Formation of Topological Defects with Explicit Symmetry Breaking

Sanatan Digal and Ajit M. Srivastava*
Institute of Physics, Sachivalaya Marg, Bhubaneswar-751005, India
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We demonstrate a novel mechanism for topological defect formation in a first order phase transition for theories with small explicit symmetry breaking terms. We perform simulations of two bubble collisions in 2 + 1 dimensions. In the coalesced region of bubble walls, field oscillations result in the production of a large number of vortices and antivortices. We discuss the implications of our results for axionic strings in the early Universe, for baryon formation in quark-gluon plasma, and for strings in liquid crystals in external electric or magnetic field.

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Production and subsequent evolution of topological defects have been of considerable interest for particle physicists in the context of the early Universe [1]. Similar techniques have also been used to study baryon formation during hadronization of quark-gluon plasma (QGP) in heavy ion collisions [2,3]. Study of topological defects has, of course, been possible in a most detailed way only in condensed matter systems where they can be experimentally studied [4].

The aforementioned defects correspond to a spontaneous breakdown of a symmetry. However, there are many situations when the symmetry is also explicitly broken. In particle physics, the Peccei-Quinn scheme for solving the strong *CP* problem of quantum chromodynamics leads to the presence of an explicit symmetry breaking term and, consequently, to axionic strings [5]. The Skyrmion picture of baryons in the context of chiral models is another example where explicit symmetry breaking terms are needed to incorporate a nonzero pion mass [3]. In condensed matter, liquid crystals provide a simple example of such systems where the presence of external electric or magnetic fields induces explicit symmetry breaking terms [6].

The study of formation of topological defects in such systems is therefore important as it has implications for a diverse set of phenomena. It has recently been argued that explicit symmetry breaking can lead to a fourfold enhancement in the production of baryons in QGP [3]. These arguments were largely qualitative and did not depend sensitively on the order of the phase transition. It was argued in Ref. [3] that a similar enhancement should occur for other topological defects as well.

In this Letter we demonstrate a new mechanism for the production of topological defects for systems with explicit symmetry breaking and with a first order phase transition, where phase transition proceeds via bubble nucleation. This mechanism leads to a much stronger enhancement in defect production, and results from a combination of the effects discussed in Ref. [3] as well as effects coming from the large field oscillations in the region of coalesced bubble walls. The net result is that wall oscillations decay by producing a large number of vortices and antivortices. For example, in one simulation we found five vortices and five antivortices produced in a single two-bubble collision.

We adopt the same numerical technique used in previous simulations of vortex formation via bubble collision; see Ref. [7]. We will study vortex formation in 2 + 1 dimensions in a field theory system described by the following Lagrangian:

$$L = \frac{1}{2}\partial_\mu \Phi^\dagger \partial^\mu \Phi - \frac{1}{4}\phi^2(\phi - 1)^2 + \epsilon\phi^3 + \kappa\phi^2 \cos\theta \,. \quad (1)$$

This Lagrangian (without the last term) is related to the one discussed in Ref. [7] by a rescaling of variables so that it is now written in terms of dimensionless field Φ and length variable. ϕ and θ are the magnitude and the phase of the complex scalar field Φ ($\Phi = \phi e^{i\theta}$). Equation (1) describes a theory where the U(1) global symmetry is spontaneously broken, except for the presence of the last term which breaks this U(1) explicitly. When κ is zero, this theory allows for the existence of a cylindrically symmetric vortex which is a solution of the time independent field equations. For a nonzero κ, the vortex loses azimuthal symmetry and is not a solution of the time independent equations of motion anymore.

For $\kappa = 0$, the process of vortex creation via bubble nucleation has been described in Ref. [7]. At zero temperature, bubbles of true vacuum nucleate via quantum tunneling in the background of the metastable vacuum with $\phi = 0$. These are described by the bounce solution which is an O(3)-symmetric, least action, solution of the Euclidean field equation [8]

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - V'(\phi) = 0 \,, \quad (2)$$

where $V(\phi)$ is the effective potential in Eq. (1) (with $\kappa = 0$), and r is the radial coordinate in the Euclidean space. In the Minkowski space the profile of the nucleated bubble is obtained from the solution of Eq. (2) by putting $t = 0$. This bubble then evolves according to the classical

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field equations obtained from the Lagrangian in Eq. (1) in Minkowski space,

$$\Box\Phi_i = -\frac{\partial V(\Phi)}{\partial \Phi_i}, \quad i = 1, 2, \tag{3}$$

where $\Phi = \Phi_1 + i\Phi_2$, $\Box$ is the d'Alembertian, and time derivatives of fields are set equal to zero at $t = 0$. In a phase transition, θ varies randomly from one bubble to another. These bubbles expand, and vortices form at the junction of three or more bubbles if the phase θ traces a nontrivial winding in that region. This is the conventional Kibble mechanism of defect formation [9] which leads to the probability of vortex formation for 2 space dimensions equal to $1/4$ per bubble [4].

