Long-Range Fluctuation-Induced Attraction of Vortices to the Surface in Layered Superconductors

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It is shown that in extremely anisotropic layered superconductors the interaction of vortex lines with a parallel planar surface, which for straight lines along the c axis decreases exponentially over the in-plane penetration depth λ, becomes a long-range dipole-dipole attraction when the vortex line is distorted randomly. This novel long-range fluctuation-induced attraction enhances the thermal fluctuations down to depths much larger than λ and may lead to flux creep towards the surface.

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Abrikosov vortex lines in layered superconductors have several unusual properties as compared to vortex lines in isotropic or weakly anisotropic superconductors. In particular, vortex lines oriented perpendicular to the superconducting layers may be considered as a stack of two-dimensional point vortices or pancakes [1–3]. In the case of large anisotropy the pancakes interact via a magnetic pair potential, which parallel to the layers decreases logarithmically and perpendicular to the layers decreases exponentially. Most importantly, the interaction of pancakes within the *same* layer is *repulsive* while between different layers it is *attractive* and reduced by a factor $s/2\lambda \ll 1$, where s is the layer spacing and $\lambda = \lambda_{ab}$ is the penetration depth for the currents in the layers. As a consequence, the interaction of two straight stacks of pancakes at distance $\rho \gg \lambda$ decays as $\exp(-\rho/\lambda)$ since at long distances the attraction and repulsion between the pancakes from both stacks compensates almost exactly. Thus, the interaction of two straight stacks is just the usual short-range repulsion of Abrikosov vortices.

The repulsive and attractive interaction of point vortices has a further important consequence, which to our knowledge has not been pointed out previously, namely, the interaction of a distorted pancake stack with a surface that is parallel to the stack. Within the linear London theory the condition of zero perpendicular current through a planar specimen surface may be satisfied by adding the magnetic fields and currents of image vortex lines [4–7]. Each vortex then is attracted to its image since the images have opposite orientation (antivortices). However, this short-range attraction applies only when the vortex line is *perfectly straight.* As soon as the vortex line is distorted, the compensation of repulsive and attractive terms in the vortex-vortex interaction is no longer ideal. As a consequence, randomly distorted vortex lines feel a *long-range attraction to the surface.*

In this paper we derive this novel long-range attraction and discuss some of its consequences. We show that the result of the isolated-layer model exactly coincides with the result of anisotropic London theory in the limit $\lambda_c \gg \lambda$, where λ_c is the penetration depth for currents perpendicular to the layers. The long-range attraction is, therefore, also present in superconductors with finite anisotropy, where it is proportional to $x^{-2}\exp(-2x/\lambda_c)$ for long distances x between vortex and surface. Since in high-T_c superconductors λ_c typically is a macroscopic length, this surface attraction is really of long range.

We first give a simple physical interpretation of the long-range fluctuation-induced attraction. Assume that only one of the pancakes of a straight stack is displaced by a small distance u away from the surface. This local distortion is formally described by adding a pancake at the new position $x + u$ and an antipancake at the equilibrium position x, which annihilates the original pancake. The same procedure has to be done with the image stack situated at the position $-x$ if the surface is at $x = 0$ (see Fig. 1). The two pancake-antipancake pairs are dipoles with a strength proportional to the displacement u and a dipole-dipole interaction energy proportional to u^2/x^2, where $1/x^2$ is the second derivative of the pancake-pancake potential $(\ln x)$. Therefore, the distorted vortex line is attracted to its image, and thus to the planar surface, by a long-range potential proportional to u^2/x^2. This long-range interaction of each displaced point vortex is in addition to the short-range interaction of a perfectly straight vortex line with its image, which is proportional to $K_0(2x/\lambda) \propto \exp(-2x/\lambda)$, where $K_0(x)$ is a modified Bessel function.

