

Tidal Stabilization of Neutron Stars and White Dwarfs

Dong Lai*

Theoretical Astrophysics, 130-33, California Institute of Technology, Pasadena, California 91125
(Received 29 January 1996)

What happens to a neutron star or white dwarf near its maximum mass limit in a close binary orbit? Using an energy variational principle the tidal field reduces its central density, making it stable against radial collapse. For a cold white dwarf, the tidal field increases the maximum stable mass, but lowers the maximum central density by 30%. A white dwarf in a close binary may be more susceptible to the general relativistic instability than that with electron capture or pycronuclear reaction. We analyze radial stability of a neutron star using post-Newtonian approximation with a degenerate neutron gas equation of state. Tidal stabilization implies that the neutron star in coalescing neutron star–neutron star or neutron star–black hole binaries does not collapse prior to merger or tidal disruption. [S0031-9007(96)00496-6]

PACS numbers: 97.80.Fk, 04.25.Nx, 04.40.Dg, 97.60.Jd

It is well established that there exist upper limits to the mass and central density of degenerate compact objects, white dwarf (WD) and neutron star (NS) [1,2]. For WD, the mass asymptotes to the Chandrasekhar limit as the central density increases, bounded by general relativistic radial instability, electron capture, or pycronuclear reaction. For NS, the existence of an absolute mass limit is truly a general relativistic phenomena: pressure, which supports the star against gravity, also acts as a source of gravitation.

WD and NS frequently appear in binary systems. Type Ia supernova is thought to arise from an accreting WD in a close binary or the merger of a WD-WD binary [3]. Of particular interest is the coalescing NS-NS binaries and NS–black hole binaries, which are the most promising sources of gravitational waves that could be detected by interferometers such as LIGO and VIRGO [4–6]. NS binary merger is also considered to be a natural engine that drives cosmological gamma-ray bursts. Although the inspiral at larger orbital radius may be treated by the post-Newtonian expansion technique [7], coupled with semianalytic studies of the hydrodynamical effects [8–13], quantitative understanding of the orbital evolution at small separation and the final merging requires full numerical simulation, which still is in its infancy [14–18].

The present study is motivated by the recent general relativistic hydrodynamical simulations of Wilson and co-workers [19] which reveal evidence that “general relativistic effects may cause otherwise stable neutron stars to individually collapse prior to merging.” Obviously, this requires the neutron star to have a mass close to its maximum value to begin with (at large separation). However, it does raise a question as to how the tidal field modifies the mass limit and central density limit of a compact object. In this Letter, we analyze the radial stability of a neutron star and white dwarf under the influence of the tidal field of a companion. We show that the tidal effect generally stabilizes the star. We adopt the energy varia-

tional principle as widely used in the analysis of stability of isolated WD and NS [1,2,20].

Let $E(\rho)$ be the energy of an isolated object of baryon mass M as a function of the central density ρ . The function $E(\rho)$ consists of an internal energy and self-gravitational energy, including their relativistic corrections. The equilibrium central density ρ_0 is obtained from the condition $\partial E/\partial \rho = 0$; radial stability requires $\partial^2 E/\partial \rho^2 > 0$.

Now for the star in a binary, the dimensionless tidal distortion is of order $\sim \varepsilon \equiv (M'/M)(R/r)^3$, where $R \propto (M/\rho)^{1/3}$ is the mean stellar radius, M' the companion mass, and r the orbital separation. The total stellar energy can be written as $\mathcal{E}(\rho) = E(\rho) + W_t(\rho)$. Note that the tidal distortion modifies the “intrinsic” energy $E(\rho)$, but we will group this correction into $W_t(\rho)$. Thus the function $W_t(\rho)$ consists of (i) the correction ΔW to the self-gravitational potential energy, (ii) the interaction energy W_i between M' and the tide-induced quadrupole, and (iii) the kinetic energy T_s of internal fluid oscillation and rotation. The first two contributions are of the same order $\sim (GM^2/R)\varepsilon^2 \propto 1/r^6$, but have opposite signs, while the third $T_s \sim MR^2\varepsilon^2\Omega^2 \propto 1/r^9$ is a factor $(1 + M'/M)(R/r)^3$ smaller, and will be neglected. [We assume the star has negligible viscosity to be tidally synchronized, as is the case for compact objects [8,9]. We also assume the star has zero intrinsic spin—finite spin tends to stabilize the star against radial collapse, and can be treated separately (e.g., [2]). Note that there is no ambiguity in our definition of the function $\mathcal{E}(\rho)$ for stability analysis: One could add to $\mathcal{E}(\rho)$ a term $T = T_s + T_{\text{orb}}$ associated with the kinetic spin and orbital energies (so that \mathcal{E} can be considered as the total energy of the binary system). However, differentiation of T with respect to ρ under fixed total angular momentum J and fluid circulation C gives $(\partial T/\partial \rho)_{J,C} = 2T_s/3\rho$, and the orbital motion does not affect the stellar structure.]

