Femtosecond X-Ray Pulses of Synchrotron Radiation

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A method capable of producing femtosecond pulses of synchrotron radiation is proposed. It is based on the interaction of femtosecond light pulses with electrons in a storage ring. The application of the method to the generation of ultrashort x-ray pulses at the Advanced Light Source of Lawrence Berkeley National Laboratory has been considered. The same method can also be used for extraction of electrons from a storage ring in ultrashort series of microbunches spaced by the periodicity of light wavelength.

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In this Letter we consider a method for the generation of femtosecond pulses of synchrotron radiation radiated by electrons in a storage ring. To obtain such a short duration of the radiation pulses we want to select the radiation of electrons only from a thin bunch slice. Such a slice can be created in the interaction of electrons with a femtosecond laser pulse. Indeed, after the interaction of electrons with the laser light we will get a bunch slice with energy modulated electrons. Because of the high electric field possible in the ultra-short light pulse, the amplitude of this energy modulation can be several times larger than the rms beam energy spread. Then, the modulation of energy can be transformed into the modulation of electron's transverse coordinates with an amplitude much larger than the rms transverse size of a beam. Finally, by a collimation of the synchrotron radiation of electrons in the beam core, we will get pulses of synchrotron radiation only from the offset electrons with approximately the same duration as the duration of the light pulses.

The process for the generation of femtosecond pulses of synchrotron radiation is schematically shown in Fig. 1. Electrons circulate in a storage ring and interact with the laser radiation in an undulator in the interaction region (IR). This interaction occurs when the undulator period λ_u satisfies the resonance condition $\lambda_u = 2\gamma^2\lambda_L/(1 + K^2/2)$ [1], where γ is the Lorentz factor, K is the deflection parameter, and λ_L is the laser light wavelength. According to the reciprocal principle the most efficient energy exchange between the electron and the light will take place when the far field laser radiation has the maximum overlap with the far field spontaneous radiation of electrons in the undulator in both the transverse and longitudinal phase space. Assuming that this condition is satisfied, we calculate the energy absorbed by the electron in the interaction with the light (or transferred to the light field) with the following technique.

A superimposed field of the laser radiation $\vec{E}_L(\omega,\vec{r})$ and the spontaneous electron undulator radiation $\vec{E}_R(\omega,\vec{r})$ is written $\vec{E}_L(\omega,\vec{r}) + \vec{E}_R(\omega,\vec{r})$, where ω is the frequency and $\vec{r}$ is the coordinate vector. Thus, the total field energy $A \sim \int\int |\vec{E}_L(\omega,\vec{r})|^2 \, dS\, d\omega$ can be written as a sum of the energy of the laser pulse A_L, the energy of the electron radiation A_R, and an exchange energy

$$A = A_L + A_R + 2\sqrt{A_L A_R \Delta\omega_L/\Delta\omega_R}\cos(\phi). \qquad (1)$$

Here $\Delta\omega_R$ is the bandwidth of the undulator radiation and $\Delta\omega_L$ is the bandwidth of the light pulse. Formula (1) is written in the assumption of $\Delta\omega_R \geq \Delta\omega_L$.

The amount of energy absorbed by the electron (or transferred to the field from the electron) is proportional to $\cos(\phi)$, where ϕ is the relative phase of the laser light wave and the electron wiggling trajectory in the undulator. Electrons, entering the undulator at different times, interact with the laser light wave at different phases, which produces the modulation of electron energies.

Spontaneous radiation of electrons in the undulator is equal [2]

