Charge Density Wave in Two-Dimensional Electron Liquid in Weak Magnetic Field

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We study the ground state of a clean two-dimensional electron liquid in a weak magnetic field where $N \gg 1$ lower Landau levels are completely filled and the upper level is partially filled. It is shown that the electrons at the upper Landau level form domains with filling factors equal to 1 and zero. The domains alternate with a spatial period of order of the cyclotron radius, which is much larger than the interparticle distance at the upper Landau level. The one-particle density of states, which can be probed by tunneling experiments, is shown to have a gap linearly dependent on the magnetic field in the limit of large N.

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The nature of the ground state of an interacting two-dimensional (2D) electron gas in a magnetic field has attracted much attention. The studies have been focused mostly on the case of very strong magnetic fields where only the lowest Landau level (LL) is occupied, so that the filling factor $\nu = k_F^2 l^2$ does not exceed unity (here k_F is the Fermi wave vector of the 2D gas in zero magnetic field and l is the magnetic length, $l^2 = \hbar/m\omega_c$). The physics at the lowest LL turned out to be so rich that, perhaps, only at $\nu = 1$ does the ground state have a simple structure; namely, it corresponds to one fully occupied spin subband of the lowest LL. The charge density in such a state is uniform. The case of a partial filling, $\nu < 1$, is much more interesting. Using the Hartree-Fock (HF) approximation, Fukuyama, Platzman, and Anderson [1] found that a uniform uncorrelated spin-polarized electron liquid (UEL) is unstable against the formation of a charge density wave (CDW) at wave vectors larger than $0.79l^{-1}$. The optimal CDW period was later found to coincide with that of the classical Wigner crystal (WC) [2].

Subsequently, however, it turned out that non-HF trial states suggested by Laughlin [3] for $\nu = 1/3$ and $1/5$ to explain the fractional quantum Hall effect are lower in energy by a few percent. The Laughlin states were further interpreted in terms of an integer number of fully occupied LL's of new quasiparticles, composite fermions [4]. This concept was then applied to even denominator fractions [5]. Thus, although the HF approximation gives a rather accurate estimate of the energy, it fails to describe important correlations in a partially filled lowest LL.

Recently, the requirement of complete spin polarization in the ground state was also reconsidered. It was found that a partially filled lowest LL may contain Skyrmions [6].

In this Letter we consider the case of weak magnetic fields or high LL numbers N. There is growing evidence from analytical and numerical calculations that fractional states, composite fermions, and Skyrmions are restricted to the lowest and the first excited LL's ($N = 0, 1$) only (see Refs. [7–9]). We will present an additional argument in favor of this conclusion. This point of view is also consistent with the experiment because none of those structures has been observed for $N > 1$.

Before we proceed to the main subject of the paper, a partially filled upper LL, note that we can use the concept of LL's only if the electron-electron interactions do not destroy the Landau quantization. For weak magnetic fields where the cyclotron gap $\hbar\omega_c$ is small, this is far from being evident. To see that the LL mixing is indeed small one has to calculate the interaction energy per particle at the upper LL and verify that its absolute value is much smaller than $\hbar\omega_c$. The largest value of the interaction energy is attained at $\nu = 2N + 1$ where the electron density at the upper LL is the largest. The interaction energy per particle is equal to $-\frac{1}{2}E_{ex}$, where E_{ex} is the exchange-enhanced gap for the spin-flip excitations [10] at $\nu = 2N + 1$ (it determines, e.g., the activation energy between spin-resolved quantum Hall resistivity peaks). Aleiner and Glazman (AG) [9] calculated E_{ex} to be

$$E_{ex} = \frac{r_s \hbar\omega_c}{\sqrt{2}\,\pi} \ln\left(\frac{2\sqrt{2}}{r_s}\right) + E_h, \qquad r_s \ll 1, \quad (1)$$

where E_h is the "hydrodynamic" term (see Ref. [11]) given by [12]

$$E_h = \hbar\omega_c \frac{\ln(Nr_s)}{2N + 1}. \quad (2)$$

The parameter r_s entering these formulas is defined by $r_s = \sqrt{2}/k_F a_B$, $a_B = \hbar^2\kappa/me^2$ being the effective Bohr radius. In realistic samples $r_s \sim 1$ but even at such r_s the ratio $E_{ex}/\hbar\omega_c$ is still rather small. Therefore even at weak magnetic fields the cyclotron motion is preserved and the mixing of the LL's is small. Note that the first term in E_{ex} linearly depends on the magnetic field, whereas E_h has an approximately quadratic dependence.

