Even and Odd Coherent States of the Motion of a Trapped Ion

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We propose a scheme for generating even and odd coherent states of the motion of a trapped ion via laser excitation of two vibronic transitions. These quantum superposition states emerge from the long-time dynamics, when the ion decouples from the laser fields and remains in a pure state given by the product of the electronic ground state with a quantum superposition of coherent states of the vibrational motion.

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In quantum mechanics the superposition principle, stating that linear superpositions of quantum states give rise to new ones, plays a fundamental role [1]. It implies that probability densities of observable quantities, in a superposition state, usually exhibit interference effects instead of simply being added. The striking consequences of this principle are particularly evident in the superpositions of coherent states: although formed by quantum states close to classical ones, they show highly nonclassical properties [2,3], such as squeezing [4,5], sub-Poissonian [6], and oscillating number statistics [7].

Representative examples for the superposition of two coherent states are the even and odd coherent states $|\alpha_{\pm}\rangle$ [8,9], which can be given in the form

$$|\alpha_{\pm}\rangle = N_{\pm}(|\alpha\rangle \pm |-\alpha\rangle), \tag{1}$$

where $|\alpha\rangle$ is a coherent state and $N_{\pm}$ are normalization constants. For a large amplitude α, they can be interpreted as quantum superpositions of two macroscopically distinguishable states, so-called Schrödinger cat states. It is worth noting that recently electronic states of Schrödinger cat type have been prepared via pulsed excitation of atomic Rydberg wave packets [10].

The experimental realization of even and odd coherent states of a quantized harmonic oscillator is of great interest, in view of both their simplicity and their interesting features. There have been proposals to examine these and closely related states in nonlinear optics [11,12]. However, the quantum coherence effects are extremely sensitive to losses [13] so that their preparation and observation in the form of radiation states is difficult. Alternatively, it has been predicted that superpositions of coherent states of molecular vibrations could be prepared by appropriately exciting a molecule with two short laser pulses [14] and the practical possibilities of realizing such an experiment have been discussed [15]. In this scheme the quantum interferences would survive on a picosecond time scale. Their observation requires a fast optical time gate and the visibility of the effects is expected to be rather small.

In this contribution we propose the generation of even and odd coherent states of the center-of-mass motion of a trapped ion. Our scheme essentially eliminates the limitations of earlier proposals and opens novel possibilities to study quantum interferences with a high degree of stability. It is important that these states are stationary ones in our scheme. Electronic damping is included in our treatment and the vibrational damping is so weak that it can be disregarded. Experimental disturbances, apparently due to heating by stray noise fields, led to a small heating rate of about 6 s^{-1} for transitions from the vibrational ground state to the first excited state [16]. Recently a measurement principle has been proposed for determining the vibrational quantum state of the ion with high overall efficiency [17], which yields a large visibility of the quantum interference effects.

Our proposal for preparing even and odd coherent states is based on bichromatic excitation of both the electronic transition (carrier) and the second vibrational sideband of an ion in the well resolved sideband regime, which can be realized using presently available experimental techniques [16,18]. Whereas the laser resonant to the carrier alone would heat the ion up and the laser on the second sideband would lead to second-sideband cooling [19], their combination will be shown to balance the system in a stationary superposition of two coherent states.

Consider a single ion trapped in a harmonic potential and interacting with laser light. The Hamiltonian of this system (in the optical rotating-wave approximation) may be given as

$$\hat{H} = \hbar\nu\hat{a}^{\dagger}\hat{a} + \hbar\omega_{21}\hat{A}_{22} + [\lambda E^{(-)}(\hat{x}, t)\hat{A}_{12} + \text{H.c.}], \tag{2}$$

where the first two terms describe the free motion of the external and internal degrees of freedom of the ion and the last one the interaction with the laser field. The operators $\hat{a}$ and $\hat{A}_{ij}$ $(i, j = 1, 2)$, respectively, are the annihilation operator of a quantum of the ionic vibrational motion and the electronic (two-level) flip operator for the $|j\rangle \rightarrow |i\rangle$ transition of frequency ω_{21} between two electronic states, ν is the trap frequency, λ is the electronic coupling matrix element, and $E^{(-)}(\hat{x}, t)$ the negative frequency part of the classical electric field of the driving light. The

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ion is assumed to be driven by two lasers, tuned to the electronic transition and to the lower second sideband, so that $E^{(-)}(\hat{x},t)$ is given by

$$E^{(-)}(\hat{x},t) = E_0 e^{-i(k_0\hat{x}-\omega_{21}t)} + E_2 e^{-i[k_2\hat{x}-(\omega_{21}-2\nu)t]}, \quad (3)$$

where $\hat{x}$ is the operator of the center-of-mass position and may be written as

$$\hat{x} = \frac{\eta}{k_L}(\hat{a} + \hat{a}^\dagger), \quad (4)$$

η being the Lamb-Dicke parameter and $k_L \approx k_0 \approx k_2$ the wave vector of the driving light field.

