Kinetic Ballooning Mode with Negative Shear

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Kinetic analysis on the ballooning mode in tokamaks has indicated the existence of a residual ballooning mode in the negative shear region ($s < 0$). The instability has a small threshold in α (the ballooning parameter), requires a finite ion temperature gradient (η_i), and is characterized by a broad eigenfunction $\phi(\theta)$ extending to $\theta \simeq 50$ in the ballooning space.

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The ideal magnetohydrodynamic (MHD) ballooning mode in tokamaks is known to be stable in discharge regions where the magnetic shear s is negative [1,2]. This may qualitatively be seen from the simplified dispersion relation for the ballooning mode,

$$\omega^2 k_\perp^2 = V_A^2 k_\parallel k_\perp^2 k_\parallel - \omega_{*p}\omega_{Di}/\rho_i^2 \,, \tag{1}$$

where V_A is the Alfvén speed, $\omega_{*p} = \omega_{*e}(1+\eta_e) + \omega_{*i}(1+\eta_i)$ is the total diamagnetic frequency, ω_{Di} is the ion magnetic drift frequency due to the magnetic curvature, and ρ_i is the ion Larmor radius. For a simple trial eigenfunction in the ballooning space θ, $\phi \simeq 1 + \cos\theta$, the norms of the differential operators are [3,4]

$$\langle k_\parallel k_\perp^2 k_\parallel \rangle_\theta \simeq \frac{k_\theta^2}{(2qR)^2}\left(1 + \frac{\pi^2 - 1.5}{3} s^2 - \frac{8}{3} s\alpha + \frac{3}{4}\alpha^2\right), \tag{2}$$

$$\langle \omega_{Di} \rangle_\theta = 2\epsilon_n \omega_{*i}\left(\tfrac{2}{3} + \tfrac{5}{9} s - \tfrac{5}{12}\alpha\right), \tag{3}$$

where $s = d\ln q/d\ln r$ is the shear parameter and α is the ballooning parameter defined by

$$\alpha = q^2 \frac{R}{L_n}[\beta_e(1+\eta_e) + \beta_i(1+\eta_i)], \tag{4}$$

with $\eta_{i,e} = d\ln T_{i,e}/d\ln n$ the temperature gradient. Then the stable-unstable boundary on which $\omega = 0$ is described by

$$1 + \frac{\pi^2 - 1.5}{3} s^2 - \frac{8}{3} s\alpha + \frac{3}{4}\alpha^2 - 4\alpha\left(\tfrac{2}{3} + \tfrac{5}{9} s - \tfrac{5}{12}\alpha\right) = 0 \,. \tag{5}$$

A negative shear $s < 0$ is clearly stabilizing. It effectively enhances the Alfvén frequency through the increase in $k_\parallel$ and reduces the interchange drive $\langle \omega_{Di}\rangle_\theta$. In more quantitative analysis, stabilization occurs at small positive s below which the ballooning mode is stable [5].

Recently, it has been shown that the MHD second stability regime at large enough α (plasma pressure gradient) is subject to kinetic ballooning modes driven by a finite ion temperature gradient η_i [6]. (The coexistence of a MHD ballooning mode and η_i driven kinetic mode in the MHD unstable regime had been noted earlier by Cheng [7].) The instability is due to the ion magnetic drift resonance,

$$\omega + \hat{\omega}_{Di}(\mathbf{v}) = 0 \,, \tag{6}$$

where

$$\hat{\omega}_{Di}(\mathbf{v}) = \frac{Mc}{eB^3}\left(\tfrac{1}{2} v_\perp^2 + v_\parallel^2\right)(\nabla B \times \mathbf{B})\cdot \mathbf{k}$$

is the velocity dependent ion magnetic drift frequency. Ideal MHD evidently overlooks such resonance in the velocity space and thus is unable to describe the kinetic instability. Two-fluid approximation [8] qualitatively recovers the instability. However, the underlying assumption of long cross-field wavelength $(k_\perp \rho_i)^2 = (k_\theta \rho_i)^2[1 + (s\theta - \alpha\sin\theta)^2] \ll 1$ tends to be violated because of broad eigenfunctions in the ballooning space θ. Using the gyrofluid approximation for the ion response [9], Nordman *et al.* [10] have identified a kinetic instability in the negative shear region, which is similar to that in the MHD second stability regime. The purpose of the present Letter is to determine the stable-unstable boundary in the (s, α) plane and asses the critical temperature gradient in terms of a more accurate fully kinetic approach.