Since we chose to rely on the HF approximation, a natural turn of thought is to consider a WC-type state whose wave function is given by [9,13]

$$|\Psi\rangle = \prod_i c^\dagger_{\mathbf{R}_i} |0_N\rangle, \quad (3)$$

where $|0_N\rangle$ stands for N completely filled LL's and $c^\dagger_{\boldsymbol{R}}$ is the creation operator for a certain one-particle state, called a coherent state [14]. The modulus of the coherent state wave function is not small only within a distance l off the classical cyclotron orbit with the center at the point $\boldsymbol{R}$ and radius $R_c = k_F l^2$. In the HF WC state $\boldsymbol{R}_i$ coincide with the sites of a triangular lattice with density $\nu_N/2\pi l^2$, where $\nu_N \equiv \nu - 2N$. From now on we consider only $\nu_N \leq \frac{1}{2}$, which suffices because of the electron-hole symmetry.

When ν_N is small, $\nu_N \ll 1/N$, the cyclotron orbits at neighboring lattice sites do not overlap, and the concept of the WC is natural. However, this concept was applied for overlapping orbits as well. According to AG, at $N \gg r_s^{-2} \gg 1$ and not too small ν_N, $\nu_N \gg 1/Nr_s^2$, the cohesive energy of the WC; i.e., the energy per particle at the upper LL with respect to that in the UEL of the same density, is given by [15]

$$E_{\rm coh}^{\rm WC} = -\frac{\hbar\omega_c}{16\pi N}\left[\frac{\sqrt{2}}{r_s} + \frac{3}{2\pi}\ln(N\nu_N)\right] - \frac{1-\nu_N}{2}E_{\rm h}\,. \tag{4}$$

Assuming that the WC is the ground state, AG found that the one-particle density of states (DOS) consists of two narrow peaks separated by the gap $E_{\rm g} = E_{\rm h}$ (see also Ref. [11]). In the limit of large N, both E_g and $|E_{\rm coh}^{\rm WC}|$ are much smaller than $E_{\rm ex}$, and so AG concluded that there are two different scales for spin and charge excitations.

In this Letter we claim that for $\nu_N \gg 1/Nr_s^2$ the ground state is not the WC, but another type of CDW whose period is of order R_c. In contrast to the lowest LL, the optimal CDW period is *much larger* than the average distance between the electrons at the upper LL. The cohesive energy of the CDW has the scale $E_{\rm ex}$ and is given by

$$E_{\rm coh}^{\rm CDW} \approx -f(\nu_N) r_s \hbar\omega_c \ln\left(1 + \frac{0.3}{r_s}\right) - \frac{1-\nu_N}{2}E_{\rm h}\,, \tag{5}$$

where $f(\nu_N) \approx 0.03$ at $\nu_N = \frac{1}{2}$ and $f(\nu_N) \propto \nu_N$ at $1/Nr_s^2 \ll \nu_N \ll \frac{1}{2}$. The DOS consists of two peaks (Van Hove singularities) at the edges of the spectrum, the distance between them for $\nu_N \sim \frac{1}{2}$ being equal to

$$E_{\rm g} \approx \frac{r_s\hbar\omega_c}{\sqrt{2}\,\pi}\ln\left(1 + \frac{0.3}{r_s}\right) + E_{\rm h}\,. \tag{6}$$

Hence, we claim that all the important properties of the Nth LL are determined by the *single* scale $E_{\rm ex}$.

Let us compare $E_{\rm coh}^{\rm WC}$ and $E_{\rm coh}^{\rm CDW}$. The "hydrodynamic" term is the same in both. Hence one has to compare only the remaining terms. It is easy to see that the CDW state wins over the WC provided $\nu_N \gtrsim 1/Nr_s^2$.