To calculate W_t , we model the star as a polytrope (with equation of state $P = K\rho^{1+1/n}$; n is the polytropic index). Approximating the tidally deformed star as an ellipsoid with axes $a_i = R(1 + \alpha_i)$ (a_1 is along the tidal bulge), the tidal distortion can be calculated in Newtonian theory. To leading order in ε , we have [21]

$$\begin{aligned}\alpha_1 &= \frac{5}{2} q_n \frac{M'}{M} \left(\frac{R}{r}\right)^3, \\ \alpha_2 &= \alpha_3 = -\frac{5}{4} q_n \frac{M'}{M} \left(\frac{R}{r}\right)^3,\end{aligned}\quad (1)$$

where $q_n = \kappa_n(1 - n/5)$, and κ_n is defined such that the moment of inertia of an isolated object is $I = (2/5)\kappa_n MR^2$ (κ_n specifies the mass concentration within the star). Note that $\alpha_1 + \alpha_2 + \alpha_3 = 0$ to leading order in ε . The two contributions to W_t are

$$\begin{aligned}\Delta W &= \frac{3}{5-n} \frac{GM^2}{R} \frac{4}{45} \\ &\quad \times (\alpha_1^2 + \alpha_2^2 + \alpha_3^2 - \alpha_1\alpha_2 - \alpha_2\alpha_3 - \alpha_1\alpha_3) \\ &= \frac{3}{4} \kappa_n q_n \frac{GM'^2 R^5}{r^6}\end{aligned}\quad (2)$$

and

$$\begin{aligned}W_i &= -\frac{GM'}{2r^3} \left(\frac{2}{5} \kappa_n MR^2\right) (2\alpha_1 - \alpha_2 - \alpha_3) \\ &= -\frac{3}{2} \kappa_n q_n \frac{GM'^2 R^5}{r^6}.\end{aligned}\quad (3)$$

Thus, up to order ε^2 , the total tidal energy is

$$W_t = -\lambda \frac{GM'^2 R^5}{r^6} \sim -\lambda \frac{GM'^2}{r^6} (M/\rho)^{5/3}, \quad (4)$$

with $\lambda = (3/4)\kappa_n q_n$. Note that the negative sign in W_t is crucial for the tidal stabilization of the radial mode.

The equilibrium condition in the presence of a tide requires $\partial\mathcal{E}/\partial\rho = 0$. Thus the density change $\delta\rho = \rho - \rho_0$ (where ρ_0 is the equilibrium density of the isolated object) due to the tidal field is given by

$$\delta\rho = \frac{5W_t}{3\rho} \left(\frac{\partial^2 E}{\partial\rho^2}\right)^{-1} \Big|_{\rho_0}, \quad (5)$$

with the expression evaluated at ρ_0 . Since $\partial^2 E/\partial\rho^2 > 0$ for stable configuration, we see that $\delta\rho < 0$, i.e., *the tidal field reduces the central density of a stable object*. If the star is not too close to the stability limit, we have an estimate $\delta\rho/\rho_0 \sim -\varepsilon^2$, which is of second order in the tidal deformation ε . Heuristically, a tidally distorted object is less bound gravitationally (compared to a spherical object of the same mass); thus its volume expands in order to satisfy hydrostatic equilibrium.