The interaction of a distorted vortex line with its image may be calculated from the interaction of pancakes separated by $\mathbf{r}_{mn} = (x_m - x_n, y_m - y_n, z_m - z_n) = (x_{mn}, y_{mn}, z_{mn})$ [8,9],

$$\mathcal{E} \approx \epsilon_0 \begin{cases} 2\ln(\rho_{mm}/\xi)\,, & n = m\,, \\ -\frac{s}{2\lambda}\exp\left(-\frac{|z_{mn}|}{\lambda}\right)\ln\left(\frac{\rho_{nm}}{\lambda}\right), & n \neq m\,, \end{cases} \quad (1)$$

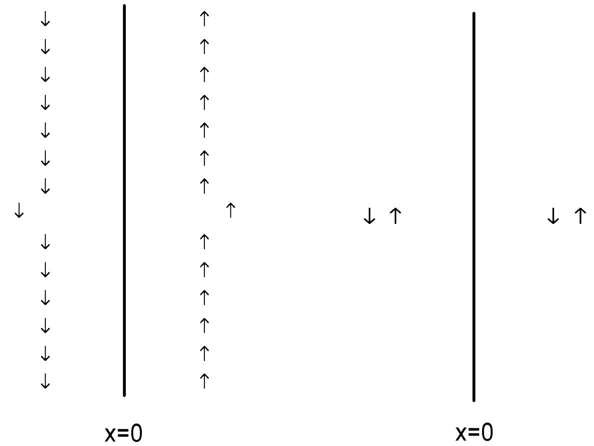

FIG. 1. Left: A distorted vortex line and its image composed of pancakes (↑) and antipancakes (↓). Right: The two dipoles generated by the displacement cause a long-range attraction between the distorted vortex and the surface (indicated by a vertical bold line).

with $\epsilon_0 = s\Phi_0^2/4\pi\mu_0\lambda^2$, $\rho_{mn}^2 = x_{mn}^2 + y_{mn}^2$, and $z_m = ms$ where s is the layer spacing and ξ the core radius of the pancake. For $z_m = z_n$ Eq. (1) applies to all distances $\rho_{mn} > \xi$, but for $z_m \neq z_n$, $\rho_{mn} \gg \lambda$ was assumed. The total energy of a vortex line is composed of its self-energy and the interaction with its image line of opposite orientation, namely,

$$\frac{1}{2}\sum_m \sum_n [\mathcal{E}(x_m - x_n, y_m - y_n, z_m - z_n) - \mathcal{E}(x_m + x_n, y_m - y_n, z_m - z_n)], \quad (2)$$

where the sums are over the pancakes of the *real* vortex in the superconducting half space $x > 0$, thus $x_m, x_n > 0$. For a distorted vortex line parallel to the surface $x = 0$ we define pancake displacements $\mathbf{u}_m = \mathbf{u}_m(z_m) = (u_{xm}, u_{ym})$ by writing $x_m = x + u_{xm}$ and $y_m = u_{ym}$. Keeping only the terms quadratic in the $\mathbf{u}_m$ we obtain from (2) the linear elastic energy of the vortex.

We first consider random and isotropic displacements with ensemble averages $\langle u_{xm}\rangle = \langle u_{ym}\rangle = 0$, $\langle u_{xm}u_{xn}\rangle = \langle u_{ym}u_{yn}\rangle = f(|m - n|)$, $\langle u_{xm}u_{yn}\rangle = 0$. The long-range interaction with the image line for $2x \gg \lambda$ then becomes from (1) and (2)

$$E_{\rm int} = -\frac{\Phi_0^2 L s}{64\pi\mu_0\lambda^3 x^2}\sum_l \exp\left(-\frac{s}{\lambda}|l|\right)\langle(\mathbf{u}_l - \mathbf{u}_0)^2\rangle, \quad (3)$$

where L is the vortex length. This fluctuation-induced interaction is attractive and depends on the relative displacements $\langle(\mathbf{u}_l - \mathbf{u}_0)^2\rangle$ over a vortex length of order λ. Like a dipole-dipole interaction it decreases as $1/x^2$. Since $s \ll \lambda$, one may approximate the sum in (3) by

$$E_{\rm int} = -\frac{\Phi_0^2 L}{32\pi\mu_0\lambda^3 x^2}\int_0^\infty dz\, \exp\left(-\frac{z}{\lambda}\right) g(z), \quad (4)$$

where $g(z)$ is the correlation function

$$g(z) = \langle[\mathbf{u}(z) - \mathbf{u}(0)]^2\rangle. \quad (5)$$

The integral (4) shows that the long-range interaction does not depend on the layer separation s.

