## **Quantum-Limited Electrometer Based on Single Cooper Pair Tunneling**

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We propose a system comprising the small-capacitance double Josephson junction shunted as a whole by a small resistance ( $R_S \ll R_Q \equiv h/4e^2$ ). We have calculated the dynamic and noise characteristics of the system as functions of a polarization charge on the central electrode. It has been shown straightforwardly that the energy resolution of such an electrometer implying the "backaction noise" approaches the fundamental quantum limit  $\hbar/2$  when  $T \rightarrow 0$ . [S0031-9007(96)00173-1]

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The normal-state single electron tunneling (SET) transistor has gained general recognition as a highly sensitive electrometer [1] which is used in delicate experiments with single electron charging [2]. Its operation is based on the gate modulation of the current of electrons tunneling one by one through two small-capacitance tunnel junctions. The superconducting state of the transistor opens up the possibility to exploit single Cooper pair tunneling (SCPT) [3,4], in particular, a modulation of the critical current  $I_c$  of the system by polarization charge of the island  $Q_0$  with a period of the pair charge 2e. In the I-V characteristics (see the recent experiments [5] and [6]), the dependence  $I_c(Q_0)$  is, however, manifested in a modulation of the switching current  $I_0$  ( $< I_c$ ) at which the onset of voltage occurs. This is because the small tunnel junctions used in experiments usually have a high resistance R and a large stray capacitance of the leads  $C_L$  and hence fall into the category of underdamped Josephson junctions which have a hysteretic I-V characteristic [7]. In contrast to  $I_c$ , the switching current  $I_0$  is crucially sensitive to temperature variations and the electromagnetic impedance seen by the double junction [5,8]. The dependence  $I_0(Q_0)$ can, therefore, hardly be used for electrometry.

Here we consider the system of two small superconducting junctions shunted by a small resistor  $R_S$  which changes dramatically the system dynamics, turning it into an overdamped mode of operation with a nonhysteretic *I-V* characteristic. When the biasing current exceeds the critical current  $I_c(Q_0)$ , the finite dc voltage across the junctions arises and reacts to variations of polarization charge  $Q_0$ . We calculate dynamic characteristics of such a superconducting SCPT transistor, in particular the charge-to-voltage transfer function. Although in practice the sensitivity of any SET electrometer is deteriorated by low-frequency fluctuations of the background charge in a chip substrate and oxide barriers of junctions, the determination of the intrinsic noise of the system presents an important study. Such a study of the SET transistor in the framework of the orthodox theory [3] has been made by Korotkov et al. [9] who have calculated the noise of the device in classical limit. The main goal of the present

work was to examine the noise in the SCPT transistor in a general case.

Figure 1 shows the schematic of the SCPT transistor, where two small-capacitance  $(C_{1,2})$  Josephson junctions,  $E_C, E_{J1,2} \gg k_B T$ , form inside a small superconducting island which is polarized by the gate electrode via a small capacitance  $C_0 \ll C_{1,2}$ . The charging energy  $E_C = e^2/2C$ , where  $C = C_1 + C_2 + C_0$ , and the Josephson coupling strengths  $E_{J1,2} = (\Phi_0/2\pi)I_{c1,2}$ , where  $I_{c1,2}$  $(I_{c1} \ge I_{c2})$ , are critical currents of individual junctions and  $\Phi_0 = h/2e$  is the flux quantum. Both junctions are shunted by a common resistor  $R_S$  which, in practice, is supposed to be placed on the same chip. High resistance  $(\gg R_S)$  of the current source I is ensured by another resistor (not shown), and the requirements imposed on its size are not stringent because we keep the capacitance of on-chip leads  $C_L \gg C_{1,2}$ . We will omit the tunneling of single quasiparticles in the junctions implying that at  $E_C \ll \Delta$  ( $\Delta$  is the superconducting energy gap) it can safely be suppressed by the parity effect [10,11]. Although the normal resistance of the junctions need not be much larger than  $R_0 \equiv h/4e^2 = 6.47...k\Omega$ , we assume that the pregap (at  $V < 2\Delta/e$ ) quasiparticle resistance is high,  $R_{qp} \gg R_Q$ , and then cotunneling of electrons through two junctions can be neglected. The Hamiltonian of the system then reads

$$H = H_0 + H_C + H_R$$

where

$$H_0 = \frac{(Q + Q_0)^2}{2C} - E_J(\phi) \cos[\varphi + \gamma(\phi)], \quad (1)$$



FIG. 1. The equivalent circuit of the SCPT transistor where the Josephson junctions are denoted by crossed boxes. The signal source (not shown) is coupled to the island via small capacitance (included in  $C_0$ ).