$$A_R \simeq \pi\alpha\hbar\omega_L \frac{K^2/2}{1 + K^2/2}, \qquad (2)$$

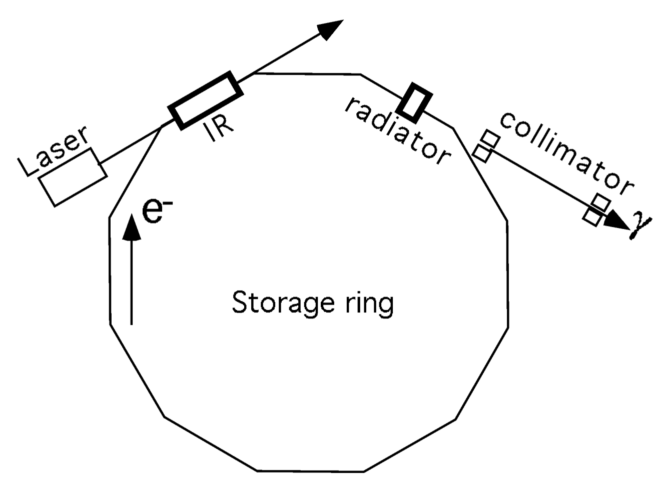

FIG. 1. Schematic of the generation of the femtosecond pulses of synchrotron radiation.

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where $\hbar$ is the Planck's constant, α is the fine structure constant, $\omega_L = 2\pi c/\lambda_L$, and c is the speed of light. Therefore, according to Eq. (1), an amplitude of energy modulation ΔE is written

$$(\Delta E)^2 \simeq 4\pi\alpha A_L \hbar\omega_L \frac{K^2/2}{1 + K^2/2} \frac{\Delta\omega_L}{\Delta\omega_R}. \tag{3}$$

As follows from Eq. (3), ΔE has a weak dependence from K. Thus one can use the relatively short IR even in a high energy ring by using the undulator with a large K.

Assuming $K > 1$ and using approximations $\Delta\omega_R/\omega \simeq 1/M_u$ [2] and $\Delta\omega_L/\omega \simeq 1/M_L$, where M_u is the number of undulator periods and M_L is the number of waves in the laser pulse, we get

$$(\Delta E)^2 \simeq 4\pi\alpha A_L \hbar\omega_L \frac{M_u}{M_L}. \tag{4}$$

Formula (3) is written in the assumption of $M_u \leq M_L$.

An electron slips over M_u laser waves in the undulator with M_u periods. Therefore, one needs $M_u \cong M_L$ to allow for electrons to slip over the entire laser pulse from the front end to the back end. For the same reason an undulator with $M_u > M_L$ does not result in the larger ΔE, since after the electron slips over the laser pulse it does not interact with the light any further.

Transverse separation of electron orbits naturally occurs when energy modulated electrons proceed into a region with a nonzero dispersion function. With a large ΔE, one can reach a condition when a population of electrons at large transverse coordinates is dominated by the electrons from the bunch slice that had interacted with the light.

Finally, by selecting synchrotron radiation of electrons from transverse bunch tails, we can get radiation pulses with a desired timing structure. The pulses will have a high intensity peak whose duration is $\tau_S = M_L\lambda_L/c$ and a low intensity pedestal whose duration is $\tau_b = \ell_b/c$, where ℓ_b is the bunch length. Depending from a wavelength of the radiation used in the experiment, such a selection can be performed either by a collimator system or by optical means.

As a source of synchrotron radiation (radiator) one can use an undulator or a bending magnet. The actual choice depends from the application of the described method.

Intensity of synchrotron radiation.—The number of electrons per second involved in the interaction with the light pulses is $N_b f_L \tau_S/\tau_b$, where N_b is the number of bunch electrons and f_L is the interaction frequency. Only a fraction of these electrons will interact with the light near the optimal phase of the electric field oscillations and get energy kicks suitable for creating the large transverse offsets. We will define this fraction with a symbol η. Typically $\eta \simeq 0.2$; an explicit expression for η will be given later.

There are two main factors limiting f_L and, accordingly, the average intensity of femtosecond radiation. The first factor is related to the growth of the beam energy spread in the storage ring. Each time the electron bunch interacts with the light pulse, a large fraction of electrons from a bunch slice gets a strong energy kick. Beam stability requires that these electrons are mixed within the bunch before the next interaction of the same bunch with the light. Consequently, the energy kicks can be considered as random kicks leading to a growth of the beam energy spread with a growth time τ_g defined as

$$\frac{1}{\tau_g} = \frac{p^2\tau_S}{2\,\tau_b}\frac{f_L}{n}. \tag{5}$$

Here n is the number of bunches and p is normalized amplitude of energy modulation $p = (\Delta E/E)/\sigma_e$, where E is the equilibrium beam energy and σ_e is the equilibrium relative beam energy spread.