For a calculation applying also to $\rho_{mn} < \lambda$ we use the general expression for the two-pancake interaction [8,9],

$$\mathcal{E} = -\frac{\Phi_0^2 s^2}{\mu_0}\int \frac{d^3k}{8\pi^3}\frac{1}{1 + k^2\lambda^2}\frac{k^2}{q^2}e^{i\mathbf{k}\cdot\mathbf{r}_{mn}}, \quad (6)$$

where $\mathbf{k} = (k_x, k_y, k_z)$, $\mathbf{q} = (k_x, k_y)$, $p = k_z$, thus $q^2 = k_x^2 + k_y^2$ and $k^2 = q^2 + p^2$, and as above $\mathbf{r}_{mn} = (x_{mn}, y_{mn}, z_{mn})$. Since (6) is valid for all $\mathbf{r}_{mn}$, we may obtain the total elastic energy of the distorted vortex line $E_{\rm tot} = E_{\rm self} + E_{\rm int}$ from (2) and (6). Expanding this to quadratic terms in $\mathbf{u}$, introducing the Fourier transform

$$\mathbf{u}(z_l) = \int \frac{dp}{2\pi}\tilde{\mathbf{u}}(p)e^{ipz_l}, \quad (7)$$

and using $s\sum \exp(ipz_l) = 2\pi\delta(p)$ (for $|p| \leq \pi/s$) and $\int(dp/2\pi)|\tilde{\mathbf{u}}(p)|^2 = \int dz\,\mathbf{u}(z)^2 = \langle\mathbf{u}^2\rangle L$ we obtain

$$E_{\rm tot} = \frac{\Phi_0^2}{4\mu_0}\int \frac{d^3k}{8\pi^3}|\tilde{\mathbf{u}}(p)|^2(f_{\rm self} + f_{\rm int}), \quad (8)$$

$$f_{\rm self} = \frac{k^2}{1 + k^2\lambda^2} - \frac{q^2}{1 + q^2\lambda^2}, \quad (9)$$

$$f_{\rm int} = \left(\frac{k^2}{1 + k^2\lambda^2}\frac{k_x^2}{q^2} - \frac{1}{2}f_{\rm self}\right)e^{2ik_x x}. \quad (10)$$

Remarkably, exactly the same result (8)–(10) is obtained from the anisotropic London theory in the limit $\lambda_c \to \infty$. Namely, the interaction energy of two London vortices at positions $\mathbf{r}_1(z)$ and $\mathbf{r}_2(z)$ is

$$\int d\mathbf{r}_{1\alpha}\int d\mathbf{r}_{2\beta}\, V_{\alpha\beta}(\mathbf{r}_1 - \mathbf{r}_2), \quad (11)$$

where the anisotropic interaction $V_{\alpha\beta}(\mathbf{r}_1 - \mathbf{r}_2)$ $(\alpha, \beta = x, y, z)$ is given in Ref. [10] as a Fourier integral over $V_{\alpha\beta}(\mathbf{k})$. In the limit $\lambda_c \to \infty$, the general expression $V_{\alpha\beta}(\mathbf{k}) = (\Phi_0^2/\mu_0)(1 + k^2\lambda^2)^{-1}g_{\alpha\beta}(\mathbf{k})$ simplifies to a diagonal matrix with $g_{xx} = k_x^2/q^2$, $g_{yy} = k_y^2/q^2$, and $g_{zz} = 1$ if z is along the c axis of the uniaxial superconductor. Inserting this $V_{\alpha\beta}(\mathbf{k})$ into (11) and integrating over the vortex and its image, we reproduce the result (8)–(10) of the pancake approach.

