.* [13] studied the conductance in a mesoscopic system due to the Berry phase, and found that the Berry phase associated with the Aharonov-Bohm effect can induce persistent spin and mass currents. Along this line of study of the spin phase effects on the electron transport problem, Stern subsequently discovered that a nonelectromotive force can exist for a spin [14]. By noting the close similarity between the spin Berry phase and the Aharonov-Bohm flux, Stern showed that conductance of a mesoscopic ring is affected by the time-independent Berry phase, and also that a spin motive force is induced when the Berry phase varies in time. He found, furthermore, that there is an analog of the Faraday law for the time-dependent Berry phase causing this spin motive force. That is, the relationship between the spin motive force and the Berry phase for the time-dependent Zeeman coupling was found to be analogous to the Faraday law for the electromotive force and the magnetic flux. Immediately after this discovery, Balatsky and Altshuler identified another type of spin motive force following the Faraday law in the spin-orbit coupling [15]. They showed that time variation of the AC flux can generate an effective spin dependent "electric field" which interacts with the spin. Balatsky and Altshuler demonstrated, in addition, that the AC effect can induce persistent spin and mass current, similar to the AB effect.

Since the essential difference between the AB effect and the AC effect arises from the spin degree of freedom, the spin Faraday laws found previously may be better understood in terms of the interaction between the spin charge and the SU(2) spin gauge field. The purpose of

 0031-9007/96/76(6)/968(3)$06.00

this Letter is to show that a unified understanding of the various spin motive forces and the associated Faraday laws is, in fact, possible based on the spin gauge theory. We discuss the spin motive forces induced by the time-dependent electric and magnetic field for a mesoscopic ring.

Consider first a spin $1/2$ electron in an electromagnetic field. At low energies, the Lagrangian operator considering the spin interaction alone becomes

$$L = \frac{1}{2}mv^2 + \frac{\mu}{2c}\mathbf{v} \cdot \sigma \times \mathbf{E} + \mu\sigma \cdot \mathbf{B}, \quad (1)$$

where $\mu = \frac{e\hbar}{2mc}$ is the magnetic moment, e (<0) is the electric charge, and σ are the Pauli spin matrices. $\mathbf{E}$ and $\mathbf{B}$ are the electric field and magnetic field, respectively. The Lagrangian operator gives rise to the Hamiltonian

$$H = \frac{1}{2m}\left(\mathbf{p} - \frac{\mu}{2c}\sigma \times \mathbf{E}\right)^2 - \mu\sigma \cdot \mathbf{B}, \quad (2)$$

from which the Schrödinger equation for the two-component spinor can be obtained as

$$i\hbar\frac{\partial}{\partial t}\psi = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla - \frac{\mu}{2c}\sigma \times \mathbf{E}\right)^{\mathbf{2}}\psi - \mu\sigma \cdot \mathbf{B}\psi. \quad (3)$$

The Zeeman interaction term and the spin-orbit coupling term in Eq. (3) behave as if they are due to the interaction between the charge μ and the SU(2) gauge field $b_\mu = (b_0, \mathbf{b})$, where $b_0 = -\boldsymbol{\sigma} \cdot \mathbf{B}$ and $\mathbf{b}_i = (\boldsymbol{\sigma} \times \frac{\mathbf{E}}{2})_i$ (see Ref. [9]).

The solution for Eq. (3) becomes

$$\Phi(x^\mu) = P\exp\left(-\frac{i\mu}{\hbar c}\int b_\mu dx^\mu\right)\Phi(x_0^\mu), \quad (4)$$

where P represents the path ordering.