Now consider how the tidal field changes the maximum mass M_m of the object. Let ρ_{m0} and M_{m0} be the zero-tide values of the maximum density and max-

imum mass, at which $\partial E/\partial\rho = \partial^2 E/\partial\rho^2 = 0$ is satisfied. Taking the difference between $\partial\mathcal{E}/\partial\rho|_{\rho_m, M_m} = 0$ and $\partial E/\partial\rho|_{\rho_{m0}, M_{m0}} = 0$, we obtain

$$\delta M_m = M_m - M_{m0} = \frac{5W_t}{3\rho} \left(\frac{\partial^2 E}{\partial\rho\partial M}\right)^{-1} \Big|_{\rho_{m0}, M_{m0}}. \quad (6)$$

Since we can show $\partial^2 E/\partial\rho\partial M < 0$, we find $\delta M_m > 0$, i.e., *the tidal field increases the maximum mass for stability*. [Consider a sequence of stellar models parametrized by the central density. Differentiating $\partial E/\partial\rho = 0$ we obtain $\partial^2 E/\partial\rho^2 + (\partial^2 E/\partial\rho\partial M)dM/d\rho = 0$. The stable branch of the sequence satisfies $\partial^2 E/\partial\rho^2 > 0$ and $dM/d\rho > 0$, while the unstable branch satisfies $\partial^2 E/\partial\rho^2 < 0$ and $dM/d\rho < 0$. In both cases we have $\partial^2 E/\partial\rho\partial M < 0$.] An order of magnitude estimate gives $\delta M_m/M_{m0} \sim \lambda\varepsilon^2$.

Note that two implicit assumptions have been made in our analysis: (i) The star ‘‘relaxes’’ to its equilibrium shape even when the density is out of equilibrium. This separation of the radial motion and the ‘‘shape adjustment’’ makes our analysis of radial instability more transparent. It is valid because near the stability limit, the radial oscillation has almost zero frequency. (ii) The displacement of a fluid element inside the star is a linear function of the fluid position. This is exact only for the $n = 3$ polytrope and in the incompressible limit, but otherwise corresponds to an approximate trial wave function in the variational principle.

We now consider the stability of white dwarfs. Near the maximum mass, the stellar density profile resembles that of a $n = 3$ polytrope. The intrinsic energy can be written as [1,2]

$$E = E_{\text{int}} + W + \Delta E_{\text{int}} + \Delta E_{\text{GR}}, \quad (7)$$

where $E_{\text{int}} \sim M\rho^{1/3}$ is the internal energy of the ultrarelativistic electrons, $\Delta E_{\text{int}} \sim M\rho^{-1/3}$ is the correction due to finite electron mass, $W \sim -M^2/R \sim -M^{5/3}\rho^{1/3}$ is the potential energy of self-gravity, and $\Delta E_{\text{GR}} \sim -(M^2/R)M/R \sim -M^{7/3}\rho^{2/3}$ is the general relativistic (post-Newtonian) correction. For isolated cold WD, this gives for the maximum central density and maximum mass $\rho_{m0} = 2.737 \times 10^{10} \text{ g cm}^{-3}$, $M_{m0} = 1.4156M_\odot$, with the corresponding minimum radius $R_{m0} = 1110 \text{ km}$. (We assume that the number of electron per nucleon is $Y_e = 0.5$.)

We parametrize the strength of the tidal field by the dimensionless ratio $\beta \equiv (M'/M_{m0})(R_{m0}/r)^3$. To avoid tidal disruption we require $\beta \lesssim 0.1$. With $\lambda \approx 0.01$ (for $n = 3$ polytrope) we then have

$$W_t \approx -0.01\beta^2 \frac{GM_{m0}^2}{R_{m0}} \left(\frac{M}{M_{m0}}\right)^{5/3} \left(\frac{\rho}{\rho_{m0}}\right)^{-5/3}. \quad (8)$$

Expressing mass in the units of M_\odot , density in $10^{10} \text{ g cm}^{-3}$, energy in 10^{51} ergs , the total energy

can be written as

$$\begin{aligned} \mathcal{E} = & 3.7129M\rho^{1/3} - 2.8895M^{5/3}\rho^{1/3} \\ & + 0.0457M\rho^{-1/3} - 0.0105M^{7/3}\rho^{2/3} \\ & - 0.14\beta^2(M/\rho)^{5/3}. \end{aligned} \quad (9)$$

Solving $\partial\mathcal{E}/\partial\rho = \partial^2\mathcal{E}/\partial\rho^2 = 0$ yields the maximum density and maximum mass as a function of β . The result is shown in Fig. 1. The increase in the maximum mass is rather small ($\leq 0.1\%$). However, *the maximum density decrease can be rather substantial, reaching as much as $\sim 30\%$* . For small β , we have $\delta M_m/M_{m0} \approx 0.05\beta^2$, and $\delta\rho_m/\rho_{m0} \approx -17\beta^2$. Figure 1 also shows the central density of three constant-mass sequences with mass slightly below M_{m0} . We see that as the binary separation decreases (β increases), the central densities also decrease, and always remain smaller than the maximum density $\rho_m(\beta)$ allowed for radial stability.