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with

$$E_J(\phi) = (E_{J1}^2 + E_{J2}^2 + 2E_{J1}E_{J2}\cos\phi)^{1/2};$$
  

$$\tan\gamma(\phi) = -[(E_{J1} - E_{J2})/(E_{J1} + E_{J2})]\tan(\phi/2)$$
(2)

is the Hamiltonian of the subsystem, island + junctions [4]. The overall Josephson phase across two junctions  $\phi = \varphi_1 + \varphi_2$ . The island charge operator Q is a variable conjugate to the operator of the semidifference phase,  $\varphi = (\varphi_1 - \varphi_2)/2$ , and they obey the commutational rule  $[\varphi, Q] = 2ie$  [4]. The gate source  $V_g$  fixes the bias charge  $Q_0 = C_0 V_g$  whose variations  $\delta Q_x$  are associated with an input charge [9].  $H_R$  is a Caldeira-Leggett Hamiltonian [12] of the electromagnetic environment consisting of resistor  $R_S$  and capacitance  $C_L/2$  connected in parallel. The term  $H_C = (\Phi_0/2\pi)\phi(I_R - I)$  provides coupling of the subsystem variable  $\phi$  to those of the environment  $I_R$ (operator of the sum current flowing through  $R_S$  and  $C_L$ ) and the source I.

The role of the shunt is crucial. We require  $R_S \ll R_Q$ and this condition, first, makes the characteristic Josephson oscillation frequency  $\omega_c = (2\pi/\Phi_0)I_cR_s$  smaller than the other two basic frequencies in the system,  $E_J/\hbar$ and  $E_C/\hbar$ , and, secondly, guarantees smallness of the quantum fluctuations of the phase  $\phi$  [13]. Hence, variations of  $\phi$  in Eq. (1) can be considered as adiabatic and fluctuations as a perturbation. Moreover, small values of  $\omega_c$  and a correspondingly large electromagnetic wavelength make it possible to fabricate the on-chip bias resistors of rather large size and to define that part of the on-chip wiring which is not yet cut by these resistors and hence contributes to capacitance  $C_L$ . The corresponding McCumber parameter of the effective Josephson junction,  $\beta_c = \omega_c R_S C_L/2$ , can be made  $\leq 1$ , that eliminates the hysteresis in the I-V curve if a self-inductance L of the shunt is small enough,  $\omega_c L \ll R_S$  [7].

For fixed  $\phi$  and  $Q_0$  the Schrödinger equation for the Hamiltonian  $H_0$  [Eq. (1)] is reduced to that for the sole junction, i.e., the Mathieu equation,

$$4\frac{d^2\psi}{d\chi^2} + (\lambda\cos\chi + \varepsilon)\psi = 0, \qquad (3)$$

where  $\lambda$  denotes the ratio of the Josephson and charging energies  $E_J/E_C$ . When the eigenvalue  $\varepsilon = E/E_C$  lies within the allowed band  $\varepsilon^{(n)}(k, \lambda)$ , the solution is a Bloch wave  $\psi_k^{(n)}(\chi)$  where n = 0, 1, 2, ... is the band index, k = q/2e the Bloch vector, and q the quasicharge [14]. The ground state is n = 0 (we will omit this index) and  $k = k_0 = Q_0/2e$  with  $\chi = \varphi + \gamma$ , and while temperature and perturbations are small the system remains in this state. Averaging over the ground state allows two quantities important for further analysis to be calculated, the supercurrent  $I_s = \langle I_{c1} \sin \varphi_1 \rangle = \langle I_{c2} \sin \varphi_2 \rangle$  and the potential of the island  $U = (\Phi_0/2\pi) \langle \dot{\varphi} \rangle$ , both of which are  $2\pi$ -periodic functions of  $\phi$  and 2e-periodic functions of  $Q_0$ . Making use of Eq. (2) and performing a trigonometric transformation, we finally get

