Direct Measurement of Diffusion Rates in High Energy Synchrotrons Using Longitudinal Beam Echoes

L. K. Spentzouris, J.-F. Ostiguy, and P. L. Colestock
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
(Received 6 June 1995)

We have made a direct determination of the diffusion rates in a stored, coasting antiproton beam by observing the decay rates associated with beam echoes in the longitudinal plane. The beam echoes, similar to those observed in other fields of physics, are generated by a sequential impulse excitation at harmonics of the beam revolution frequency. The echo envelope follows a characteristic response which, however, can be modified by the presence of even a very weak scattering process, permitting a sensitive determination of the longitudinal diffusion rate in the beam.

PACS numbers: 29.20.Lq, 29.27.Bd

Of importance to the stored beam lifetime and maximum achievable beam density in a storage ring is the diffusion rate produced by Coulomb interactions between particles or by external noise sources. Typically this is measured by equilibrium emittance measurements or by long-time-scale observations of comparatively slow beam-size growth over a fairly wide range of beam parameters.

However, it is possible to obtain an incremental measurement of the diffusion rate with the use of a phenomenon widely observed in a number of areas of physics where scattering processes are sufficiently weak, namely, beam echoes [1–3]. Echo generation is formally a nonlinear process, whereby a medium is first given an impulse excitation followed by decay of the perturbation through phase mixing of an ensemble of particles; a second impulse is then applied to reconstruct a portion of the original perturbation after a specific delay. Since the echo reconstruction depends sensitively on the long-time structure of the particle distribution, even weak scattering processes can cause an observable effect after a relatively short time interval [4,5].

Echoes have been studied extensively in other areas of physics, and it has recently been suggested that similar phenomena might be observed in bunched beams [6–8]. Given the extremely weak scattering processes known to exist in high energy hadron beams, it can well be expected that long-lived echo phenomena can be made to occur. Moreover, the long-time behavior of the echoes depends directly on the diffusion rate and, as such, can be used to give a direct measurement of the same. In this paper we report the first observation of echoes in a coasting beam and use the results to determine the absolute diffusion rates under various operating conditions. A model is developed which is manifestly in agreement with the observations.

The echo phenomenon can be viewed as a nonlinear mixing of two waves propagating around the ring. If a short duration rf excitation is applied to the beam at revolution harmonic $n\omega_0$, where ω_0 is the revolution frequency of an ideal particle and n is any integer, then a longitudinal wave of the form $\exp(in\omega_0 t)$ will be readily induced. However, following the excitation pulse, the wave will evolve according to $\exp(in[\omega_0 + k_0\varepsilon]t)$, owing to the energy spread in the beam distribution, where ε is the energy deviation from the mean energy, and k_0 is the proportionality between energy deviation and frequency deviation. Specifically, $k_0 \equiv -\eta\omega_0/\beta^2 E_0$, where E_0 is the energy of a particle at the center of the distribution, $\beta \equiv v/c$ is the relativistic β factor, and η is a machine dependent parameter called the "slip factor." The energy spread results in the decay of the wave amplitude in a so-called Landau damping time. If, however, a second excitation pulse is applied at, say, $m\omega_0$, after a delay Δt, product perturbations can be excited at the difference frequency $(n - m)\omega_0$, by virtue of the amplitude nonlinearity. These second-order currents evolve in time according to $\exp\{i[m - n][\omega_0 + k_0\varepsilon](t - \Delta t) - in[\omega_0 + k_0\varepsilon]\Delta t\}$, and have the property that the energy dependence of the phase can disappear at a time $t_{\mathrm{echo}} = m/(m - n)\Delta t$. Thus, at $t = t_{\mathrm{echo}}$, the phase-mixing process has been effectively unwound, resulting in a reconstruction of a portion of the original perturbation.