The covariant derivative of the potential term $U = -\frac{\mu}{2c}\mathbf{v} \cdot \boldsymbol{\sigma} \times \mathbf{E} - \mu\boldsymbol{\sigma} \cdot \mathbf{B}$ in the Lagrangian operator of Eq. (1) defines the covariant force on a spin as (Refs. [16] and [17])

$$\mathbf{F} = \mu\left(\mathbf{E}_s + \frac{\mathbf{v}}{c} \times \mathbf{B}_s\right), \quad (5)$$

where

$$\mathbf{E}_s = -\nabla b_0 - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{b} + i\frac{\mu}{\hbar c}[\mathbf{b}, b_0],$$

$$\mathbf{B}_s = \nabla \times \mathbf{b} - i\frac{\mu}{\hbar c}\mathbf{b} \times \mathbf{b}.$$

In the analogy to the electromotive force, the spin motive force can be defined from this covariant force as

$$\epsilon = \frac{1}{\mu}\oint \mathbf{F} \cdot d\boldsymbol{l}. \quad (6)$$

We now consider a situation where a spin rotates around an electric charge in a two-dimensional plane. In this case, since $\mathbf{E}_s = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{b}$ and $\mathbf{B}_s = \nabla \times \mathbf{b}$ with $\mathbf{E} = \frac{e}{r^2}\hat{\mathbf{r}}$, the Yang-Mills fields in Eq. (5) become simplified, resulting in

$$\mathbf{F} = -\mu\frac{1}{2c}\frac{\partial}{\partial t}(\boldsymbol{\sigma} \times \mathbf{E}) + \frac{\mu}{2c}\mathbf{v} \times [\nabla \times (\boldsymbol{\sigma} \times \mathbf{E})]. \quad (7)$$

In the case of $\frac{v}{c} \ll 1$, the second term in Eq. (7) is negligible and the spin motive force is obtained as

$$\epsilon = \oint \mathbf{E}_s \cdot d\boldsymbol{l} = -\frac{1}{2c}\frac{\partial}{\partial t}\oint \boldsymbol{\sigma} \times \mathbf{E} \cdot d\boldsymbol{l}. \quad (8)$$

Here $\oint d\boldsymbol{l}$ is the line integral around the loop. For spin up or spin down, the path ordering in Eq. (4) can be simplified to give rise to the AC flux $\Phi^{\mathrm{AC}} = \frac{\mu}{\hbar c}\oint \mathbf{E}_s \cdot d\boldsymbol{l}$.

The spin motive force ϵ in Eq. (8), then, can be written in terms of Φ^{AC} as

$$\epsilon = -\frac{\hbar}{\mu}\frac{d}{dt}\Phi^{\mathrm{AC}}. \quad (9)$$

The results of Eqs. (8) and (9) state that the time-dependent AC flux generates a spin motive force and the spin "electric field" $\mathbf{E}_s$ in the form of the Faraday law. This gauge theoretic result agrees with the previous result in Ref. [15].

We note that, when the adiabatic condition is satisfied, i.e., when the spin gently precesses along the circuit with a precession frequency small compared to the splitting between the two eigenstates, then the path integral can define correctly the AC flux whose time derivative gives the spin motive force, but also note that, if the adiabatic condition is strongly broken, then the spin Faraday law is sometimes not valid. We illustrate the latter observation by considering the AC flux for a fixed spin (an impulse approximation) rotating in a ring where the time-dependent electric field is applied in the $\mathbf{z}$ direction. Aronov and Lyanda-Geller considered a quasi-one-dimensional ring of radius $\mathbf{r}$ defined in the two-dimensional electron gas (2DEG) of a semiconductor heterostructure [18]. The normal to the 2DEG plane can be taken along $\mathbf{z} \parallel (100)$. Then the AC flux can be obtained from Eq. (4) as (Refs. [19] and [20])

$$\Phi^{\mathrm{AC}(\pm)} = -\pi\left(1 \mp \sqrt{\left(\frac{\mu E r}{\hbar c}\right)^2 + 1}\right), \quad (10)$$

where $\pm$ on the left-hand side denotes the spin up and down. Time derivative of this AC flux is apparently not zero in general.