The fact that the maximum density $\rho_m(\beta)$ for stability decreases with decreasing r may have some interesting astrophysical consequences. The maximum central density of normal carbon-oxygen white dwarfs is set by the threshold of electron capture on ^{16}O , at $\rho_{\text{cap}} = 1.9 \times 10^{10} \text{ g cm}^{-3}$ (the pycnonuclear reaction between ^{12}C may set in at smaller density, of order $10^{10} \text{ g cm}^{-3}$; but this increases as the ^{12}C abundance decreases [22]). From Fig. 1 we see that $\rho_m(\beta)$ can drop below ρ_{cap} ; therefore, the general relativistic (GR) effect becomes more important than neutronization. This implies that an accreting white dwarf in close binary can be more susceptible to collapsing to NS by general relativistic radial instability. Of course, the real situation may be complicated by the finite temperature due to the high accretion rate.

We now consider neutron stars. The equation of state of nuclear matter is uncertain. For the purpose of illustra-

tion, we adopt an equation of state to be that of an ideal degenerate neutron gas and analyze the radial stability in the post-Newtonian approximation. Assuming the stellar density profile to be that of the $n = 3/2$ polytrope, the intrinsic energy can still be written as in Eq. (7), except that $E_{\text{int}} \sim M\rho^{2/3}$ is the internal energy of nonrelativistic neutrons, $\Delta E_{\text{int}} \sim -M\rho^{4/3}$ is the correction due to special relativistic effect. The maximum baryon mass of an isolated NS based on this model is $M_{m0} = 1.1108M_\odot$, the maximum central density $\rho_{m0} = 7.415 \times 10^{15} \text{ g cm}^{-3}$, and the corresponding minimum radius $R_{m0} = 10.48 \text{ km}$. We again parametrize the tidal strength by the ratio $\beta \equiv (M'/M_{m0})(R_{m0}/r)^3$. The tidal energy can then be written in the form of Eq. (8) except that we use $\lambda \approx 0.1$ appropriate for $n = 3/2$ polytrope. (The uncertainty can be readily absorbed into the definition of β .) The GR correction to the tidal potential energy is of order M'/r smaller, and is neglected. Expressing mass in the units of M_\odot , density in $10^{15} \text{ g cm}^{-3}$, and energy in 10^{53} ergs , the total energy function can be written as

$$\begin{aligned} \mathcal{E} = & 0.85168M\rho^{2/3} - 1.5968M^{5/3}\rho^{1/3} \\ & - 0.02887M\rho^{4/3} - 0.16774M^{7/3}\rho^{2/3} \\ & - 7.356\beta^2(M/\rho)^{5/3}. \end{aligned} \quad (10)$$

Figure 2 shows the maximum density ρ_m and maximum mass M_m as a function of β . We see that, as in the white dwarf case, ρ_m decreases and M_m increases with increasing β . For small β (≤ 0.15), we have

$$\frac{\delta M_m}{M_{m0}} \approx -\frac{15W_t}{4(2E_{\text{in}} + W)} = 0.33\beta^2, \quad (11)$$

$$\frac{\delta\rho_m}{\rho_{m0}} \approx -\frac{5W_t(28E_{\text{in}} + 15W)}{2W(2E_{\text{in}} + W)} = -2.7\beta^2. \quad (12)$$

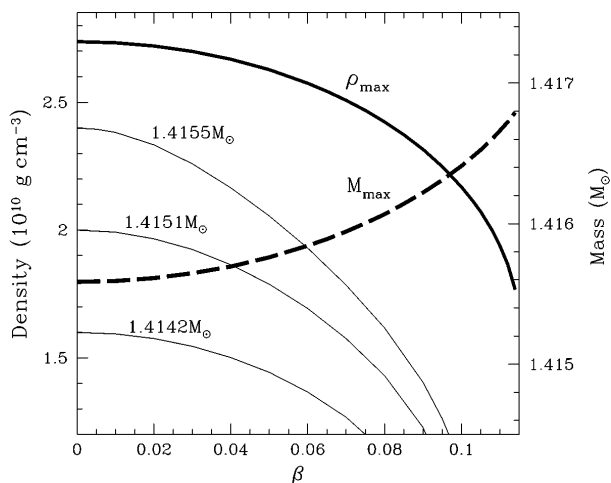


FIG. 1. The maximum central density (heavy solid line) and maximum mass (dashed line) of a white dwarf as a function of the tidal ratio $\beta \equiv (M'/M_{m0})(R_{m0}/r)^3$. The solid lines show the central densities of three sequences with mass slightly smaller than maximum mass of a isolated WD $M_{m0} = 1.4156M_\odot$.