A quantitative model of this phenomenon can be developed for the case where wakefields are negligible, i.e., the free-streaming case, by an elementary construction of the distribution function following each impulse [3]. The effect of collisions has recently been included in this model [4] as an expansion in orders of the kick parameter δ, where δ is the ratio of the energy gained during each impulse to the beam momentum spread. However, the presence of wakefields modifies the beam response from the simple energy shift associated with free streaming and a quantitative solution requires a perturbation approach, which we follow in this work. We shall return to the free-streaming case when evaluating the echo response in the Fermilab Antiproton Accumulator, since wakefields appear to be negligible in this ring.

A perturbation approach including the effects of wakefields and collisions can be developed using the Vlasov

 0031-9007/96/76(4)/620(4)$06.00

equation in the longitudinal plane in conjunction with a phase-diffusion term in the following form:

$$\frac{\partial f_{n-m}}{\partial t} + i(n-m)(\omega_0 + k_0\varepsilon)f_{n-m} - \nu\varepsilon_0^2\frac{\partial^2 f_{n-m}}{\partial\varepsilon^2} = \frac{e\omega_0}{2\pi}\frac{\partial f_0}{\partial\varepsilon}U_{n-m} + \frac{e\omega_0}{2\pi}\sum_{n'\neq 0}U_{-n'+n-m}\frac{\partial f_{n'}}{\partial\varepsilon}, \quad (1)$$

where f_0 is the unperturbed distribution, ν is the collision rate, $\varepsilon_0 = \sqrt{2}\Delta E$, where ΔE is the rms beam energy spread, f_{n-m} is the $(n-m)$th Fourier component of the perturbed distribution function, and U_{n-m} is the potential energy loss per turn, given by

$$U_{n-m} = V_{n-m}^{\mathrm{ext}} + \frac{e\omega_0}{2\pi}Z_{n-m}\int_{-\infty}^{\infty} f_{n-m}\,d\varepsilon\,, \quad (2)$$

Z_{n-m} is the longitudinal machine impedance, and V_{n-m}^{ext} is the external excitation. To produce a solution of Eq. (1), we first Laplace transform this equation with respect to time, yielding

$$\{s + i(n-m)[\omega_0 + k_0\varepsilon]\}\tilde{f}_{n-m}(s,\varepsilon) = \frac{e\omega_0}{2\pi}\tilde{U}_{n-m}(s)\frac{\partial f_0}{\partial\varepsilon} + \nu\varepsilon_0^2\frac{\partial^2\tilde{f}_{n-m}(s)}{\partial\varepsilon^2} + \frac{e\omega_0}{2\pi}\int_{\sigma-i\infty}^{\sigma+i\infty}\sum_{n'\neq 0}\tilde{U}_{-n'+n-m}(s-s')\frac{\partial\tilde{f}_{n'}(s')}{\partial\varepsilon}\frac{ds'}{2\pi i}\,, \quad (3)$$

where σ is taken to be to the right of all poles.

A solution for the perturbed current may be found by a perturbation expansion, whereby the nonlinear term in Eq. (3) is initially ignored to permit first-order solutions for f_{n-m} and U_{n-m} to be found. These are then used to construct an expression for the second-order current using the right-hand side of Eq. (3). Employing a second Laplace transform (in energy ε), an expression for the perturbed current can be found,

$$I_{n-m}(s) = -\left(\frac{e\omega_0}{2\pi}\right)^3\sum_{n'\neq 0}\frac{1}{(n-m)n'k_0^2}\int_{\sigma-i\infty}^{\sigma+i\infty}\frac{ds'}{2\pi i}\,\frac{V_{-n'+n-m}^{\mathrm{ext}}(s-s')V_{n'}^{\mathrm{ext}}(s')}{D_{-n'+n-m}(s-s')D_{n'}(s')D_{n-m}(s)} \times\int_{-\infty}^{\infty}d\varepsilon\,\mathcal{F}_{n-m}(\varepsilon,s)\frac{\partial}{\partial\varepsilon}\left[\frac{\partial f_0}{\partial\varepsilon}\mathcal{F}_{n'}(\varepsilon,s')\right], \quad (4)$$