In the resolved sideband limit the vibrational frequency ν is much larger than the other characteristic frequencies of the problem. In this case the interactions of the ion with the two lasers can be treated separately, using a nonlinear Jaynes-Cummings Hamiltonian [20]. This approach simplifies the interaction part of the Hamiltonian (2), in the interaction picture, to

$$\hat{H}_{\rm int} = \hbar\Omega_2 e^{-\eta^2/2}\hat{A}_{21}\left[\sum_{k=0}^{\infty}\frac{(i\eta)^{2k+2}}{k!\,(k+2)!}(\hat{a}^\dagger)^k\hat{a}^{k+2} + \frac{\Omega_0}{\Omega_2}\sum_{k=0}^{\infty}\frac{(i\eta)^{2k}}{(k!)^2}(\hat{a}^\dagger)^k\hat{a}^k\right] + \text{H.c.}, \quad (5)$$

with Ω_0 and Ω_2, respectively, being the Rabi frequency of the laser tuned to the electronic transition and to the second sideband. The time evolution of the whole system can be described by the master equation

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H}_{\rm int},\hat{\rho}] + \frac{\Gamma}{2}(2\hat{A}_{12}\hat{\tilde{\rho}}\hat{A}_{21} - \hat{A}_{22}\hat{\rho} - \hat{\rho}\hat{A}_{22}), \quad (6)$$

where the last term describes spontaneous emission with the energy relaxation rate Γ, and

$$\hat{\tilde{\rho}} = \frac{1}{2}\int_{-1}^{1} ds\, W(s) e^{i\eta(\hat{a}+\hat{a}^\dagger)s}\hat{\rho}\, e^{-i\eta(\hat{a}+\hat{a}^\dagger)s} \quad (7)$$

accounts for changes of the vibrational energy due to spontaneous emission. $W(s)$ is the angular distribution of spontaneous emission and $\hat{\rho}$ the vibronic density operator.

Let us consider the behavior of the ion for small values of the Lamb-Dicke parameter η; additionally we assume that $\Omega_0 \ll \Omega_2$. In this limit the leading terms of $\hat{H}_{\rm int}$ are those corresponding to $k = 0$ and $\hat{\tilde{\rho}}$ can be replaced by $\hat{\rho}$ (after a spontaneous emission process the ion remains with large probability in the same vibrational state). Hence, the master equation is simplified to

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}[\hat{H}'_{\rm int},\hat{\rho}] + \frac{\Gamma}{2}(2\hat{A}_{12}\hat{\rho}\hat{A}_{21} - \hat{A}_{22}\hat{\rho} - \hat{\rho}\hat{A}_{22}), \quad (8)$$

with

$$\hat{H}'_{\rm int} = -\frac{1}{2}\hbar\eta^2\Omega_2 e^{-\eta^2/2}\hat{A}_{21}\left(\hat{a}^2 - \frac{2\Omega_0}{\eta^2\Omega_2}\right) + \text{H.c.} \quad (9)$$

Note that these equations of motion, although derived for small η, go beyond the Lamb-Dicke approximation which neglects second and higher order terms in η.

To derive the steady-state solution $\hat{\rho}_s$ of Eq. (8) we use the ansatz

$$\hat{\rho}_s = |1\rangle|\psi\rangle\langle\psi|\langle 1|, \quad (10)$$

where $|1\rangle$ is the electronic ground state and $|\psi\rangle$ is the vibrational state of the ion. Setting $d\hat{\rho}/dt = 0$ in Eq. (8) and using Eq. (10) we readily derive that the state $|\psi\rangle$ is a right-hand eigenstate of the operator $\hat{a}^2$,

$$\hat{a}^2|\psi\rangle = \frac{2\Omega_0}{\eta^2\Omega_2}|\psi\rangle. \quad (11)$$

The complex eigenvalue $2\Omega_0/\eta^2\Omega_2$ is determined by the intensities and the phase difference of the driving lasers and can be easily controlled. Note that $H'_{\rm int}|1\rangle|\psi\rangle = 0$, so that the ion no longer interacts with the laser fields. In the number representation Eq. (11) reads as

$$\langle n+2|\psi\rangle = \frac{2\Omega_0}{\eta^2\Omega_2}\frac{1}{\sqrt{(n+1)(n+2)}}\langle n|\psi\rangle. \quad (12)$$

It turns out that the particular form of $|\psi\rangle$ depends on the initial conditions. Since the master equation (8) contains only even powers of $\hat{a}$ and $\hat{a}^\dagger$, it will preserve the "parity" (oddness or evenness) of the initial state.