Synchrotron radiation damping will limit energy growth, thus the equilibrium energy spread can be defined as follows:

$$\sigma_e^2 = \sigma_{e0}^2 + \sigma_e^2 \frac{\tau_D}{\tau_g}, \tag{6}$$

where σ_{e0} is the beam energy spread in the storage ring without beam interaction with light pulses and τ_D is the damping time.

At this point we should mention that the limitation of f_L described above is not inevitable and can be overcome in a more elaborate scheme. In fact, consider a storage ring with two identical interaction regions (one downstream to another with a radiator in between) where the electron beam interacts with the light. Assume that these IR's are separated by the isochronous lattice. Then, energy modulation of electrons in the first IR can be compensated in the second IR if the relative phase of the light waves in these IR's is adjusted to 180°. Since energy losses in the bending magnets between the IR's are small, this compensation can be fairly accurate.

Assuming that Eq. (6) defines the maximum energy spread acceptable for a normal machine operation, we can find τ_g from Eq. (6) and maximum permissible f_L from Eq. (5). Now, we can calculate the flux of photons radiated by $\eta N_b f_L \tau_S/\tau_b$ electrons in the bending magnet from a segment of the circular trajectory in the bandwidth $\Delta\lambda/\lambda$ [3]:

$$\frac{d\mathcal{F}(\lambda)}{d\theta} = \frac{\sqrt{3}}{\pi}\alpha\,\gamma\,\frac{\eta I}{p^2 e f_0 \tau_D}\left[1 - \left(\frac{\sigma_{e0}}{\sigma_e}\right)\right]^2 F\left(\frac{\lambda_c}{\lambda}\right)\frac{\Delta\lambda}{\lambda}. \tag{7}$$

Here $d\mathcal{F}(\lambda)/d\theta$ is the photon flux (number of photons per second and per unit of the azimuthal angle θ), I is the average beam current, e is the electron charge, f_0 is the revolution frequency, λ_c is the critical wavelength of radiation, and F is the spectral function $F(y) = y\int_y^\infty K_{5/3}(\xi)d\xi$, where $K_{5/3}$ is the modified Bessel function of the second kind. The photon flux for an undulator radiation is defined by an expression very similar to Eq. (7).

The second factor limiting f_L and, accordingly, the average intensity of the femtosecond radiation is related to the available laser power. For a given laser pulse energy A_L, the available output power of the optical amplifier P_L defines the maximum interaction frequency $f_L = P_L/A_L$. If this frequency is less than the frequency defined by the growth of the beam energy spread, then the photon flux is defined by

$$\frac{d\mathcal{F}(\lambda)}{d\theta} = \frac{\sqrt{3}}{2\pi}\,\alpha\,\gamma\,\frac{\eta\,I}{n\,ef_0}\,\frac{\sigma_\tau c}{\sigma_z}\,\frac{P_L}{A}\,F\left(\frac{\lambda_c}{\lambda}\right)\frac{\Delta\lambda}{\lambda}\,, \qquad (8)$$

and the beam energy spread in Eq. (6) must be evaluated using the actual f_L.

Pulse duration.—What is the minimum duration of the synchrotron radiation pulses that is achievable with the discussed technique? Of course, it is defined by the light pulse shape, but there is also a stretching of the electron bunch slice on the way from the IR to the radiator and in the radiator itself. This process is related to the nonisochronisity of the beam line and can be characterized by the differences of the path lengths of electron trajectories:

$$\Delta\ell = (I_U^2\sigma_x^2 + I_V^2\sigma_{x'}^2 + I_D^2\sigma_e^2)^{1/2}, \qquad (9)$$

where $\Delta\ell$ is the rms spread of the path lengths, σ_x and $\sigma_{x'}$ are the horizontal beam size and divergence of the electron beam in the IR, and I_U, I_V, I_D are the following integrals:

$$I_U = \int_0^\ell \frac{U(z)}{\rho}\,dz,\; I_V = \int_0^\ell \frac{V(z)}{\rho}\,dz,$$
$$I_D = \int_0^\ell \frac{D_x(z)}{\rho}\,dz\,, \qquad (10)$$