We wish to study the case when κ is nonzero. First, we briefly recall the physical picture described in Ref. [3]. Consider a two-bubble collision with the phase θ in the two bubbles taking values $\pi + \alpha$ and $\pi - \alpha$, where α is small. (We mention that here, as well as for the results discussed later in this paper, it is not necessary that θ in the two bubbles be equally spaced from π. We take this just for simplicity.) As the bubbles collide, θ in the coalesced portion will assume a value π due to the geodesic rule (essentially to minimize energy) and will keep evolving towards zero inside the bubbles (and in the walls, which, at later times, forces θ in the coalescing regions to change to zero). It is then easy to see that this leads to a winding one being created on one end of the coalesced wall and winding minus one on the other end [3]. It was argued in Ref. [3] that this leads to roughly fourfold enhancement in the number density of vortex production per bubble.

However, it turns out that the actual dynamics of vortex creation has a much richer structure, especially for a first order phase transition. As θ in both the bubbles evolves towards zero, the coalesced portion of the walls undergoes large oscillations. Such oscillations have been described in Ref. [7], where it was shown (for the case of subcritical bubbles) that, as ϕ undergoes large oscillations, it passes through $\phi = 0$, forcing θ to change to $\theta + \pi$.

This flip in the orientation of Φ has very important effects on the process of vortex formation. The evolution of θ towards zero inside the bubbles tends to create a winding one near one end and an antiwinding one near the other end inside the coalesced region. The flip in the orientation of Φ in the central region completes these windings and results in the nucleation of a vortex-antivortex pair in the coalesced wall.

This explains the formation of the first vortex-antivortex pair. Subsequent pairs are created due to the oscillation of θ about $\theta = 0$. As θ in the two bubbles evolves towards zero, it overshoots the true vacuum (i.e., θ = 0). It is easy to convince oneself that this evolution of θ in the two bubbles, combined with the flipping of the orientation of Φ due to wall oscillations, will result in the creation of another vortex-antivortex pair. This process continues as long as θ and ϕ oscillate about $\theta = 0$ and $\phi = 0$, respectively.

It is important to mention that the first vortex-antivortex pair always forms, given appropriate initial θ values for the two bubbles. However, subsequent vortices will not be nucleated if ϕ oscillations did not continue to have large enough amplitudes (as may happen due to damping in the presence of a thermal bath). Even with large ϕ oscillations, vortices are not nucleated unless there are appropriate θ oscillations. We see this in our simulations where many oscillations of the wall, and that of θ, may pass by before a given pair gets nucleated. Also, the sequence of vortices and antivortices created on one end of the coalesced region is quite arbitrary, depending on the details of ϕ and θ oscillations; although, the net winding number is always zero. This implies that the annihilation of vortex-antivortex pairs may be very ineffective.

We now proceed to describe our numerical results. We find the bubble profile by solving Eq. (2) for $\kappa = 0$ in $V(\phi)$. This bubble profile will be an approximate solution of the field equations obtained from Eq. (1) for small nonzero values of κ and provides an adequate starting point as bubbles collide only after undergoing large expansions. (In fact, our choice of the specific form of the explicit symmetry breaking term was motivated by this consideration, as well as by simplicity. With this term, $\phi = 0$ is also a local minimum of the full effective potential so the asymptotic form of the bubble profile calculated with $\kappa = 0$ is suitable for nonzero κ as well.) It is important to mention that our results do not sensitively depend on the specific form of the explicit symmetry breaking term, as long as there is a unique true vacuum. [It is interesting to investigate the case of degenerate vacua, e.g., with $\cos(n\theta), n > 1$ in the last term of Eq. (1); we hope to discuss it in a future work.] We have verified that similar enhancement in vortex production results for other types of symmetry breaking terms as well (e.g., $\kappa\phi\cos\theta$).

We use a natural system of units with $\hbar = c = 1$ and choose the value of $\epsilon = 0.05$. We have studied a range of values of κ. Large values of κ (>0.03) do not give any vortex formation as θ in bubbles rolls down and settles to zero before bubbles can effectively coalesce. For all other values of κ, vortices form (with smaller κ leading to vortex formation at a later stage). The figures shown in this paper correspond to the choice of $\kappa = 0.015$.