where

$$\mathcal{F}_{n-m}(\varepsilon,s) = \int_0^{\infty}du \times\exp\left\{i\frac{[s + i(n-m)(\omega_0 + k_0\varepsilon)]u}{(n-m)k_0} + i\frac{\nu\varepsilon_0^2u^3}{3(n-m)k_0}\right\}$$

and $D_{n-m}(s)$ is the dispersion function [9,10] for longitudinal mode $n-m$ given by

$$D_{n-m}(s) = 1 - \left(\frac{e\omega_0}{2\pi}\right)^2 Z_{n-m} \times\int_{-\infty}^{\infty}d\varepsilon\frac{\partial f_0/\partial\varepsilon}{s + i(n-m)(\omega_0 + k_0\varepsilon)} \quad (5)$$

in the limit where wakefield effects are negligible, that is, when the roots of the dispersion relations are sufficiently removed from the real axis that collective effects are strongly damped when the echo occurs. In this case, the current may be explicitly evaluated as

$$I_{n-m}^{(2)}(t) = \frac{i}{nk_0}(t-\Delta t)\left(\frac{e\omega_0}{2\pi}\right)^3\int_{-\infty}^{\infty}d\varepsilon\frac{\partial f_0}{\partial\varepsilon}\exp\Big(-i(n-m)[\omega_0 + k_0\varepsilon](t-\Delta t) - in[\omega_0 + k_0\varepsilon]\Delta t - \frac{\nu k_0^2\varepsilon_0^2}{3}[(n-m)^2(t-\Delta t)^3 - n^2\Delta t^3]\Big). \quad (6)$$

We note that the integral over energy vanishes except where the exponential phase vanishes, namely, when $t_{\mathrm{echo}} = m\Delta t/(m-n)$. The echo occurs at t_{echo}, and it is evident from Eq. (6) that the echo shape in time is proportional to the energy derivative of the distribution function and decays with a time constant proportional to $\nu^{1/3}$. Hence a measurement of the echo decay time gives a direct measurement of the collision rate ν. Moreover, owing to the fine structure in the distribution required to reconstruct the echo, even a weak scattering process can have a significant effect, as indicated by the large multiplicative factor in the damping decrement of Eq. (6).

In the Fermilab Antiproton Accumulator, experiments were carried out to investigate echo formation and the effect of scattering on the echo evolution. The hardware configuration used is shown in Fig. 1. Two successive, equal intensity, longitudinal impulses of 5 msec duration were applied at different revolution harmonics

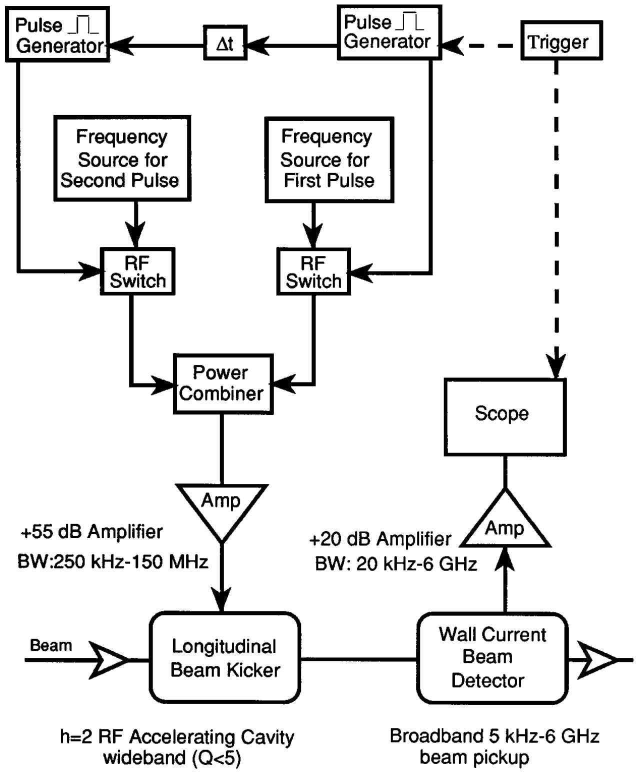

FIG. 1. Experimental setup.