$$I_{s}(\phi, Q_{0}) = I_{s}^{0} \langle \cos \chi \rangle = I_{s}^{0} \frac{\partial \varepsilon}{\partial \lambda} \Big|_{\lambda = E_{J}(\phi)/E_{C}, \ k = k_{0}}, \quad (4)$$

$$I_{s}^{0}(\phi) = \frac{2\pi}{\Phi_{0}} \frac{E_{J1}E_{J2}}{E_{J}(\phi)} \sin\phi , \qquad (5)$$

where  $I_s^0(\phi)$  stands for the usual  $(\lambda \to \infty)$  supercurrent of a double junction while the factor  $\langle \cos \chi \rangle$  describes its partial suppression by the charging effect (in the limit cases of  $\lambda \ll 1$  and  $\lambda \gg 1$  the expression for supercurrent has been earlier derived by Likharev [15]). The second equality in Eq. (4) is justified by differentiating Eq. (3) which has been first multiplied by  $\psi^*(\chi)$  and integrated over the period of  $2\pi$ . Voltage *U* is expressed via  $\varepsilon(k, \lambda)$ in a way similar to that in Ref. [14] in the single-band approximation,

$$U(\phi, Q_0) = \frac{e}{4C} \frac{\partial \varepsilon}{\partial k} \bigg|_{\lambda = E_I(\phi)/E_C, \ k = k_0}, \tag{6}$$

and its variations can be expressed in units of input charge via the differential capacitance defined as  $C_d = (\partial U/\partial Q_0)^{-1} = 8C(\partial^2 \varepsilon/\partial k^2)^{-1}$ . We have developed a numerical method of solving Eq. (3) for arbitrary  $\lambda$  and computed the quantities  $\langle \cos \chi \rangle = \partial \varepsilon/\partial \lambda$  and  $\partial \varepsilon/\partial k$  as functions of the Bloch vector k. The plots are presented in Fig. 2.

When the bias current exceeds the critical current,  $I > I_c = \max_{\phi} [I_s(\phi, Q_0)]$ , the phase slowly varies (with a rate of  $\sim \omega_c$ ). Using adiabaticity and first neglecting fluctuations, we obtain for the classical  $\phi$  the regular Josephson equation of motion ( $\beta_c \ll 1$ ),

$$(\Phi_0/2\pi R_S)\phi + I_s(\phi, Q_0) = I.$$
 (7)

As can be seen from Eqs. (2), (4), and (5), the supercurrent  $I_s(\phi)$  is strictly of  $\sin\phi$  shape when junctions are highly dissimilar,  $E_{J1} \gg E_{J2}$ . In this case the effective Josephson coupling  $E_J \approx E_{J1}$ ,  $\lambda \approx \lambda_0 = E_{J1}/E_C$ , and  $I_c = I_{c2}\langle\cos\chi\rangle$ . Note that if  $Q_0$  is not close to the values  $(2m + 1)e, m = 0, \pm 1, \ldots$ , even for another extreme case of similar junctions,  $E_{J1} = E_{J2} (\sim E_C)$ , the supercurrent, as is proved by calculation, can be fairly approximated by  $\propto \sin\phi$ . Then Eq. (7) is the resistively shunted junction (RSJ) model equation which is integrated explicitly giving the hyperbolic I-V curve, i.e., dependence of the average voltage upon I,  $V = R_S[I^2 - I_c^2(Q_0)]^{1/2}$ . The charge-to-voltage response function is

$$\eta = \frac{\partial V}{\partial Q_x} = -\frac{R_d I_{c2}^2}{2eI} g(k_0, \lambda_0), \qquad (8)$$

where  $R_d = \partial V / \partial I = R_s^2 I / V$  is the differential resistance and  $g = \partial^2 \varepsilon / \partial k \partial \lambda$ . For practical use, Eq. (8) can be rewritten as

$$\eta = -\frac{\pi}{4} \left( \frac{R_S I_{c2}}{V} \right) \left( \frac{E_{J2}}{E_{J1}} \right) \left( \frac{R_S}{R_Q} \right) \lambda g C^{-1}$$

The numerical evaluation of  $\max_k [\lambda g(\lambda, k)]$  shows that it gradually rises up to 1.54... (at  $k \rightarrow 1$ ) when  $\lambda$  decreases