From Eq. (10) one reproduces the long-distance interactions (3) and (4). Moreover, from the self-interaction (9) the line tension P of an isolated flux line [11] or stack of pancakes is obtained,

$$P(p) = \frac{\Phi_0^2}{8\pi\mu_0\lambda^2}\frac{\ln(1 + p^2\lambda^2)}{p^2\lambda^2}, \quad (12)$$

which determines the linear elastic self-energy of a distorted vortex line,

$$E_{\rm self} = \frac{1}{2} \int \frac{dp}{2\pi} \, p^2 P(p) \, |\tilde{\mathbf{u}}(p)|^2. \quad (13)$$

In real space this energy looks similar to Eq. (3),

$$E_{\rm self} = \frac{\Phi_0^2 L}{16\pi \mu_0 \lambda^4} \sum_{l \neq 0} \exp\left(-\frac{s}{\lambda}|l|\right) \frac{\langle (\mathbf{u}_l - \mathbf{u}_0)^2 \rangle}{|l|}, \quad (14)$$

and since $s \ll \lambda$ it can be approximated as

$$E_{\rm self} = -\frac{\Phi_0^2 L}{8\pi \mu_0 \lambda^4} \int_0^\infty dz \, \exp\left(-\frac{z}{\lambda}\right) \frac{g(z)}{z}. \quad (15)$$

The similarity of (4) and (15) leads to the following useful relationship. If the correlation function (5) increases algebraically, $g(z) = \text{const} \times |z|^\gamma$, one has (still for $2x \gg \lambda$)

$$E_{\rm int} = -(\gamma \lambda^2 / 4x^2) E_{\rm self}. \quad (16)$$

A flux line diffusing in a random pinning potential may exhibit $\gamma \approx 1$ [9], thus $E_{\rm int} \approx -(\lambda^2/4x^2) E_{\rm self}$ is not a very small correction to $E_{\rm self}$.

To obtain the thermal fluctuations one has to consider the pronounced dispersion ("nonlocality") of the line tension $P \propto 1/p^2$ (12) at large $p \gg \lambda^{-1}$. This means that a single displaced vortex "feels" a parabolic potential of curvature $p^2 P(p) \approx \Phi_0^2 \ln(\alpha\lambda/s)/4\pi\mu_0\lambda^4$ (with $\alpha = 1.16$ for $s \ll \lambda$) originating from the global interaction with all pancakes within a distance of order of λ along the stack. Therefore, the self-energy (3) caused by short-wavelength fluctuations is simply

$$E_{\rm self} = \frac{\Phi_0^2 L \ln(\alpha\lambda/s)}{8\pi\mu_0\lambda^4} \langle \mathbf{u}^2 \rangle. \quad (17)$$

Each pancake only weakly feels the usual "local" (nondispersive) line tension that originates from the nearest-neighbor interaction. For long tilt wavelengths $2\pi/q \gg \lambda$, however, the line tension (12) of the pancake stack is local, $P = \Phi_0^2/8\pi\mu_0\lambda^2$, as can be seen also by inserting a uniform tilt $\mathbf{u}_l - \mathbf{u}_0 \propto l$ in (14).

In terms of the Fourier transform (7) the correlation function $g(z)$ in (4) and (15) may be expressed as

$$g(z) = \frac{2}{L} \int \frac{dp}{2\pi} \langle |\tilde{\mathbf{u}}(p)|^2 \rangle [1 - \cos(pz)]. \quad (18)$$

Writing the self-energy (11) for an isolated vortex of finite length L as a sum over discrete modes with wave vectors p_n and amplitudes $\mathbf{u}_n$ and ascribing to each mode the average thermal energy $k_B T$ (since $\mathbf{u}_n$ has two components), one obtains $\langle |\tilde{\mathbf{u}}(p)|^2 \rangle$ and with (18),

$$g(z) = k_B T \, \frac{32\pi\mu_0\lambda^4}{\Phi_0^2} \int \frac{dp}{2\pi} \frac{1 - \cos(pz)}{\ln(1 + p^2\lambda^2)}. \quad (19)$$