If the initial vibrational state of the ion is a combination of only even number states, the long-time solution will not contain any odd number state. In this case $|\psi\rangle$ reduces to

$$|\psi_e\rangle = \cosh^{-1/2}\left\{\left|\frac{2\Omega_0}{\eta^2\Omega_2}\right|\right\}\sum_{n=0}^{\infty}\left(\frac{2\Omega_0}{\eta^2\Omega_2}\right)^n\frac{1}{\sqrt{(2n)!}}|2n\rangle, \quad (13)$$

which is the even coherent state $|\alpha_+\rangle$ with $\alpha = (2\Omega_0/\eta^2\Omega_2)^{1/2}$. On the other hand, if the initial vibrational state contains only odd number states, the long-time solution $|\psi\rangle$ is reduced to the odd coherent state

$$|\psi_o\rangle = |\alpha_-\rangle, \qquad \alpha = \left(\frac{2\Omega_0}{\eta^2\Omega_2}\right)^{1/2}. \quad (14)$$

In the more general case of an initially incoherent combination of even and odd vibrational number states, the long-time solution will be a statistical mixture of both solutions:

$$\hat{\rho}_s = P_e|1\rangle|\psi_e\rangle\langle\psi_e|\langle 1| + P_o|1\rangle|\psi_o\rangle\langle\psi_o|\langle 1|, \quad (15)$$

where P_e and P_o, respectively, are the weights of the even and odd number states in the initial state.

For preparing an even coherent state of the vibrational motion, it is useful to cool the ion to its vibrational ground state via sideband cooling [16], which has recently been achieved with a probability of 98% [18]. Subsequently,

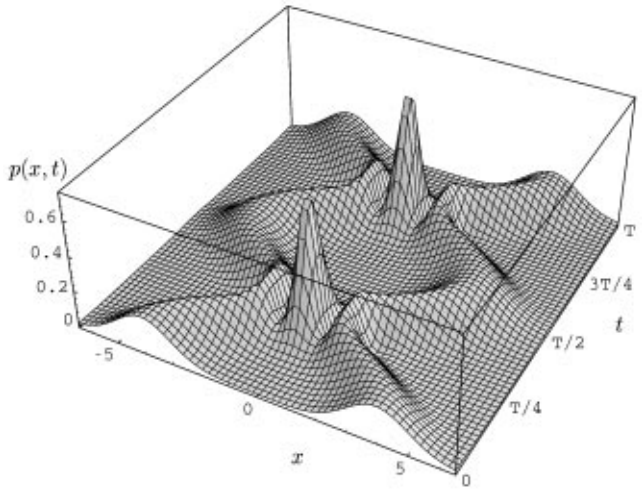

FIG. 1. Spatial probability distribution $p(x,t)$ for the ion initially in the vacuum state, $\eta = 0.05$ and $2\Omega_0/\eta^2\Omega_2 = 4$. The stationary state is very close to a pure one: $\mathrm{Tr}\{\hat{\rho}_s^2\} \approx 0.97$.

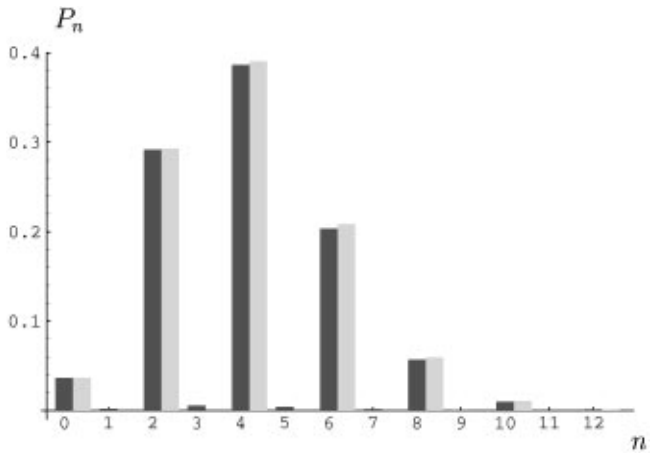

FIG. 3. The vibrational number distribution P_n is shown for the same situation as in Fig. 1 (dark bars) and is compared with that of an even coherent state of the same amplitude (bright bars).

the ion is driven on both the carrier and the second sideband as proposed above. The steady state will be reached when the ion stops to fluoresce. In the following we present throughout results obtained from a numerical integration of the master equation (6), which still contains all powers of the Lamb-Dicke parameter η. In this manner we confirm the validity of the analytical treatment developed above. In Fig. 1 we show the time dependence of the spatial probability distribution $p(x,t)$ of the ion, which has a periodicity time $T = 2\pi/\nu$. The spatial distribution consists of two coherent waves which are phase shifted by π with respect to each other. Instead of simply overlapping, quantum interferences appear when they come close together. The corresponding fringe structures are significantly narrower than the vacuum distribution. Figure 2 shows the corresponding Wigner function $W(x,p)$, which also displays the quantum interferences. The negative values of the Wigner function are a signature of the nonclassical nature of the effects. The vibrational number statistics of the ion is compared in Fig. 3 with that of an even coherent state. Small differences between them are due to processes changing the vibrational state of the ion via spontaneous emission of a photon, which have been disregarded in the analytical treatment.