where $U(z)$ and $V(z)$ are the two independent cosinelike and sinelike solutions of the homogeneous equation of the electron motion, ρ is the bending radius, ℓ is the distance from the IR to the radiator, and D_x is the dispersion function [4]:

$$D_x(z) = V(z)\,I_U(z) - U(z)\,I_V(z)\,. \qquad (11)$$

All three integrals I_U, I_V, and I_D can vanish simultaneously only if taken over the achromat lattice [4]. But the lattice from the IR to the radiator cannot be the achromat, since a nonzero dispersion in the radiator is the integral part of the method. In a nonachromat lattice either I_U or I_V will be nonzero and using Eqs. (10) and (11) the associated path length difference can be estimated as

$$\Delta\ell \simeq \frac{\epsilon_x}{\sigma_e}\,, \qquad (12)$$

where ϵ_x is the horizontal beam emittance.

Notice that the pulse stretching can be significantly smaller if a vertical dispersion can be used in the radiator instead of the horizontal dispersion. Then the pulse stretching will be defined by the vertical beam emittance which is typically much smaller than the horizontal one.

Signal and background.—We define as a background a synchrotron radiation propagated to the experimental area emitted by the electrons not belonging to the bunch slice. If the bunch frequency is larger than the frequency of the light pulses, then a gated input, triggered with the light pulse frequency, should be used. Then, the signal-to-background ratio $\mathcal{R}$ will be the intensity of the synchrotron radiation integrated over the femtosecond pulse divided by the intensity of the synchrotron radiation integrated over the bunch length except the femtosecond pulse.

Electrons contributing to the background radiation are electrons that appear in the transverse beam tails due to the natural distribution of the beam particle density. Therefore, a better $\mathcal{R}$ will correspond to conditions with larger offsets of electrons in the bunch slice and, correspondingly, larger amplitudes of the energy modulation of electrons by the light. But, larger modulation amplitudes correspond to a lower flux of the femtosecond radiation because $d\mathcal{F}/d\theta \sim 1/p^2$. Therefore, the actual value of the modulation amplitude will depend upon the tolerance to the background in the experiment utilizing the femtosecond synchrotron radiation.

Example.—To illustrate the above-described technique, consider the generation of femtosecond x-ray pulses at the Advanced Light Source (ALS) of Lawrence Berkeley National Laboratory [5]. Beam parameters for this storage ring are listed in Table I.

As a radiator, we considered the superconducting bending magnet with 5 T magnetic field currently under development for the ALS [6]. The critical wavelength of the synchrotron radiation in this magnet at 1.5 GeV is 1.6 Å.

For the interaction region, we chose one of the ALS undulators, which has 19 periods, period length $\lambda_u = 16$ cm, and adjustable undulator parameter K in the range

TABLE I. Summary of the ALS beam parameters.

Beam energy	E (GeV)	1.5
Revolution frequency	f_0 (MHz)	1.5
Total beam current	I (A)	0.4
Number of bunches	n	40
Damping time	τ_D (ms)	10.7
Energy spread	σ_{e0}	8×10^{-4}
Bunch length	ℓ_b (cm)	0.8
Horizontal emittance	ϵ_x (nm rad)	4
Vertical emittance	ϵ_y (nm rad)	0.1
	Beam parameters in the radiator	
Total horizontal beam size	σ_x (mm)	0.13
Dispersion function	D_x (m)	0.13
Dispersive beam size	σ_{xs} (mm)	0.10
Vertical beam size	σ_y (mm)	0.012
	Beam parameters in the IR	
Horizontal beam size	σ_x (mm)	0.2
Horizontal divergence	$\sigma_{x'}$ (mrad)	2×10^{-2}
Vertical beam size	σ_y (mm)	0.04
Dispersion function	D_x (m)	0

of K =1–30 [7]. (We need $K \simeq 13$ for first harmonic radiation at $\lambda_L = 0.8\ \mu$m.)

We further assume a FWHM duration of 50 fs for the laser pulse. Then $M_L \simeq M_u$.