We consider collisionless electromagnetic modes in the intermediate frequency regime such that $\omega_{Ti} < |\omega| < \omega_{Te}$, where $\omega_{Ti(e)} = k_\parallel v_{Ti(e)}$ is the ion (electron) transit frequency. In this regime, ion dynamics becomes electrostatic and ion density perturbation can be found from the gyrokinetic equation as follows:

$$n_i = -\frac{e\phi}{T_i} n_0 + \int \frac{\omega + \hat{\omega}_{*i}(v^2)}{\omega + \hat{\omega}_{Di}(\mathbf{v})} J_0^2\left(\frac{k_\perp v_\perp}{\Omega_i}\right) f_{Mi}\, d\mathbf{v}\, \frac{e\phi}{T_i} n_0 = (-1 + I_i)\frac{e\phi}{T_i} n_0 \,, \tag{7}$$

where

$$\hat{\omega}_{*i}(v^2) = \omega_{*i}\left[1 + \eta_i\left(\frac{Mv^2}{2T_i} - \frac{3}{2}\right)\right],$$

$$\omega_{*i} = \frac{cT_i}{eB^2}(\nabla \ln n_0 \times \mathbf{B})\cdot \mathbf{k} \tag{8}$$

is the energy dependent ion diamagnetic frequency, J_0 is the Bessel function, ϕ is the scalar potential, and $f_{Mi}(v^2)$

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is the Maxwellian ion distribution. The ion magnetic drift resonance is contained in the integral I_i. The electron density and parallel current perturbation in the assumed low frequency regime, $|\omega| < \omega_{Te}$, are [6]

$$n_e = \left(\phi - \frac{\omega - \omega_{*e}}{ck_\parallel} A_\parallel\right) \frac{e}{T_e} n_0 , \tag{9}$$

$$J_{\parallel e} = \frac{n_0 e^2}{k_\parallel T_e}\left((\omega_{*e} - \omega)\phi + \frac{(\omega - \omega_{*e})(\omega - \omega_{De}) + \eta_e \omega_{*e}\omega_{De}}{ck_\parallel} A_\parallel\right), \tag{10}$$

where $A_\parallel$ is the vector potential and

$$\omega_{*e} = \frac{cT_e}{eB^2}(\nabla \ln n_0 \times \mathbf{B}) \cdot \mathbf{k}, \qquad \omega_{De} = \frac{2cT_e}{eB^3}(\nabla B \times \mathbf{B}) \cdot \mathbf{k} \tag{11}$$

are the electron diamagnetic and magnetic drift frequencies. Eliminating the vector potential between the charge neutrality condition $n_i = n_e$ and parallel Ampere's law

$$\nabla^2 A_\parallel = -\frac{4\pi}{c} J_{\parallel e} , \tag{12}$$

we obtain the following mode equation:

$$k_\parallel k_\perp^2 k_\parallel \phi + \left(\frac{k_{De}}{c}\right)^2 \left[\frac{(\omega - \omega_{*e})^2}{1 + \tau - \tau I_i} - (\omega - \omega_{*e})(\omega - \omega_{De}) - \eta_e \omega_{*e}\omega_{De}\right]\phi = 0 , \tag{13}$$

where $k_{De}^2 = 4\pi n_0 e^2/T_e$ and $\tau = T_e/T_i$ is the electron/ion temperature ratio. For simplicity we assume $\tau = 1$ and $\eta_i = \eta_e$. In the tokamak magnetic geometry with shifted circular magnetic surfaces, the differential form of Eq. (13) is

$$\frac{d}{d\theta}\left([1 + (s\theta - \alpha \sin\theta)^2]\frac{d\phi}{d\theta}\right) + \frac{\alpha}{4\epsilon_n(1 + \eta)}\left((\Omega - 1)[\Omega - f(\theta)] + \eta f(\theta) - \frac{(\Omega - 1)^2}{2 - I_i(\theta)}\right)\phi = 0 , \tag{14}$$

where $\Omega = \omega/\omega_{*e}$, $f(\theta) = 2\epsilon_n[\cos\theta + (s\theta - \alpha\sin\theta) \times \sin\theta]$, and $\eta = \eta_i = \eta_e$. In numerical evaluation of the two-dimensional integral $I_i(\theta)$, the Gaussian-Hermite quadrature method [11] is used. An isothermal discharge with $T_e = T_i$ is assumed throughout this Letter.