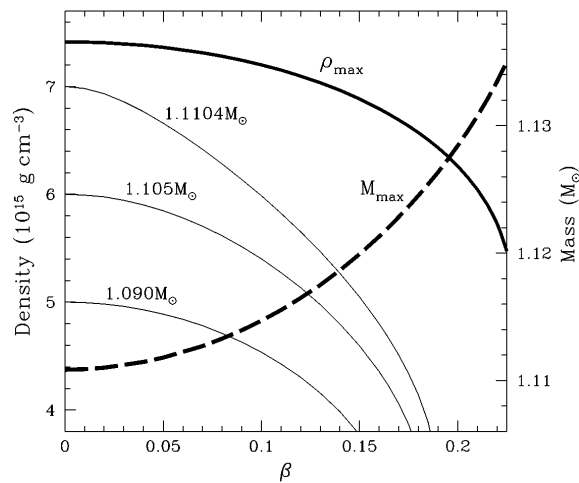


FIG. 2. The maximum central density (heavy solid line) and maximum mass (dashed line) of a neutron star as a function of the tidal ratio $\beta \equiv (M'/M_{m0})(R_{m0}/r)^3$. The solid lines show the central densities of three sequences with mass slightly smaller than maximum mass of an isolated NS $M_{m0} = 1.1108M_\odot$.

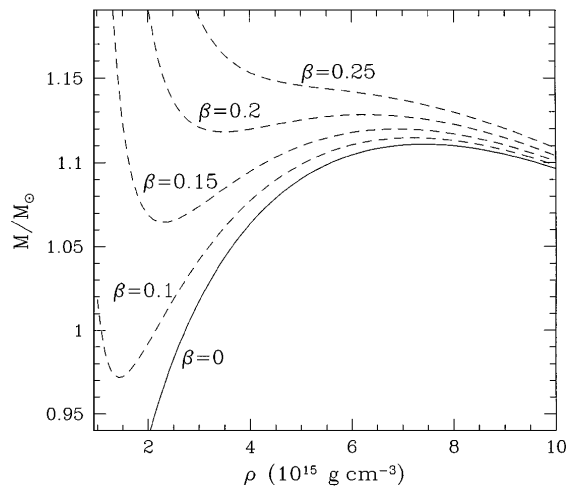


FIG. 3. The mass-central density curves of sequences of NS models with constant values of β . ($\beta = 0$ corresponds to isolated NS.)

Figure 2 also depicts the densities of constant-mass sequences with M slightly below M_{m0} . Again, we see that these densities $\rho(\beta)$ always remain below $\rho_m(\beta)$; i.e., these stars are stable against radial perturbation until the tidal limit $\beta \approx 0.23$ is reached.

Another way to look at the tidal effect on the radial stability is to construct a sequence of stellar models with varying central densities at a given fixed β . Figure 3 shows the M - ρ curves of several such sequences. Only models in the $dM/d\rho > 0$ branch are stable, and the maximum density ρ_m and maximum mass M_m are given by the values at the turning point $dM/d\rho = 0$. One can readily see that the effects of the tidal field is to raise M_m and lower ρ_m . For $\beta \geq 0.23$, no stable configuration exists, and this corresponds to the tidal disruption limit.

The above analysis demonstrates that the tidal effect tends to stabilize a neutron star and white dwarf against radial collapse, at least within the framework of post-Newtonian theory. Our method is accurate for white dwarfs, but only approximate for neutron stars. However, we think it is unlikely that higher order GR corrections or the use of a more sophisticated nuclear equation of state [19] will change our qualitative results for neutron stars, although the precise numbers can certainly change.

In the cases of NS-NS binaries, one might still consider the possibility of neutron star collapse prior to merging when there is stable mass transfer from its lower mass companion. However, by the time mass transfer starts, the binary must already have encountered the orbital dynamical instability as a result of strong tidal interaction [10]. This tidal instability, enhanced by general relativistic effects [23], leads to rapid coalescence of the binary within a few orbits [13,24].

