Up-Down Quark Mass Difference Effect in Nuclear Many-Body Systems

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A charge-symmetry-breaking nucleon-nucleon force due to the up-down quark mass difference is evaluated in the quark cluster model. It is applied to the shell-model calculation for the isovector mass shifts of isospin multiplets in $1s0d$-shell nuclei. We find that the contribution of the quark mass difference effect explains the systematic behavior of experiment. This contribution is large and may explain the Okamoto-Nolen-Schiffer anomaly, alternatively to the meson-mixing contribution, which is recently predicted to be reduced by the large off-shell correction.

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A charge-symmetry-breaking (CSB) term is required in the nuclear force for explaining several phenomena, e.g., the difference between the proton-proton and neutron-neutron scattering lengths [1] and the anomaly of mass differences of several mirror nuclei, called the Okamoto-Nolen-Schiffer (ONS) anomaly [2–5]. These experimental data imply that the nuclear force between two neutrons is slightly more attractive than between two protons.

Three of the present authors have made an extensive analysis of isovector mass shifts and isospin-mixing matrix elements in $1s0d$-shell nuclei. It was shown that experimental values of these quantities are well explained [6] by a short-range CSB force, but not by a long-range force. According to these findings, we look for the origin of a short-range CSB force in this Letter.

The experimental isovector mass shift $b(\nu, T)$ is calculated by letting the isospin multiplet mass equation,

$$E(\nu, T, T_z) = a(\nu, T) + b(\nu, T)T_z + c(\nu, T)T_z^2,$$

reproduce the masses, $E(\nu, T, T_z)$, of the $2T + 1$ members of the nuclear isospin multiplet. In the above equation, ν represents the quantum numbers other than the isospin T and its third component T_z.

Several well-known CSB sources have contributions to the isovector mass shifts. Before considering the nuclear-CSB force, we subtract the contributions of the electromagnetic interactions (EM) and of the isovector single particle energy (ISPE). They are calculated by the analysis of $1s0d$-space shell model [6,7]. The EM contribution is evaluated by taking into account the Coulomb force between the protons with the charge form factor correction, the electromagnetic spin-orbit force, and the magnetic spin-spin contact interaction with the magnetic form factor of the nucleon. The explicit formulas are given in Ref. [6]. The ISPE represents the CSB interactions between the valence nucleon and the ^{16}O core. Subtracting them, we obtain the "experimental" data, shown in Fig. 1, which are to be compared with the nuclear-CSB contribution.

We find systematic behavior, called "zigzag behavior," i.e., that the experimental values of the isodoublets are reduced for $A = 4n + 1$, while for $A = 4n + 3$ they are enhanced. It is demonstrated in the previous studies [6,7] that a short-range CSB force is the most probable source of the zigzag behavior.

A short-range nuclear-CSB force can be provided by the exchange of the mixed ρ-ω complex. It is extremely short ranged, and explains at least half of the zigzag behavior [7]. The meson-mixing potential is also used to explain other phenomena, e.g., the ONS anomaly [8–10]. Goldman, Henderson, and Thomas [11] argued, however, that an off-shell correction reduces the meson-mixing amplitude by a large factor. The correction is so large as to eliminate the meson-mixing contribution. Other calculations [12–14] also verify such an off-shell effect. It is, however, still controversial, because another analysis indicates the strong off-shell effect being inconsistent with the observed q^2 dependence of ρ-γ^* coupling [15]. Further studies seem to be necessary.

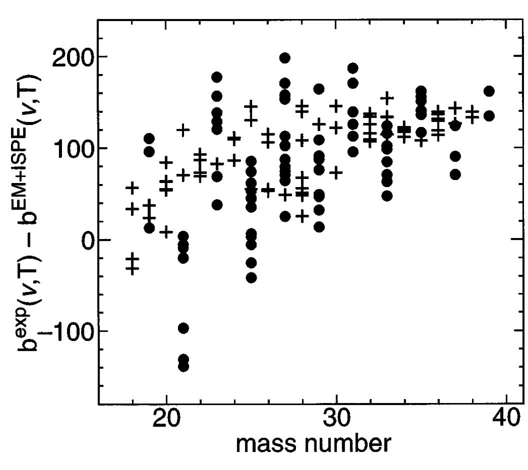

FIG. 1. The experimental isovector mass shifts [22–24] after subtracting the electromagnetic contributions and the isovector single-particle-energy contribution in keV. The isodoublets ($T = 1/2$) are denoted by the filled circles, and the others ($T > 1/2$) by crosses.

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In the present Letter, we assume that the meson-mixing contribution is negligible, and examine another short-range CSB force due to the up-down quark mass difference, called the quark effect (QE). Precisely speaking, it is a contribution of the quark mass difference in the gluon-exchange interaction between the valence quarks, while the meson mixing may contain quark mass difference effects as well [1]. Such a quark CSB force has been used to explain the difference between the proton-proton and neutron-neutron scattering lengths [16] and the ONS anomaly [17]. In this study, we apply the quark CSB force to the shell-model calculation for the mass shifts of nuclear isospin multiplets.