Figure 1(a) shows both the mode frequency and growth rate normalized by $\omega_A = V_A/qR$ as functions of α when $s = -0.2$, $b_0 = (k_\theta \rho)^2 = 0.01$, $\eta_i = \eta_e = 1$, $\epsilon_n = L_n/R = 0.1$. The mode frequency ω_r is of order $V_A/2qR$ (Alfvén frequency). It may be more appropriate to call the instability a destabilized Alfvén mode to distinguish it from the MHD ballooning mode. The critical α for the instability is small, $\alpha \gtrsim 0.1$. Stabilization at large α is likely due to deactivation of the interchange drive similar to the second stabilization of the conventional MHD ballooning mode. Dependence of the growth rate on the shear parameter s is shown in Fig. 1(b) for the case $\alpha = 1$. There exists an unstable window in s, $-0.4 \lesssim s \lesssim -0.1$ for instability. Stabilization in the region $s \lesssim -0.4$ is again due to deactivation of the interchange drive.

Unstable domain in the (s, α) plane is depicted in Fig. 2(a) and dependence of the growth rate on the temperature gradient $\eta_i = \eta_e$ in Fig. 2(b). As seen in Fig. 2(b), the instability requires a finite (ion) temperature gradient, similar to the case of the kinetic ballooning mode in the MHD second stability regime. The threshold in η is modest, $\eta_i \gtrsim 0.5$ for instability. A typical eigenfunction is shown in Fig. 3. It extends to $\theta \simeq 50$ where the ion finite Larmor radius parameter, $(k_\theta \rho)^2[1 + (s\theta - \alpha\sin\theta)^2]$, becomes of order unity which necessitates the use of kinetic formulation. The eigenfunction is similar to that of the kinetic ballooning mode in the second stability regime. In both cases, eigenfunctions are broad and have peaks off the center ($\theta = 0$). The norms of the differential operators corresponding to the eigenfunction shown in Fig. 3 are

$$\langle k_\perp^2 \rangle_\theta = k_\theta^2 \int [1 + (s\theta - \alpha\sin\theta)^2]|\phi|^2 d\theta \simeq 6.3 k_\theta^2 ,$$

$$\langle k_\parallel^2 \rangle_\theta = -\frac{1}{(qR)^2}\int \phi^* \frac{d^2\phi}{d\theta^2} = \frac{1}{(qR)^2}\int \left|\frac{d\phi}{d\theta}\right|^2 d\theta \simeq \frac{0.34}{(qR)^2} ,$$

$$\langle \omega_D \rangle_\theta / 2\epsilon_n \omega_* = \int [\cos\theta + (s\theta - \alpha\sin\theta)\sin\theta]|\phi|^2 d\theta \simeq 0.21 ,$$

where ϕ is normalized such that $\int |\phi|^2 d\theta = 1$. As expected, the norm of $k_\perp^2$ is large because of the extended eigenfunction. The norm of $k_\parallel^2$ is primarily determined by local derivative of ϕ and remains of order $1/(qR)^2$.

Since the growth rate is small compared with that of the ideal MHD ballooning mode, it is important to check whether the instability may be suppressed or not by stabilizing effects. One such effect is the stabilizing