Assuming that the amplitude of electron's energy modulation produced in the interaction with the laser light must be four and a half times bigger than the equilibrium beam energy spread, i.e., $p = 4.5$, and considering the upper limit for a growth of the beam energy spread being $\sigma_e/\sigma_{e0} = 1.5$, we calculate (i) required energy of the light pulse $A_L = 75\ \mu$J and (ii) laser repetition rate $f_L = 115$ kHz. Thus the average power of the laser beam is 8.6 W. Lasers with the 1 W average beam power are widely available [8] and current prospects are that lasers with 10 W average beam power will be available soon. However, since there is no energy transfer from the light beam to the electron beam [see Eq. (1)], the laser requirements can be relaxed if each light pulse will be reused for $\sim 10-20$ interactions. In this case, the whole interaction region of electrons with the light must be placed in the optical cavity.

For calculations of the photon flux and the background we assume a condition when the actual size of the radiation source seen in the experiment is limited to one σ_x. Then we obtain

$$\eta = \frac{1}{\pi}\cos^{-1}\left(1 - \frac{\sigma_x}{2p\sigma_{xs}}\right) \simeq 0.15 \tag{13}$$

and a signal-to-background ratio of

$$\mathcal{R} = \frac{2\eta\tau_S}{\tau_b}\left[\Phi\left(\frac{p\sigma_{xs}}{\sigma_x} + \frac{1}{2}\right) - \Phi\left(\frac{p\sigma_{xs}}{\sigma_x} - \frac{1}{2}\right)\right]^{-1} \simeq 2\,, \tag{14}$$

where Φ is the error function and the Gaussian particle density distribution is implied.

Finally, with all the above parameters we find an x-ray flux at the critical wavelength of

$$\frac{d\mathcal{F}(\lambda)}{d\theta} = 5\times 10^9\frac{\Delta\lambda}{\lambda}\ (\text{x-rays/sec/mrad})\,. \tag{15}$$

For the pulse duration calculation, we must know I_U, I_V, and I_D, which in the ALS are -0.012, -0.39 m, and -0.0045 m, respectively. Therefore, the electron bunch slice stretches in a $\Delta\ell \simeq 25\ \mu$m between the IR and the radiator. Thus, the FWHM duration of the x-ray pulses will be

$$\tau_{\text{x-ray}} = \sqrt{\tau_S^2 + (\Delta\ell/c)^2} \simeq 100\ \text{fs}\,. \tag{16}$$

High frequency chopper.—The above described method is also suitable for extraction of electrons from a storage ring in an ultrashort series of microbunches with a well-defined period. Here one uses a septum magnet instead of the radiator in a high dispersion region. Then, electrons with a large energy deviation ($p \simeq 10$) from the equilibrium energy can be shifted into the septum. Of course, only electrons which were grouped around the optimal phase of the electric field in the IR will get necessary energy kicks to be shifted into the septum. (We will call such groups of electrons microbunches because they are much shorter than the light wavelength.) Other electrons remain inside the main storage ring vacuum chamber or hit the septum knife. As we know, electrons that were within one microbunch in the IR do not remain there when they appear in the septum (due to nonisochronisity of the lattice between the IR and septum). But, the temporal structure of microbunches can be restored after the septum in the extraction beam line. Since only microbunches are extracted, then at the end of the extraction beam line we can get a train of microbunches of electrons separated by λ_L. The overall duration of the electron macropulse will be as short as the duration of the light pulse.

In conclusion, we have proposed the method to generate femtosecond pulses of the synchrotron radiation. The method involves the interaction of the electron beam with femtosecond light pulses and is substantially based on the availability of lasers capable to the high energy light pulses. The method utilizes progress in femtosecond laser technology, currently achieved in the visible light spectrum, and allows its extension to a much broader spectral range accessible with synchrotron radiation.

As an example, we considered the generation of the femtosecond x-ray pulses at the Advanced Light Source. We have shown that the flux of the x rays in this storage ring is approximately $d\mathcal{F}/d\theta = 5\times 10^9\Delta\lambda/\lambda$ x-rays/sec mrad and that the FWHM duration of x-ray pulses is approximately 100 fs.

We also suggest that the same method can be used for an extraction of electrons from a storage ring in an ultrashort series of microbunches spaced by the periodicity of a light wavelength.

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