Our CDW state can be roughly approximated by a state (3), with $\boldsymbol{R}_i$ forming patterns shown in Fig. 1. The aggregation of many particles in large domains of size R_c allows the system to achieve a lower value of the exchange energy. At the same time, due to the fact that the domain separation is chosen according to the special ringlike shape of the wave functions at the upper LL, the actual charge density variations are not too large (of order 20%). Hence, the increase in the Hartree energy due to the domain formation is small. According to our numerical simulations for $N = 5$ and $r_s = 0.5$, at $\nu_N > 0.3$ the optimal CDW has a "stripe" structure [Fig. 1(a)]. At $\nu_N < 0.3$ a "bubble" pattern [Fig. 1(b)] wins. The distance between the "bubbles" in this pattern is of order R_c and remains approximately the same as ν_N decreases. Correspondingly, their diameter is given by $\sim R_c\sqrt{\nu_N}$. At $\nu_N \sim 1/N$ where it becomes of order l the "bubbles" consist of single electrons, i.e., the CDW state becomes indistinguishable from the WC [Fig. 1(c)]. With further decrease in ν_N, the distance between the electrons increases.

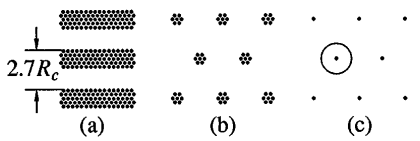

FIG. 1. CDW patterns. (a) Stripe pattern. (b) Bubble pattern. (c) WC. One cyclotron orbit is shown.

At this point we would like to address the issue of the fractional states at high LL's. We believe that at $\nu_N \gg 1/N$ the fractional states cannot compete with the CDW state. Indeed, the CDW state has a very low energy because of the correlations in the positions of the guiding centers on the length scale R_c, which is the largest length scale in a not too dilute system. In the fractional states, just like in the WC, these correlations have the length scale l. Based on the example of the WC, it seems very plausible that the correlations of this type are much less effective. On the other hand, there is no doubt that at $\nu_N \ll 1/N$ the WC is the ground state. This leaves only a narrow window in the vicinity of $\nu_N = 1/N$, where the fractional states may or may not appear.

The novel ground state enables us to explain two interesting experimental findings. One is the magnitude of a pseudogap in the tunneling DOS, first observed in experiments on a single quantum well [16] and, recently, on double quantum well high-mobility GaAs systems [17,18]. The pseudogap $E_{\rm tun}$ appears to be linear in magnetic field for $1 \leq N \leq 4$ [18]. Theoretically, the pseudogap is given by $E_{\rm tun} = 2E_{\rm g}$. The additional factor of 2 arises because the tunneling DOS is the convolution of the DOS of the two wells. For the parameters of Ref. [18] Eq. (6) leads to $E_{\rm tun} \approx 0.52\hbar\omega_c$, which compares favorably with the experimental value of $0.45\hbar\omega_c$ [18]. In the experimental range of parameters the "hydrodynamic" term dominates, and our result is only 35% larger than that of AG, $2E_{\rm h}$. However, in the limit $N \gg 1$ we predict a much wider pseudogap with a linear instead of an approximately quadratic dependence on the magnetic field. Note that even for $1 \leq N \leq 4$ the dependence, which we predict, is not much different from the linear one. Recently, Levitov and Shytov [19] obtained an expression for $E_{\rm tun}$ similar

but not identical to ours without studying the ground state of the system. We believe that only the CDW ground state can justify this type of expression.

Another important application of the proposed picture concerns the conductivity peak width of the integer quantum Hall effect in high-mobility structures where the disorder is believed to be long range. A semiclassical electrostatic model of Efros [20] predicts that the electron liquid is compressible in a large fraction of the sample area. If the compressible liquid is considered to be metallic, then the conductivity peaks are necessarily wide [20], which is indeed observed at relatively high temperatures [21]. However, it is well known that at low temperatures the peaks are narrow (see, e.g., Ref. [22]), which may result from the pinning of the compressible liquid [23]. The fine CDW structure of the compressible liquid (Fig. 1) makes such a pinning possible even though the disorder is long range. Note that, although the pinning prohibits sliding of the CDW as a whole, the current can still flow along the boundaries of the filled and empty regions (the "bulk edge states"). Precisely at $\nu_N = \frac{1}{2}$, the bulk edge states form a percolating network, which leads to a narrow peak in conductivity with, in certain models [24], a universal height $0.5e^2/h$.