FIG. 2. The functions  $\langle \cos \chi \rangle = \partial \varepsilon / \partial \lambda$  [the factor of the Josephson current suppression Eq. (4)] and  $\partial \varepsilon / \partial k$  (the voltage across a single Bloch junction normalized by e/4C) vs the Bloch vector  $k = k_0 = Q_0/2e$  for  $\lambda = E_J/E_C = 0.02, 0.1, 0.2, 0.4, 0.6, 0.8, 1, 1.3, 1.6, 2, 2.6, 3.2, 4, 6, and 10 [from bottom to top in (a) and in the order of decreasing the amplitude in (b)].$ 

to zero and for  $\lambda = 1$  it already approaches 1.47... (at  $k \approx 0.41$ ). On the assumption that  $E_{J1} \sim E_{J2} \sim E_C$ , we have an estimate  $\eta \sim (R_S I_c/V) (R_S/R_Q)C^{-1}$ . At a sufficiently low temperature, when thermal rounding of the *I*-V curve is small, the prefactor  $R_S I_c/V$  can be increased by reducing *V*. Thus the response of the SCPT transistor can reach and even exceed, for instance, the maximum value characterizing the symmetric  $(C_1 = C_2 \gg C_0)$  orthodox SET transistor  $\eta_{\text{SET}} \sim 2C^{-1}$ . Similar to the asymmetric  $(C_1 \gg C_2 \gg C_0)$  SET transistor, in which the better response  $(\sim C_2^{-1})$  is achieved at the expense of the smaller operating range for the dc charge bias,  $\Delta Q_0 \sim 0.5(C_2/C_1)e$ , the response of the SCPT transistor is also critical to variations of  $Q_0$ . For example, changing of  $Q_0$  from 0.8*e* (optimum value for  $\lambda = 1$ ) to 0.6*e* results in 2 times smaller value of  $\eta$ .

Since there is no quasiparticle tunneling in the system, the only source of noise is the environment represented by the impedance  $Z(\omega) = (R_S^{-1} + i\omega C_L/2)^{-1}$ . The fluctuating part of the operator of its current  $I_R$  equals

 $I_f = I_R - \langle I_R \rangle_0$ , where statistical averaging is performed over the equilibrium states of reservoir, and has a spectral density [16]

$$S_I(\omega) = (\hbar \omega / \pi R_S) \coth(\hbar \omega / 2k_B T).$$

The operator  $-I_f$  now appears on the right-hand side of Eq. (7) as a small perturbation. By linearizing this equation with respect to small fluctuations  $\phi_f$  of the phase  $\phi$  and solving it [7], we find two key noise figures: First, the spectral density of voltage fluctuations  $V_f =$  $(\Phi_0/2\pi)\dot{\phi}_f$  at low frequency  $\omega \ll \omega_J, k_BT/\hbar$ , where  $\omega_J = (2\pi/\Phi_0)V$  is the Josephson oscillation frequency (see [7] and [13] for details),

$$S_V(0) = \frac{2R_d^2}{\pi R_S} \left( k_B T + \frac{I_c^2}{2I^2} eV \coth\frac{eV}{k_B T} \right), \quad (9)$$

and, second, the low-frequency spectral density of the island potential fluctuations  $U_f = (2/\pi)gR_0I_{c2}\sin\phi\phi_f$ ,

$$S_U(0) = \frac{1}{4\pi} \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{g^2 R_S I_{c2}^2}{eV} \coth \frac{eV}{k_B T}.$$
 (10)

The first term on the right-hand side of Eq. (9) is noise generated at frequency  $\omega$ , while the second term as well as whole expression (10) are noise generated near  $\omega_J$ those are converted down to  $\omega$  by the nonlinearity of the supercurrent. The correlation between  $V_f$  and  $U_f$  and, hence, the cross spectrum  $S_{VU}(0)$  turns out to be zero.