Following the techniques developed in Ref. [7], we study the case of a two-bubble collision by prescribing the nucleation centers for the two bubbles. This amounts to replacing a portion of the false vacuum region (with $\Phi = 0$) by the profiles of two bubbles. The field configuration is then evolved by using a discretized version of Eq. (3). Simulation is implemented by using a stabilized leapfrog algorithm of second order accuracy in both space and time. We use a 1000×1600 lattice, with the lattice spacing in spatial directions, Δx, equal to 0.1 and lattice spacing in the temporal direction, Δt, equal to $\Delta x/\sqrt{2}$. With these values, the evolution was completely stable, and energy was conserved within a few percent during the simulation. For details of the numerical technique, see Ref. [7]. Simulations were carried out on a HP-735 workstation at the Institute of Physics, Bhubaneswar.

Bubble centers were chosen to lie at the y boundaries of the lattice (and at the midpoint of the x axis) so that the initial bubble profiles are that of half bubbles. We use free boundary conditions. Figure 1(a) shows the plot of Φ and the contour plot of ϕ for the initial field configuration of one of the bubbles. Values of θ for the two bubbles are taken as 1.12π and 0.88π (for the lower and upper bubbles, respectively) and are uniform inside each bubble. This leads to the development of a region of $\theta = \pi$ in the region where bubbles coalesce. Figure 1(b) shows the plot of Φ at an intermediate stage. The bubbles have significantly coalesced, and θ inside the bubbles has started rotating towards zero. The rotation of θ is smaller near the bubble walls due to the dependence of the explicit symmetry breaking term in Eq. (1) on ϕ^2. Due to this, even for large θ difference between the two bubbles, a region of $\theta = \pi$ develops in the coalesced region and vortices are produced. (However, then subsequent wall oscillations are not very prominent [7] so the number of vortices produced is small.) The only requirement for initial values of θ in the two bubbles for vortex production is that the geodesic connecting these two θ values on S^1 should pass through the value π. (Actually we get one pair even when this geodesic passes through $\theta = 0$; however, then the vortex and the antivortex do not separate out.)

Figures 1(c) and 1(d) show the configurations after the first pair has been nucleated. The plot of Φ clearly shows that θ has overshot the true vacuum ($\theta = 0$). Afterwards, θ starts climbing towards π first and then again rolls back towards zero. As described earlier, this will cause the creation of subsequent pairs for appropriate ϕ oscillations. Figures 1(e) and 1(f) show the plots at a stage when there is a total of ten vortices and antivortices. Out of these,