We start our analysis by writing down the HF cohesive energy of the electrons at the upper partially filled LL (cf. Refs. [1,2]),

$$E_{\rm coh}^{\rm CDW} = \frac{n_{\rm L}}{2\nu_N} \sum_{\boldsymbol{q}\neq 0} \tilde{u}_{\rm HF}(\boldsymbol{q})\, |\tilde{\Delta}(\boldsymbol{q})|^2. \tag{7}$$

Here and below we use tilde for Fourier transformed quantities, L is the size of the system, $n_{\rm L} = (2\pi l^2)^{-1}$, and $\Delta(\boldsymbol{r})$ is the CDW order parameter. It is proportional to the guiding center density at the point $\boldsymbol{r}$. For instance, the WC corresponds to $\Delta(\boldsymbol{r})$ in the form [2]

$$\Delta(\boldsymbol{r}) \approx \frac{2}{L^2} \sum_i \exp\left[-\frac{(\boldsymbol{r} - \boldsymbol{R}_i)^2}{l^2} \right]. \tag{8}$$

The HF interaction potential $\tilde{u}_{\rm HF}(\boldsymbol{q})$ entering Eq. (7) is the difference of the direct and the exchange terms, $\tilde{u}_{\rm HF}(q) = \tilde{u}_{\rm H}(q) - \tilde{u}_{\rm ex}(q)$, which are further defined by

$$n_{\rm L}\tilde{u}_{\rm ex}(q) = u_{\rm H}(ql^2), \qquad n_{\rm L}\tilde{u}_{\rm H}(q) = \frac{e^2F^2(q)}{q\varepsilon(q)l^2}, \tag{9}$$

$$F(q) = e^{-q^2l^2/4} L_N(q^2l^2/2), \tag{10}$$

L_N being the Laguerre polynomial. Following Ref. [9] (see also Ref. [25]), the screening by the lower LL's is explicitly taken into account with the help of the dielectric constant

$$\varepsilon(q) = \kappa\left\{1 + \frac{2}{qa_{\rm B}}[1 - J_0^2(qR_c)]\right\}. \tag{11}$$

From Eqs. (9) and (10) an asymptotic expression for $\tilde{u}_{\rm HF}(q)$ can be derived,

$$n_{\rm L}\tilde{u}_{\rm HF}(q) \approx \frac{\hbar\omega_c}{\pi}\left\{\frac{1}{2qR_c} - \frac{r_s}{\sqrt{2}} \ln\left(1 + \frac{r_s^{-1}}{\sqrt{2}\,qR_c}\right) + \frac{\sin(2qR_c)}{2qR_c[1 + (r_s/\sqrt{2})]}\right\} - E_{\rm h}\,. \tag{12}$$

We want to find the distribution of the guiding center density $\Delta(x, y)$ that minimizes the energy. Generally, this is a nontrivial problem because the HF equations have to be solved self-consistently. However, if the CDW is unidirectional, i.e., if $\Delta(x, y)$ does not depend on y, the self-consistency condition is simply

$$\Delta(x) = \Theta[-\epsilon_{\rm HF}(x)]/L^2, \tag{13}$$

$$\epsilon_{\rm HF}(x) = \sum_{q\neq 0} n_{\rm L}\tilde{u}_{\rm HF}(q)\tilde{\Delta}(q\hat{\boldsymbol{x}})e^{{\rm i}qx}, \tag{14}$$

where $\epsilon_{\rm HF}(x)$ is the HF self-energy, $\Theta(x)$ is the step function, and $\hat{\boldsymbol{x}}$ is a unit vector in the x direction. The meaning of this condition is that the states above the Fermi level are empty and below the Fermi level are filled.

For $N > 0$ the Hartree potential $\tilde{u}_{\rm H}(q)$ necessarily has zeros due to the factor $F(q)$ containing the Laguerre polynomial [Eqs. (9) and (10)]. The first zero, q_0, is approximately given by $q_0 \approx 2.4/R_c$. The exchange potential is always positive; hence, there exist q where the total HF potential $\tilde{u}_{\rm HF}$ is negative [in agreement with Eq. (12)]. This leads to the CDW instability because the energy can be reduced by creating a perturbation at any of such wave vectors (cf. Ref. [1]).