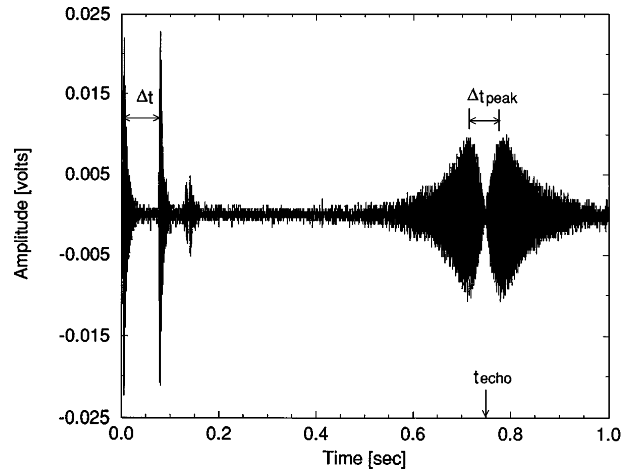

FIG. 2. Beam response to impulse excitation at $h = 9$, followed by $h = 10$. An echo at $h = 1$ occurs centered at 0.75 sec after the initial impulse. The beam parameters were beam current $I_0 = 147$ mA, $\eta = 0.023$, total beam energy $E_0 = 8696$ MeV, beam energy spread obtained from Schottky pickup $\Delta E = 3.2$ MeV, transverse normalized emittances $\epsilon_H = 1.75\pi$ mm mrad, $\epsilon_V = 0.56\pi$ mm mrad, and peak separation of the echo $\Delta t_{\text{peak}} = 0.07$ sec. Note the presence of a higher-order echo immediately following the second excitation pulse.

($f_{\text{rev}} = 687$ kHz) to unbunched, stored antiproton beams of intensities from 30 to 150 mA using a broadband rf cavity. The induced beam oscillations were monitored using a wideband, longitudinal beam current pickup. Typical results are shown in Fig. 2.

The initial excitation at the ninth harmonic $h \equiv f/f_{\text{rev}} = 9$ is applied at $t = 0$, and the second excitation at the tenth harmonic $h = 10$ is applied at $t = 0.075$ sec. The frequency of the echo is at the difference frequency of the applied pulses; and further, the time of the echo follows the expected dependence on the excitation frequencies and the time separation of the initial pulses. This is demonstrated in Fig. 3.

The spacing between the peaks of the echo may be obtained analytically by finding the maxima of the perturbed current. It is found to be inversely proportional to the energy spread of the beam, and is given by

$$\Delta t_{\text{peak}} = \frac{\beta^2}{(n - m)\pi f_{\text{rev}} \mid \eta \mid \epsilon_0/E_0}.$$

We note that the perturbation approach we have taken breaks down when $mk_0\varepsilon_0\delta\Delta t$ is of order unity, i.e., when the mean displaced particle has slipped $\pi/2$ rad in wavelengths of the second impulse relative to the central energy particle. In this case, higher order terms in the perturbation expansion must be retained, and it can be shown [3,4] that the free-streaming response shows a $J_1(2m\delta k_0\varepsilon_0\Delta t)$ dependence, of which Eq. (6) represents the first expansion term. Although we initially attempted to carry out these experiments under conditions where Eq. (6) is valid, later calibrations have shown that the kick parameter $\delta \simeq 0.01$; therefore the argument of the J_1 function is of order unity.