However, since $\boldsymbol{\sigma} \times \mathbf{E} = (\sigma_y\hat{\mathbf{x}} - \sigma_x\hat{\mathbf{y}})E$, the spin motive force becomes

$$\epsilon = -\frac{1}{2c}\frac{\partial}{\partial t}\oint [\sigma_x\cos(\phi) + \sigma_y\sin(\phi)]Eld\phi = 0. \quad (11)$$

Thus we find that a time-dependent electric field applied in the $\mathbf{z}$ direction does not induce any spin motive force in a mesoscopic ring in this case of fixed spin.

From Eqs. (10) and (11), we find that the Faraday law is not applicable since

$$\epsilon \neq -\frac{\hbar}{\mu}\frac{d}{dt}\Phi^{\mathrm{AC}}. \quad (12)$$

From this example, we see that the Faraday law for a spin motive force can sometimes not be valid. The spin Faraday law seems to be valid only under a condition satisfying $\Phi^{\mathrm{AC}} = \frac{\mu}{\hbar c}\oint \mathbf{E}_s \cdot d\boldsymbol{l}$. This specific condition seems possible only when the path ordering allows a simple integral in Eq. (4) so that the AC flux can be written as $\frac{\mu}{\hbar c}\oint \mathbf{E}_s \cdot d\boldsymbol{l}$.

Next we consider the spin motive force and Faraday law associated with the Berry phase due to a time-dependent magnetic field, based on the same gauge theory. We consider the magnetic field configuration taken in Ref. [14]. The nonuniform magnetic field is obtained by superposing a uniform magnetic field $\mathbf{B}_z$ in the $\mathbf{z}$ direction on a time-dependent $\mathbf{B}_\phi(t)$ induced by a current flowing along the $\mathbf{z}$ axis in a wire. For $\mathbf{B} = \mathbf{B}_\phi(t)\sin(\phi)\hat{\mathbf{x}} + \mathbf{B}_\phi(t)\cos(\phi)\hat{\mathbf{y}} + \mathbf{B}_z\hat{\mathbf{z}}$ and $\mathbf{E} = 0$, the gauge covariant flux defined by $P\exp(-\frac{i\mu}{\hbar c}\int b_\mu dx^\mu)$ gives the type of effect obtained by Stern [14] in a general situation when the applied field has the effect of producing instantaneous eigenvectors of the spin, which gently precesses as the circuit is followed. However, when the spin is fixed (an impulse approximation), the force on an electron spin can be reduced to

$$\mathbf{F} = \mu\nabla(\boldsymbol{\sigma}\cdot\mathbf{B}) \quad (13)$$

and the spin motive force simply becomes

$$\epsilon = \oint \nabla(\boldsymbol{\sigma}\cdot\mathbf{B})\cdot d\boldsymbol{l} = 0. \quad (14)$$

Then the spin motive force in a ring disappears and the spin Faraday law does not hold. Thus when the adiabatic condition is satisfied, Stern's effect is valid, but, if the adiabatic condition is strongly broken, then the spin motive force vanishes. Evidently, it appears that there is a nasty in-between region where neither approximation is good. Presumably in this region the spin motive force is suppressed, but not entirely eliminated.

Summarizing, we have rigorously defined the spin motive force in a mesoscopic ring associated with the time-dependent Aharonov-Casher effect by employing a gauge theoretic analysis. The gauge interaction approach seems to describe all cases of spin-motive force and spin Faraday law, including Stern's effect. We have shown, in particular, that the time-dependent AC flux for a spin rotating around an electric charge generates a spin-motive force according to the Faraday law, as previously found, and that the Faraday law for the spin motive force associated with the AC flux is sometimes not valid. The Faraday law associated with the spin-motive force seems more complicated than that for the electromotive force. This seems basically due to the non-Abelian nature of the spin Berry phase. Thus it may be concluded that there is a certain similarity between the spin-motive force and the electromotive force, but that exact parallelism may not exist between the two motive forces.

Valuable discussions with Taeyeon Lee, Kyoung Wan Park, Seongjae Lee, and Mincheol Shin are gratefully appreciated. Support from El-Hang Lee is also gratefully acknowledged. This work was supported by Electronics and Telecommunications Research Institute, through Contract No. 95038.

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