In the parameter range $0.06 < r_s < 1$ and $N < 50$ well covering all cases of practical interest, the HF potential is negative at all $q > q_0$ and reaches its lowest value near $q = q_0$ (see Fig. 2). One can guess then that the lowest energy CDW is the one with the largest possible [under the conditions (13) and (14)] value of $|\tilde{\Delta}(q_0\hat{\boldsymbol{x}})|$. The CDW having this property consists of alternating strips $\Delta(x) = 0$ and $\Delta(x) = 1/L^2$ [Fig. 1(a)], and nonzero $\tilde{\Delta}(\boldsymbol{q})$ are given by

$$\tilde{\Delta}(q\hat{\boldsymbol{x}}) = \frac{q_0}{\pi q} \sin\left(\frac{\pi\nu_N q}{q_0}\right), \tag{15}$$

provided q is an integer multiple of q_0. Our numerical simulations showed that at ν_N close to $\frac{1}{2}$ this is indeed the correct type of the solution in the specified above range of r_s and N, but q_0 should be replaced by a slightly

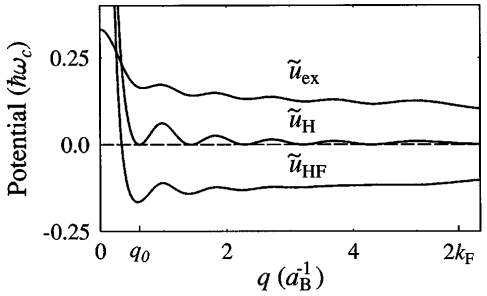

FIG. 2. The Hartree, exchange, and HF potentials in q space for $N = 5$ and $r_s = 0.5$.

smaller value of $2.3/R_c$ corresponding to the spatial period of $2.7R_c$.

Having established the functional form of $\Delta(x)$, let us estimate the cohesive energy $E_{\rm coh}^{\rm CDW}$. Performing the summation in Eq. (7) with the help of Eqs. (12) and (15), one recovers Eq. (5). As for the DOS, it is given by $(n_{\rm L}q_0/\pi)\,|d\epsilon_{\rm HF}/dx|^{-1}$. It can be verified that $\epsilon_{\rm HF}(x)$ reaches its lowest and largest values at $x = 0$ and $x = \pi/q_0$, respectively. These extrema result in the Van Hove singularities at the edges of the spectrum separated by the gap $E_g = 2|\epsilon(0)|$. Equation (6) now follows from Eqs. (12), (14), and (15).

So far we discussed the unidirectional CDW, which can be analyzed at least partially analytically. 2D CDW patterns were studied numerically. We restricted the choice of $\Delta(\boldsymbol{r})$ to the form (8) suggested by the WC state. Recall that in the WC state $\boldsymbol{R}_i$ coincide with the sites of a triangular lattice with density $\nu_N n_{\rm L}$. In the simulations we used a different set of $\boldsymbol{R}_i$, corresponding to the triangular lattice with the density $n_{\rm L}$. The fraction ν_N of the total of 50×50 lattice sites was initially randomly populated, and then the energy was numerically minimized with respect to different rearrangements of the populated sites. The expression for the energy follows from Eqs. (7) and (8):

$$E \approx \frac{1}{2}\sum_{i,j} g_{\rm HF}(\boldsymbol{R}_i - \boldsymbol{R}_j)\,(n_i - \nu_N)\,(n_j - \nu_N)\,, \quad (16)$$

where $\tilde{g}_{\rm HF}(q) = \exp(-\frac{1}{2}q^2l^2)\,\tilde{u}_{\rm HF}(q)$ and n_i is the occupancy of the ith site. In this notation the energy has a transparent interpretation of pairwise interaction among the single-electron states $|c^\dagger_{\boldsymbol{R}_i}\rangle$. In the actual simulations we used a slightly more accurate expression with $g_{\rm HF}(r)$ replaced by $g_{\rm HF}(r)/[1 - \exp(-r^2/2l^2)]$ (cf. Refs. [9,13]). The patterns obtained from the simulations are schematically shown in Fig. 1 and were discussed above.

In conclusion, we have argued that the ground state of a partially filled upper LL in a weak magnetic field is a CDW with a large period of order R_c. Based on this, we were able to explain several important experimental results.

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