Hence, the best estimate for the diffusion rate is found by fitting the echo envelope to a function of the form

$$I_m = AJ_1(2m\delta k_0\varepsilon_0\Delta t)\exp\left(-\nu k_0^2\varepsilon_0^2\frac{n^2[n - m]^2}{3m^2}\right),$$

where A, δ, and ν are considered unknowns to be found.

The experimental data along with the best theoretical fit are shown in Fig. 4. The effective collision rate is approximately $(3.0 \pm 0.8) \times 10^{-4}$ Hz. It is worthwhile

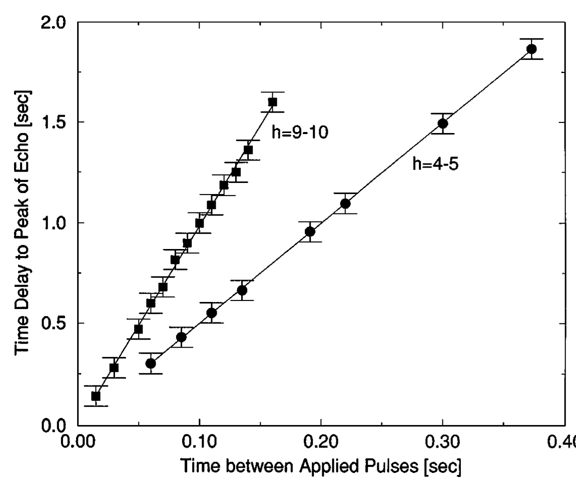

FIG. 3. Measured echo delay time as a function of the second impulse delay time. Upper curve: first pulse at $h = 9$, second pulse at $h = 10$, giving expected time dependence $t_{\text{echo}} = [10/(10 - 9)]\Delta t = 10\Delta t$. Lower curve: first pulse at $h = 4$, second pulse at $h = 5$, giving $t_{\text{echo}} = [5/(5 - 4)]\Delta t = 5\Delta t$.

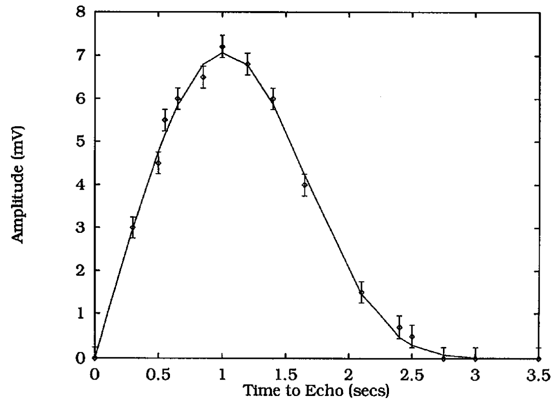

FIG. 4. Peak echo response as a function of the time to echo following the initial impulse. The solid line represents a theoretical fit corresponding to a collision rate $\nu = (3.0 \pm 0.8) \times 10^{-4}$ Hz. The beam parameters were $I_0 = 147$ mA, $\eta = 0.023$, $E_0 = 8696$ MeV, $\Delta E = 4.0$ MeV, $\epsilon_H = 0.84\pi$ mm mrad, $\epsilon_V = 0.34\pi$ mm mrad, and $\delta = 0.01$.

to note that the form of the echo is well represented by the above expression. Moreover, the response did not reappear at any later times than those shown in Fig. 4, indicating the clear influence of a diffusion mechanism.

We expect that diffusion within the beam can be caused by Coulomb, a.k.a. intrabeam, scattering, or noise in the electronics, including the power supplies, stochastic cooling, or damper systems. Gain measurements of the cooling system indicate that diffusion caused by this system should correspond to an effective collision rate on the order of the value observed, but detailed information on the frequency dependence of the system gain is not available. Moreover, power supply and damper system noise have been determined to be a negligible source of diffusion [11]. We have confirmed that no observed change in the measured diffusion rate occurred with the damper systems in the on or off states. Intrabeam scattering based on theoretical estimates [12,13] is also low by approximately a factor of 3. Some uncertainty in the measured parameters place a 20% error on these estimates. While the measurements are in rough agreement with these expected sources of longitudinal diffusion, we cannot exclude the possibility that other mechanisms, such as low-level instabilities, might be playing a role.