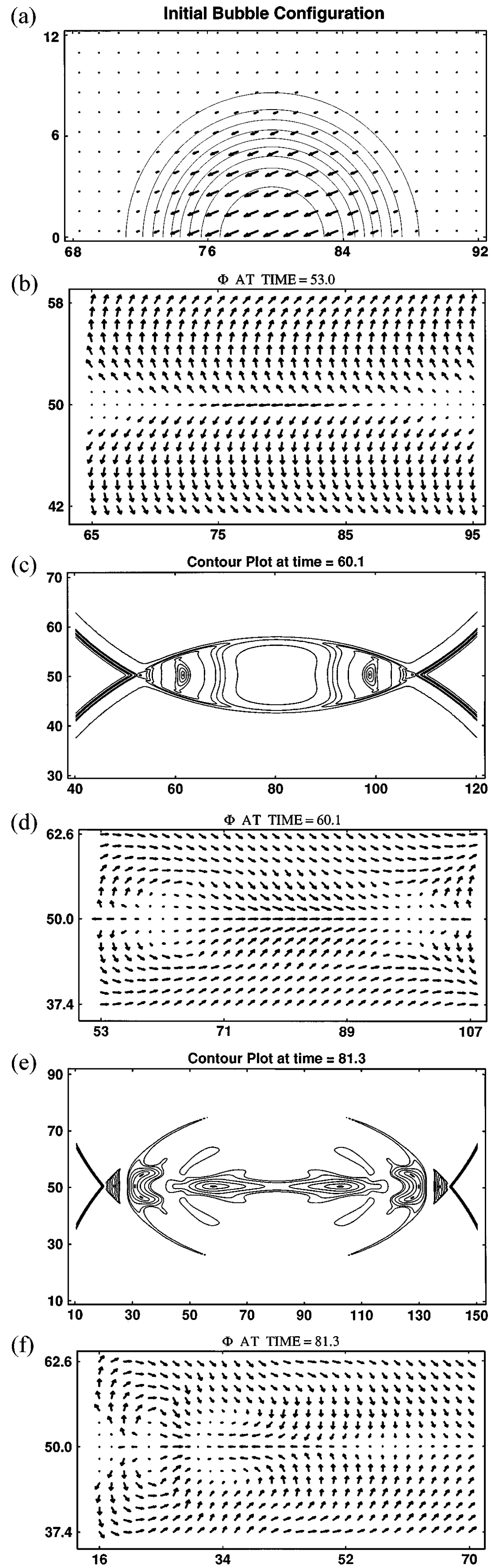

FIG. 1. (a) Plot of Φ and contour plot of ϕ for the initial configuration of one of the bubbles with center at the boundary. For all Φ plots, the orientation of the arrows from positive x axis gives the phase θ of Φ while the length of arrows is proportional to ϕ. (b) Plot of Φ for the coalesced region at $t = 53.0$ showing that θ has significantly rotated towards zero in bubble interiors. (c) Contour plot of ϕ at $t = 60.1$ showing a vortex-antivortex pair. (d) Winding numbers of the vortex and the antivortex are clear from the Φ plot. (e) Contour plot of ϕ at $t = 81.3$ showing ten vortices and antivortices. There are two groups of three overlapping vortices each, one near $x = 33$ and the other near $x = 127$. These groups have net windings of $+1$ and -1, respectively. (f) Φ plot for a portion of lattice showing winding numbers of at least two vortices and one antivortex which are well separated. The winding $+1$ region near $x = 33$ actually consists of close by configurations of two vortices and one antivortex.

there are two groups, containing three vortices each (as confirmed by detailed plots of Φ of these regions) which are not well separated. Overall there are at least six vortices and antivortices which are well separated. Note that, due to the presence of explicit symmetry breaking terms, the profiles of these vortices are highly deformed as shown by the contour plots.

In conclusion, we have demonstrated a new mechanism for the formation of topological defects in the presence of explicit symmetry breaking which may dominate over other mechanisms of defect production. A somewhat modified version of this mechanism (due to the absence of a coalesced portion of bubble walls) may also be applicable for the case of second order phase transitions. (In this context we mention that in Ref. [7] it was found that the number of vortices produced was roughly twice the estimate based on the Kibble mechanism; though many pairs annihilated quickly. In view of our results in this paper, it seems interesting to investigate whether the excess production of vortices in Ref. [7] can be due to a nontrivial dynamics of θ coupled with ϕ oscillations, even though explicit symmetry breaking was absent there.) The most interesting aspect of this mechanism is that it is literally a pair creation process, though still governed by classical equations of motion. In this sense, it resembles the pair creation of vortices in the flow of superfluid ^{4}He through a small orifice, as discussed in Ref. [10], although actual mechanisms are completely different. In a subsequent paper we will present the study of full phase transition by nucleating a large number of bubbles [11].

Implications of these results are many. Using the ideas described above, it is possible to argue that, in two-bubble collisions in $3 + 1$ dimensions, field oscillations should lead to string loops being emitted out from the coalesced region. For axionic strings in the early Universe, earlier studies have assumed that the formation mechanism is the same as for other cosmic strings, namely, via the Kibble mechanism [5]. The above discussion shows that the dominant mechanism may be via the mechanism discussed in this paper, at least for a first order phase transition. Therefore, the final distribution of axionic strings, and hence the frequency distribution of emitted axions, can be drastically different from what is conventionally taken. Also, as one now expects small string loops to be produced, axionic domain walls may not survive for long. This may make a larger class of axionic models viable.

For the case of liquid crystals in the presence of electric field, our results suggest that, instead of long strings, small string loops should form in the coalesced region of two bubbles. However, in this case, the dynamics of string formation via this mechanism may be completely dominated by the presence of damping terms. Hence, as discussed above, the number of strings produced may not be large. The mechanism discussed in this paper should also be applicable to the production of other defects, though the details of the mechanism will depend on the type of defect (and dimensionality of physical space) under consideration. (We mention again that in the presence of thermal bath damping of field oscillations may suppress this enhancement.) Especially important is the production of baryons (in the Skyrmion picture) in quark-gluon plasma [3]. In view of our results in this paper, there is a possibility of a larger enhancement in baryon production due to explicit symmetry breaking if the phase transition is of first order.

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*Electronic address: digal,ajit@iopb.ernet.in

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