The function $g(z)$ is practically constant in the interval relevant in Eqs. (4) and (15). One has $g(0) = 0$ and for $|z| \gg s$ one finds $g(z) \approx c_1 + c_2|z|/\lambda$ with $c_2 \ll c_1$. For the example $\lambda/s = 100$ some values of the integral in (19), times λ, are 0, 4.71, 5.07, 5.16, 5.25, 5.29, 5.42, 5.53, 5.68, 5.84, 6.10, 6.85 for $z/s = 0, 1, 2, 3, 4, 5, 10, 20, 50, 100, 200, 500$. The constancy of g at z up to a few λ means that the thermal fluctuations of the pancakes of one vortex line are nearly *uncorrelated* at small wavelengths [3,12], and thus the cutoff length of several s, or the effective cutoff $|q| \leq \alpha/s$, required to obtain expressions like Eq. (17) *is not crucial.* This "elastic independence" of the pancakes originates from the strong dispersion of the line tension $P(p)$ (12). For $|z| \gg 2\pi\lambda$ and for a usual string with nondispersive P the thermal random walk yields $g(z) \propto |z|$.

The expression for $g(z)$ (19) was derived from the line energy $E_{\rm self}$ (13), which does not depend on the vortex distance x from the surface. The correct calculation, however, has to consider the total energy $E_{\rm self} + E_{\rm int}$. Since the interaction with the surface $E_{\rm int}$ depends on x, and the equipartition theorem requires constant energy $k_B T$ per mode, we find that the fluctuation amplitude depends on x. Explicitly we obtain from (4) and (15) for a vortex line at $x \gg \lambda/2$ the short-wavelength thermal fluctuations, i.e., the value $g(z)$ at $s < |z| \lesssim \lambda$,

$$\langle \mathbf{u}^2 \rangle \approx \frac{2\lambda^2}{\ln(\alpha\lambda/s)} \frac{k_B T}{\epsilon_0} \left[1 + \frac{\lambda^2}{4x^2} \frac{1}{\ln(\alpha\lambda/s)} \right]. \quad (20)$$

So we have the interesting result that the correction to the thermal fluctuations decreases away from the surface *only as a power law.* This increase of $\langle \mathbf{u}^2 \rangle$ originates from the softening of the flux-line lattice near the surface.

Considering u_x and u_y separately, we find that the surface-caused softening of a vortex line is indeed *isotropic* at large x but becomes *anisotropic* for $x < \lambda$. The general expression obtained from (2) and (6) shows that to a good approximation the contributions of each displaced pancake to the elastic energy add *independently.* Each pancake contribution is composed of the interaction between six objects, namely, the displaced pancake and the hole (antipancake) it leaves in the vortex line, their images in the *same* layer, the *undisplaced straight* vortex line, and its image. Explicitly one has

$$E_{\rm tot} = \frac{\Phi_0^2 L}{8\pi\mu_0\lambda^4} [a(x)\langle u_x^2 \rangle + b(x)\langle u_y^2 \rangle],$$

$$a(x) = \ln\frac{\alpha\lambda}{s} - \frac{\lambda^2}{4x^2} - K_0''\left(\frac{2x}{\lambda}\right),$$

$$b(x) = \ln\frac{\alpha\lambda}{s} - \frac{\lambda^2}{4x^2} - \frac{\lambda}{2x} K_0'\left(\frac{2x}{\lambda}\right), \quad (21)$$

with $\alpha = 1.16$. For large $x \gg \lambda/2$ one has $a = b = \ln(\alpha\lambda/s) - \lambda^2/4x^2$, and for $x \ll \lambda/2$, $a = b - \lambda^2/2x^2$, $b = \ln(\alpha\lambda/s)$, since the derivatives are $K_0'(r) = -1/r$ and $K_0''(r) = 1/r^2$ for $r \ll 1$. The thermal fluctuations