It is worthwhile to note that wakefields can have an important effect on the echo shape, corresponding to those cases where the complex zeroes of the dispersion functions, D_m cannot be neglected in Eq. (4). This is the usual condition for most media in which echoes have been observed. In the case of high-energy synchrotrons, both the free-streaming and wakefield dominated cases may be observed, and this affords the possibility of directly measuring the characteristics of the wakefield. Wakefields are expected to be significant when the Landau decay time is on the order of the inverse frequency spread $(k_0\varepsilon_0)^{-1}$. Expressed as a condition on the longitudinal impedance, this becomes

$$Z_m \geq \frac{\pi m k_0}{N}\left(\frac{2\pi\epsilon_0}{e\omega_0}\right)^2, \tag{7}$$

where ϵ_0 is the rms beam energy spread, Z_m is the longitudinal impedance, and N is the beam intensity. The evaluation of the second-order current follows as in Eqs. (3)–(6) above with the result that the echo shape can, under the condition in Eq. (6), follow a modified Gaussian shape, whose detailed form depends on the impedances at the excitation and echo frequencies. As such, it is possible to estimate the longitudinal impedance from the echo shape.

We note that in the Fermilab Antiproton Accumulator wakefield effects are sufficiently negligible so that the free-streaming result given by Eq. (6) is an adequate representation of the beam response.

We further note that similar echoes are possible, and have been previously investigated theoretically in bunched beams [6–8]. We have also made direct observations of bunched-beam echoes and expect to be able to use them to measure diffusion phenomena in bunched beams. One interesting variant is the generation of an echo around transition with a single excitation pulse, owing to the natural phase reversal induced in crossing transition. Such a phenomenon could be used, in principle, to diagnose collective effects suspected to occur near transition. A particularly attractive application is the possibility of measuring transverse diffusive processes, such as the strength of transverse resonances and the effect of beam-beam diffusion.

The authors would like to thank D. Olivieri for determining the gain of the momentum cooling system, M. Church and S. Werkema for their assistance in preparing the Antiproton Accumulator for these experiments, and D. Finley for his continuing support of this work.

[1] E. L. Hahn, Phys. Rev. **80**, 4 (1950).
[2] R. W. Gould *et al.,* Phys. Rev. Lett. **19**, 5 (1967).
[3] T. M. O'Neil and R. W. Gould, Phys. Fluids **11**, 1 (1968).
[4] O. S. Bruning, CERN Report No. SL/95-83 (AP), Geneva (1995).
[5] C. H. Su and C. Oberman, Phys. Rev. Lett. **20**, 9 (1968).
[6] G. V. Stupakov, Superconducting Supercollider Laboratory Report No. SSCL-579, 1992.
[7] G. V. Stupakov and K. Kauffmann, Superconducting Supercollider Laboratory Report No. SSCL-587, 1992.
[8] N. Mahale *et al.,* Superconducting Supercollider Laboratory Report No. SSCL-N-817, 1993.
[9] D. A. Edwards and M. J. Syphers, *An Introduction to the Physics of High Energy Accelerators* (Wiley, New York, 1993).
[10] A. Hofmann, CERN Report No. 77-13, 1977.
[11] M. Church, "Measurement of Ripple in the Accumulator Bend Bus Supply," Pbar Note 520, Fermilab, 1992.
[12] J. D. Bjorken and S. K. Mtingwa, Part. Accel. **13**, 115 (1983).
[13] L. Evans and B. Zotter, CERN/SPS/80-IS(DI), Geneva, 1980.