The quark CSB potential is calculated in the nonrelativistic potential quark model of baryons [16]. The model consists of the standard interaction Hamiltonian, which contains a quark confining potential as well as one-gluon exchange interaction. The confining potential is assumed to be independent of isospin, i.e., flavor of the quarks [1,16]. The one-gluon exchange interaction contains an isospin-symmetry-breaking term in the hyperfine contact interaction,

$$H_{\mathrm{HC}}^{(q_i q_j)} = -(\lambda_i \cdot \lambda_j) \frac{\pi \alpha_s}{6 m_i m_j} (\vec{\sigma}_i \cdot \vec{\sigma}_j) \delta^{(3)}(r_{ij}), \quad (1)$$

where λ_i, $\vec{\sigma}_i$, and m_i are the color SU(3) generator, Pauli spin matrix, and mass of the constituent quark q_i, and α_s is the strong coupling constant. Other terms are estimated to have negligible contributions to the isospin-dependent nucleon-nucleon (*NN*) interaction [16]. This contact interaction yields a short-range *NN* force which has the range of the nucleon size.

The two-nucleon system is represented by a quark-cluster wave function composed of two three-quark clusters [16]. The internal wave function of each nucleon is approximated by a Gaussian, and the internal variables are integrated out to obtain the *NN* potential from the quark-quark interaction. The obtained potential depends on the *NN* relative coordinate as well as the spin and isospin quantum numbers.

It is found in the calculation of Chemtob and Yang (see Fig. 2 of Ref. [16]) that the local term of the hyperfine contact interaction is the leading term and represents the whole CSB interaction approximately. In this Letter, therefore, we deal only with the local term of the hyperfine contact interaction, and the other terms are neglected.

In order to apply the potential to the shell-model calculation, we have to prepare the potentials for higher partial waves than the *S* states. They are numerically calculated [18], and we obtain a quark CSB potential,

$$v_{\mathrm{QE}}^{(N_1 N_2)} = \sqrt{\frac{3}{\pi}} \frac{\beta^3 \alpha_s}{\hat{m}^2} \exp\left(-\frac{3}{4}\beta^2 r_{12}^2\right) \times \frac{1}{4}[\tau_z(1) + \tau_z(2)] \frac{\delta m}{\hat{m}} \left[1 - \frac{5}{27} \vec{\sigma}(1) \cdot \vec{\sigma}(2)\right], \quad (2)$$

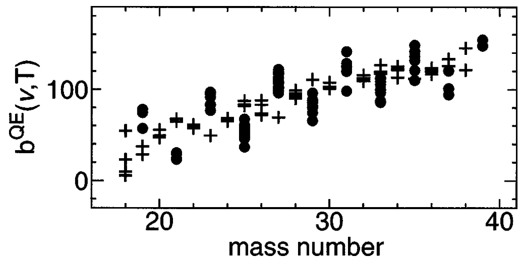

FIG. 2. The quark contribution to the isovector mass shifts in keV. Notations are the same as in Fig. 1.

where β^{-1} is the nucleon size parameter of the three-quark cluster, and r_{12} is the distance between the centers of the two clusters, $\tau_z(i)$ is the third component of the Pauli matrix for the nucleon isospin, and $\vec{\sigma}(i)$ is for the nucleon spin. $\delta m = m_d - m_u$ is the up-down quark mass difference and $\hat{m} = \frac{1}{2}(m_d + m_u)$ is the average of the masses. The input parameters are taken from Ref. [16], $\delta m = 6$ MeV, $\hat{m} = 330$ MeV, $\alpha_s = 1.624$, and $\beta^{-1} = 0.616$ fm.

The nuclear matrix element of the QE potential gives the isovector mass shift,

$$b^{\mathrm{QE}}(\nu, T) = \frac{1}{\sqrt{(2T+1)T(T+1)}} \langle \nu, T \| v_{\mathrm{QE}} \| \nu, T \rangle,$$

in the first order perturbation, where the matrix element $\langle \nu, T \| v_{\mathrm{QE}} \| \nu, T \rangle$ is reduced with respect to the isospin.

The nuclear wave function $|\nu, T\rangle$ is calculated [6,7] with Wildenthal's effective Hamiltonian [19] in the complete $1s0d$-shell space. Because the short-range QE potential is integrated, the calculation is sensitive to the short-range structure of the relative wave function of the two nucleons. We include a short-range correlation by a correlation function of Ref. [20] multiplied to the relative two-nucleon wave function.

The calculated QE contributions are shown in Fig. 2. The QE contributions are around 100 keV, and the average ratio of QE/Coulomb contributions for 143 multiplets is 5.3%. This result is consistent with the expected contribution which is introduced phenomenologically to explain the ONS anomaly in literature [4,5], and consistent with the previous calculation of the quark effect for the anomaly [17].