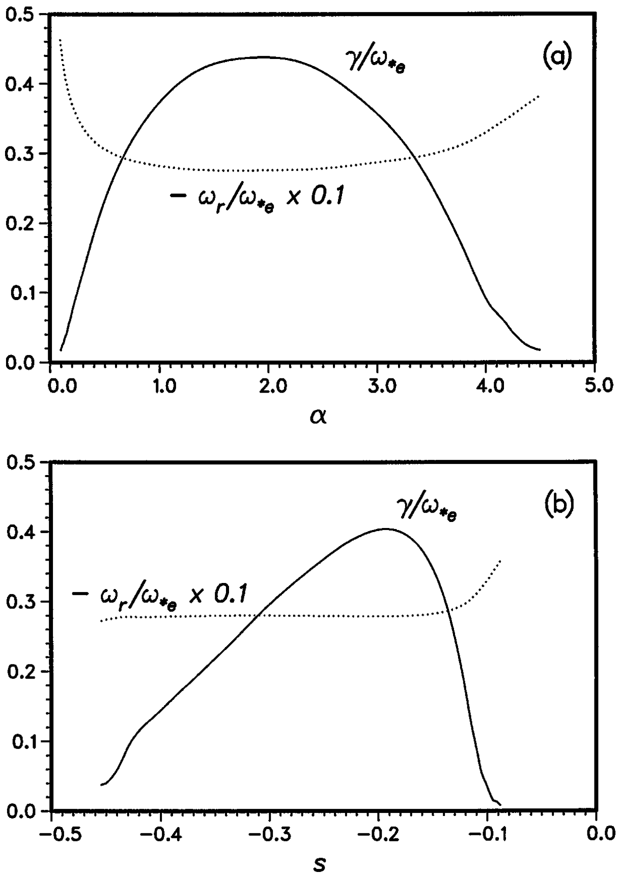

FIG. 1. Dependence of the normalized frequency $(\omega_r + i\gamma)/\omega_{*e}$ on (a) $\alpha (s = -0.2)$ and (b) s $(\alpha = 1.2)$ when $b_0 = 0.01$, $\epsilon_n = 0.1$, $\eta_i = 1$.

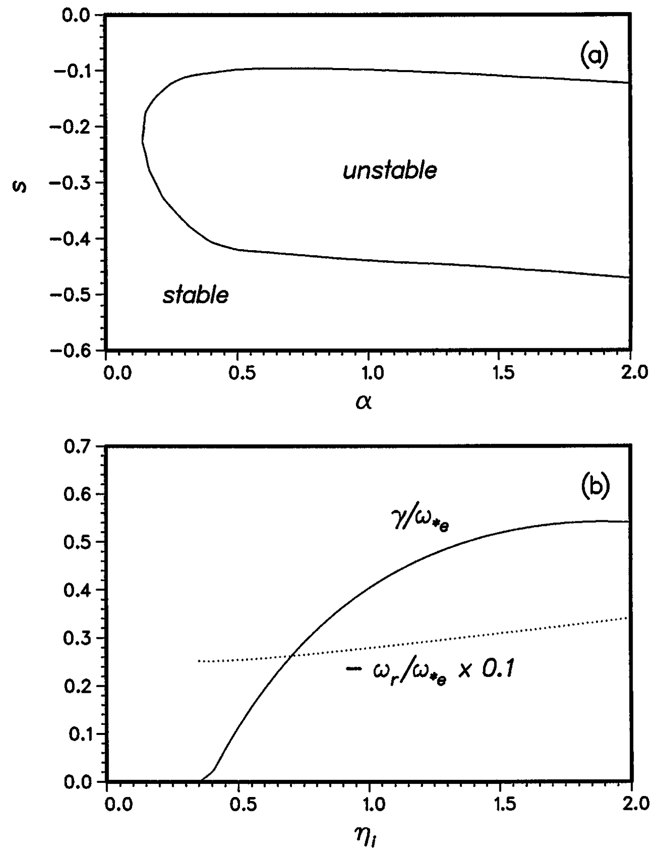

FIG. 2. (a) Stability boundary in the (s, α) plane when $\epsilon_n = 0.1$, $b_0 = 0.01$, $\eta_i = 1$. (b) Dependence of $(\omega_r + i\gamma)/\omega_{*e}$ on η_i when $s = -0.2$, $\eta_i = 1$, $b_0 = 0.01$, $\alpha = 1.2$.

role of the trapped electrons. The trapped electrons modify the mode equation as

$$\frac{d}{d\theta}\left([1 + (s\theta - \alpha \sin\theta)^2]\frac{d\phi}{d\theta}\right) + \frac{\alpha(1 - 0.6\sqrt{\epsilon})}{4\epsilon_n(1 + \eta)}\left((\Omega - 1)[\Omega - f(\theta)] + \eta f(\theta) - \frac{(1 - 0.6\sqrt{\epsilon})(\Omega - 1)^2}{2 - I_i(\theta) - I_{eT}(\theta)}\right)\phi = 0\,, \tag{15}$$

where $\sqrt{\epsilon}$ is the fraction of trapped electrons and $I_{eT}(\theta)$ is given by

$$I_{eT}(\theta) = \left\langle \frac{\omega - \hat{\omega}_{*e}(v^2)}{\omega - \hat{\omega}_{De}(\mathbf{v})} \right\rangle_{\epsilon v_\perp^2 > v_\parallel^2}. \tag{16}$$