I thank Kip Thorne for comments on an earlier version of the Letter and helpful suggestions. This research is supported by the Richard C. Tolman Fellowship in

theoretical astrophysics at Caltech, NSF Grant No. AST-9417371, and NASA Grant No. NAG 5-2756.

*Electronic address: dong@tapir.caltech.edu

- [1] Ya. B. Zel'dovich and I. D. Novikov, *Relativistic Astrophysics* (Univ. of Chicago Press, Chicago, 1971), Vol. 1.
- [2] S. L. Shapiro and S. A. Teukolsky, *Black Holes, White Dwarfs and Neutron Stars* (John Wiley & Sons, New York, 1983).
- [3] S. E. Woosley and T. A. Weaver, *Annu. Rev. Astron. Astrophys.* **24**, 205 (1986); D. Branch, M. Livio, L. R. Yungelson, F. R. Boffi, and E. Baron, *Publ. Astron. Soc. Pac.* **107**, 1019 (1995); see also I. Iben and A. V. Tutukov, *Astrophys. J. Suppl.* **54**, 335 (1984).
- [4] A. Abramovici *et al.*, *Science* **256**, 325 (1992).
- [5] C. Cutler *et al.*, *Phys. Rev. Lett.* **70**, 2984 (1993).
- [6] K. S. Thorne, in *Proceedings of Snowmass 94 Summer Study on Particle and Nuclear Astrophysics and Cosmology*, edited by E. W. Kolb and R. Peccei (World Scientific, Singapore, 1995).
- [7] L. Blanchet, T. Damour, B. R. Iyer, C. M. Will, and A. G. Wiseman, *Phys. Rev. Lett.* **74**, 3515 (1995).
- [8] L. Bildsten and C. Cutler, *Astrophys. J.* **400**, 175 (1992).
- [9] C. S. Kochanek, *Astrophys. J.* **398**, 234 (1992).
- [10] D. Lai, F. A. Rasio, and S. L. Shapiro, *Astrophys. J.* **420**, 811 (1994).
- [11] A. Reisenegger and P. Goldreich, *Astrophys. J.* **426**, 688 (1994).
- [12] D. Lai, *Mon. Not. R. Astron. Soc.* **270**, 611 (1994).
- [13] D. Lai and S. L. Shapiro, *Astrophys. J.* **443**, 705 (1995).
- [14] K. Oohara and T. Nakamura, *Prog. Theor. Phys.* **83**, 906 (1990); M. Shibata, T. Nakamura, and K. Oohara, *ibid.* **88**, 1079 (1992); **89**, 809 (1993).
- [15] F. A. Rasio and S. L. Shapiro, *Astrophys. J.* **401**, 226 (1992).
- [16] M. B. Davies, W. Benz, T. Piran, and F. K. Thielemann, *Astrophys. J.* **431**, 742 (1994).
- [17] X. Zhuge, J. M. Centrella, and S. L. W. Mcmillan, *Phys. Rev. D* **50**, 6247 (1994).
- [18] M. Ruffert, H.-T. Janka, and G. Schäfer, *Astro. Astrophys.* (to be published).
- [19] J. R. Wilson and G. J. Mathews, *Phys. Rev. Lett.* **75**, 4161 (1995); J. R. Wilson, G. J. Mathews, and P. Marronetti, *Phys. Rev. D* (to be published).
- [20] B. K. Harrison, K. S. Thorne, M. Wakano, and J. A. Wheeler, *Gravitation Theory and Gravitational Collapse* (Univ. of Chicago Press, Chicago, 1965).
- [21] D. Lai, F. A. Rasio, and S. L. Shapiro, *Astrophys. J.* **423**, 344 (1994).
- [22] See, e.g., M. Sahriling, *Astro. Astrophys.* **284**, 484 (1994); M. Hernanz, J. Isern, R. Canel, J. Labay, and R. Mochkovitch, *Astrophys. J.* **324**, 331 (1988); S. Ogata, H. Iyetomi, and S. Ichimaru, *ibid.* **372**, 259 (1991).
- [23] The precise location of the "innermost stable orbit" due to GR is still uncertain. See L. E. Kidder, C. M. Will, and A. G. Wiseman, *Phys. Rev. D* **47**, 3281 (1993); N. Wex and G. Schafer, *Classical Quantum Gravity* **10**, 2729 (1993); see also [19].
- [24] D. Lai and A. G. Wiseman (to be published).