The generation of an odd coherent state may be achieved by initially preparing the ion in the first vibrational number state [21]. Figure 4 shows a typical result, which is very close to the analytical solution (14) and exhibits pronounced quantum interferences.

Let us finally consider the situation for less perfect laser cooling of the ion, giving rise to a mixture of even and odd vibrational number states, such as a thermal initial occupation of the vibrational levels. In this case the final state will be a statistical mixture of even and odd coherent states as given in Eq. (15). In Fig. 5 we show an example for the Wigner function in the case of an initial mean thermal excitation of $\langle\hat{n}\rangle_{\mathrm{th}} = 0.3$. Although the state is no longer

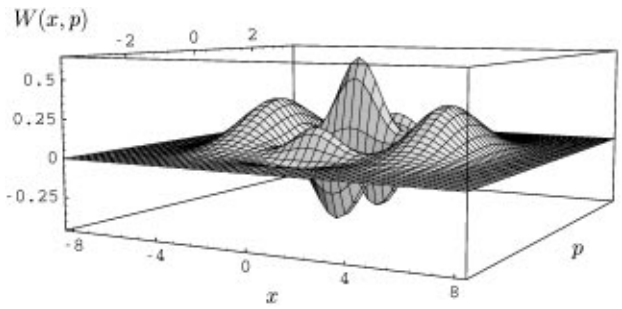

FIG. 2. Wigner function $W(x,p)$ for the same situation as in Fig. 1.

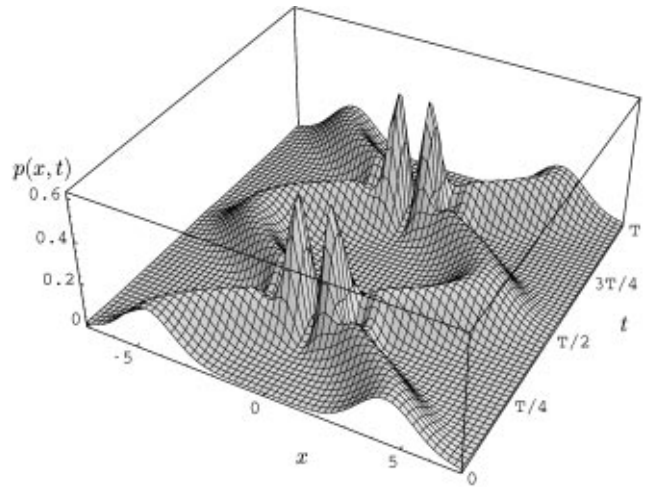

FIG. 4. Spatial probability distribution $p(x,t)$ for the ion initially prepared in the first vibrational number state, $\eta = 0.05$ and $2\Omega_0/\eta^2\Omega_2 = 5$. This state is again close to a pure one: $\mathrm{Tr}\{\hat{\rho}_s^2\} \approx 0.96$.

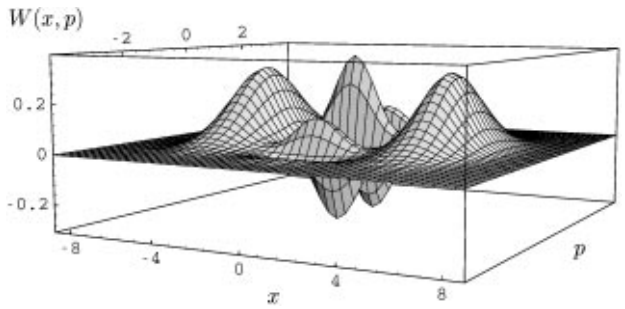

FIG. 5. Wigner function $W(x, p)$ for the ion initially prepared in a thermal vibrational distribution, $\langle \hat{n} \rangle_{\text{th}} = 0.3$ and $\eta = 0.05$. This state is not close to a pure one: $\text{Tr}\{\hat{\rho}_s^2\} \approx 0.67$.

close to a pure one, it is seen that significant quantum interferences survive. They decrease with increasing thermal excitation and eventually disappear for $P_e \approx P_o$.

In conclusion, we have proposed a scheme for the preparation of even and odd coherent states of the center-of-mass motion of a trapped ion driven by a bichromatic laser field. In appropriate limits the system will reach a steady state where the ion no longer interacts with the lasers and remains in a product of the ground state of the electronic transition with a quantum superposition of two coherent states of the vibrational motion. Based on presently available trapped-ion techniques, this would allow the experimental study of quantum interference effects at a high level of stability.

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