The zigzag behavior seen in the experimental mass shifts (Fig. 1) is also reproduced in the calculation (Fig. 2). Namely, the quark contributions are larger for the $A = 4n + 3$ isodoublets, and smaller for $A = 4n + 1$. This behavior is due to the short-range nature of the quark CSB force [6].

It should be noted that the QE contribution is not directly compared with the experimental data. The ISPE used in Fig. 1 is determined by the χ^2 fitting to data [7]. One may wonder whether the zigzag behavior has come

TABLE I. Isospin-mixing matrix elements in keV and their decompositions into the contributions of quark effect (QE), electromagnetic interactions (EM), and others. The EM consists of a large Coulomb contribution plus other small contributions, and contains the isotensor terms of the electromagnetic interactions. "Other" consists of the ISPE contribution and the isotensor contribution due to the pion mass difference [25]. Experimental values are extracted from the strengths of isospin-forbidden beta decays of the given initial states [26–30].

Initial state	^{19}O	^{20}F	^{24}Al^m	^{24}Na	^{24}Al	^{27}Mg	^{28}Mg
QE	−12.2	11.5	7.4	5.5	36.8	2.6	−8.2
EM	3.3	8.3	10.0	6.2	43.3	−16.0	−19.2
Other	25.5	−11.6	−8.3	−3.3	−21.6	41.1	67.2
Total	16.6	8.2	9.1	8.5	58.5	27.8	39.7
Experiment	20(10)	14^{+29}_{-14}	49(5)	5.4(22)	106(40)	$3.6^{+57}_{-3.6}$	20.6(16)

spuriously from the fitting process. This is not the case. We confirmed in Refs. [6] and [7] that the original data before the EM and ISPE subtraction also show the same behavior. The zigzag behavior of the experimental data is real, not spurious.

The quark CSB potential of Eq. (2) is also applied to off-diagonal matrix elements, i.e., isospin-mixing matrix elements. The results are given in Table I, with experimental values extracted from the strength of the isospin-forbidden beta decays. We find that the calculated matrix elements agree with most of the experimental values.

The quark contribution is found to be large in the off-diagonal matrix elements, comparable with the electromagnetic contributions (EM) in Table I, which are roughly equal to the contributions of the Coulomb force. By contrast, the mass shifts are dominated by the Coulomb contributions. Such large contributions to the off-diagonal matrix elements can be attributed to the short-range feature of the quark CSB force [6,21].

In particular, the beta decay of ^{24}Al (Table I) indicates a large isospin mixing in an excited state of the daughter nucleus ^{24}Mg. Our calculation also gives a large QE contribution to this matrix element, and thus is consistent with experiment.

In spite of the above successful agreements with the experiments, we are concerned that the quark model parameters have large ambiguities. Reference [1] gives three parameter sets. Models 1 and 2 use small values of δm (=4.67 and 4.21 MeV). It brings on a large reduction of the quark contributions [see Eq. (2)]. Also the small $\alpha_s = 1.08$ of model 2 causes a reduction. On the other hand, model 3 predicts an enhancement. The large $\alpha_s = 3.7$ cancel out the reduction due to the small $\delta m = 2.85$ MeV, and the large nucleon size parameter, $\beta^{-1} = 0.82$ fm makes a longer-range CSB force and brings on an enhancement. Then our result is that the quark effect explains experiment within the uncertainty of the model.

In summary, the isovector mass shifts and isospin-mixing matrix elements relevant to the isospin-forbidden beta decays are calculated considering the effect of the up-down quark mass difference in the direct gluon-exchange process. The CSB nucleon-nucleon potential due to the quark mass difference is constructed with the quark cluster model, and is applied to the shell model calculation of the $1s0d$-shell space.

The quark contributions are found to be about 5% of the Coulomb contributions to the isovector mass shifts of the nuclear isospin multiplets. It is consistent with the expected contributions [4,5] to explain the anomaly of mass differences of the mirror nuclei, known as the Okamoto-Nolen-Schiffer anomaly. Also, the quark contribution is found to have a systematic behavior in the contributions to the mass shifts of the isospin doublets; i.e., for $A = 4n + 1$ the mass shifts are reduced, while for $A = 4n + 3$ they are enhanced. This behavior is consistent with the experimental one.

The calculated values of the isospin-mixing matrix elements are consistent with most of the experimental values extracted from the isospin-forbidden beta decays. The quark CSB force is found to have large contributions to the isospin-mixing matrix elements, comparable with the Coulomb contributions. A particularly large value of the experimental matrix element of ^{24}Mg is found to be explained by the large quark contribution. These findings strongly suggest the existence of a short-range CSB interaction.

The quark mass difference effect is, of course, not a unique possible source of the short-range CSB force. However, independently of the other CSB source, it is concluded that the quark effect has a considerable contribution to the observables, and has a favorable feature for explaining the known data in the nuclear many-body systems.

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