The inverse aspect ratio $\epsilon = r/R$ has been varied up to 0.3 in solving Eq. (15). The eigenvalue ω is insensitive to ϵ, and it may be concluded that the instability persists in realistic tokamak discharges. Other stabilizing agents, such as the ion transit frequency $k_\parallel v_{Ti}$ and magnetosonic perturbation $\mathbf{A}_\perp$, are unlikely to have significant effects on the instability.

Finally, it should be pointed out that the two-fluid approximation based on the ion density perturbation [8],

$$n_i = \frac{(\omega + \frac{5}{3}\,\omega_{Di})[\omega_{*e} - \omega_{De} - (\omega + \omega_{*ip})(k_\perp \rho_s)^2] + (\frac{2}{3} - \eta_i)\omega_{*e}\omega_{Di}}{(\omega + \frac{5}{3}\,\omega_{Di})^2 - \frac{10}{9}\,\omega_{Di}^2}\,\frac{e\phi}{T_e} n_0\,,$$

is able to predict, at least qualitatively, the instability in the negative shear region even though the underlying assumption of long wavelengths $(k_\perp \rho_i)^2 \ll 1$ becomes dubious for extended eigenfunctions. This may suggest that the instability is of a reactive type rather than purely kinetic, and exact resonance as given in Eq. (2) and the corresponding Landau residue do not play essential roles.

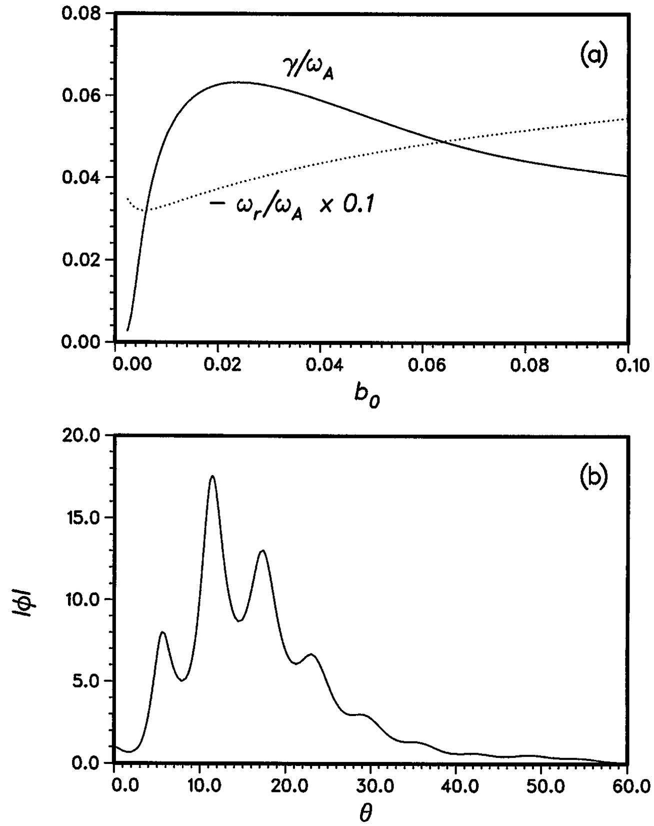

FIG. 3. (a) ω/ω_A vs b_0 when $s = -0.2$, $\alpha = 1.2$, $\epsilon_n = 0.2$, $\epsilon_i = 2$. (b) Eigenfunction when $s = -0.2$, $\alpha = 1$, $b_0 = 0.01$, $\eta_i = 1$, $\omega/\omega_{*e} = -5.03 + i0.73$.

In summary, a tokamak discharge with negative shear is subject to a kinetic Alfvén instability driven by a finite ion temperature gradient. The stability boundary in the (s, α) plane has been determined for a long wavelength mode with $(k_\theta \rho)^2 = 0.01$. The lower threshold in α is small and there is an unstable window in s, $-0.5 \lesssim s \lesssim -0.1$. The maximum growth rate is of order $0.1 V_A/qR$.

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