$$\langle u_x^2 \rangle = \frac{2k_B T\lambda^2}{\epsilon_0 a(x)}, \qquad \langle u_y^2 \rangle = \frac{2k_B T\lambda^2}{\epsilon_0 b(x)} \tag{22}$$

become thus *asymmetric* near the surface, with $\langle u_x^2 \rangle / \langle u_y^2 \rangle = b/a > 1$. Note that the curvature a in (21) turns negative at small x. To check the stability of a pancake stack close to the surface and to obtain the correct pictures of penetration and exit, one has to add the (always positive) stray-field energy [13,14] and the restoring force of the current density $j(x)$ that originates from the Meissner surface current and from other vortices, replacing a by $a + (4\pi\mu_0\lambda^4/\Phi_0)j'(x)$. Work in this direction is under way.

Large fluctuations and local tilt of flux lines can be caused by pinning. Random pinning *forces* on a single flux line would cause square displacements diverging proportional to the number of forces. More realistic random pinning *potentials* lead to finite vortex displacements [9,15]. For a crude estimate assume that random pins of density n_p are so strong that the vortex line wanders an average distance squared of order $(4n_p s)^{-1}$ as it passes to the next layer. This yields $g(z) \approx z/(n_p s^2)$, and the interaction (4) with the surface becomes

$$E_{\rm int} = -\frac{\Phi_0^2 L}{32\pi\mu_0\lambda^3 n_p s^2}\,\frac{\lambda^2}{x^2} = -\frac{\lambda^2}{4x^2}\,E_{\rm self}\,. \tag{23}$$

The resulting long-range force $-dE_{\rm int}/dx$ in principle may drive the vortex line to the surface. It has to be added to geometric forces which are exerted on the vortex line by surface screening currents when the specimen has a nonellipsoidal cross section and is not infinitely thick [16,17].

The elastic energy (8)–(10) of a *single* flux line may be generalized to the elastic energy of the distorted flux-line *lattice* in the half-space $x > 0$ by replacing in it $|\tilde{\mathbf{u}}(q)|^2$ by the three-dimensional Fourier transform $|\tilde{\mathbf{u}}(\mathbf{k})|^2$ of the lattice displacements [18]. The complete expression for $E_{\rm tot}$ slightly differs from our approximation (8), which applies if $\langle u_x^2 \rangle = \langle u_y^2 \rangle = \frac{1}{2}\langle \mathbf{u}^2 \rangle$ and $\langle u_x u_y \rangle = 0$. The presence of a flux-line lattice of density B/Φ_0 increases the fluctuation-induced attraction to the surface (3) when the displacements in the same layer are correlated over a radius $\rho_{\rm corr} > (\Phi_0/B)^{1/2}$. The interaction per vortex will then increase by a factor of order of $\pi\rho_{\rm corr}^2 B/\Phi_0$.

Similar expressions for the elastic energy of isotropic and anisotropic vortex lattices near a planar surface are derived in Ref. [13,14]. In isotropic superconductors a long-range attraction of the type (3) is absent, and in anisotropic superconductors the attraction is reduced by a factor of $\exp(-x/\lambda_c)$. Formally, the long range is due to the noncutoff factor $1/q^2$ in Eq. (10).

In conclusion, we have shown that in extremely anisotropic layered superconductors there is a long-range interaction between the sample surface and a distorted vortex line parallel to the surface and to the c axis. this interaction causes a spatial variation of the thermal fluctuations even at distances much larger than the in-plane London penetration depth, which might affect the melting process of the vortex lattice [19] and the evaporation of vortex lines into independent pancake vortices [3,12]. In the presence of sufficiently strong random pinning, a fluctuation-induced long-range force attracts the distorted vortex line to the surface. This additional force may lead to flux creep towards the surface.

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