Introducing the energy of "output" noise, associated with voltage fluctuations  $V_f$  in a unit frequency bandwidth

$$\epsilon_V \equiv (\delta Q_x)^2 / 2C\Delta f = \pi S_V(0) / C \eta^2$$

and using Eq. (9), we can find the equivalent charge noise  $\delta Q_x = V_f / \eta$  in the output bandwidth  $\Delta f$  determined by the postdevice filter,

$$\frac{\delta Q_x}{e} = a \left[ \left( \frac{k_B T}{2E_C} + \frac{eV}{4E_C} \coth \frac{eV}{k_B T} \right) \frac{R_Q^2}{R_S} C\Delta f \right]^{1/2}.$$
 (11)

This expression is obtained for  $I \approx I_c$ ; the numerical factor is  $a = (8/\pi)I_cI_{c1}/\lambda gI_{c2}^2$ . The value of *a* smoothly approaches its absolute minimum  $\approx 0.83$  for a symmetric transistor with  $\lambda \rightarrow 0$  and  $Q_0 \rightarrow e$ . Strictly speaking, in this limit the function  $I_s(\phi)$  in Eq. (7) is not of sine shape and Eqs. (9) and (11) derived from the RSJ model are not quantitatively correct. These expressions underestimate the noise because it is somewhat enhanced due to parametric conversion from the higher harmonics of the Josephson frequency  $\omega_{J,m} = 2, 3, \ldots$ , down to the observation frequency  $\omega$ . However, for  $\lambda = 1$  the closely approximating minimum value  $a \approx 0.97$  is already reached at  $Q_0 \approx 0.8e$ , i.e., when Eq. (11) is almost valid. We therefore deduce that  $a_{\min} \sim 1$  and this value is approached for  $\lambda \sim 1$ .

In the *classical* limit ( $k_BT \gg eV$ ), our *general* result (11) can be compared with a figure of the orthodox SET electrometer [9]. If we assume the similar junction capacitances of both devices and the equivalent tunneling resistances of the SET transistor to be  $R \equiv R_1 = R_2 =$ 

 $R_Q^2/R_S \ge R_Q$ , we conclude that Eq. (24) of Ref. [9] gives approximately twice as large a value for  $\delta Q_x^{\text{SET}}$  as Eq. (11) does. The classical noise characteristics of both devices are therefore nearly similar. In the *quantum* limit (T = 0), both  $\delta Q_x$  and  $\epsilon_V$  go to zero according to Eq. (11) as  $V \to 0$ . This is in contrast to the SET transistor where the noise floor is set by quantum noise due to cotunneling,  $(\epsilon_V^{\text{SET}})^{\min} \sim (R_Q/2R)\hbar$  [see the rough estimate of Eq. (28) of Ref. [9]]. Note that the energy sensitivity  $\epsilon_V$  of both devices in the considered regime of a low-frequency input signal can be made better than  $\hbar/2$ .

Finally, using Eqs. (9) and (10) we can find the potential sensitivity  $\epsilon$  of the SCPT electrometer in the case of the narrow band signal, accounting for the back fluctuation effect (see such analysis for SQUIDs in [17] and Chap. 7 in [7]). In this regime, the energy per unit bandwidth which the SCPT electrometer, as an amplifier, still resolves is

$$\boldsymbol{\epsilon} = (\boldsymbol{\epsilon}_{V}\boldsymbol{\epsilon}_{U} - \boldsymbol{\epsilon}_{VU})^{1/2}$$
$$= \left[ \left( \frac{k_{B}T}{eV} + \frac{I_{c}^{2}}{2I^{2}} \operatorname{coth} \frac{eV}{k_{B}T} \right) \frac{I^{2}C_{d}^{2}}{2I_{c}^{2}C^{2}} \operatorname{coth} \frac{eV}{k_{B}T} \right]^{1/2} \boldsymbol{\hbar},$$
(12)

where  $\epsilon_U = \pi C_d^2 S_U(0)/C$ , i.e., the backaction charge noise which is associated with fluctuations of the island potential  $(\delta Q_x)^{\text{back}} = C_d U_f$ , and  $\epsilon_{VU} = \pi S_{VU}(0)/|\eta| =$ 0, their correlation with  $V_f$ . In the most interesting case of zero temperature, Eq. (12) yields  $\epsilon = (|C_d|/C)\hbar/2$ . In the range of ultimate performance,  $\lambda \sim 1$  and  $|Q_0| \leq$ 0.8e, the capacitance ratio  $|C_d|/C \geq 1$ , and it can be made close to unity when  $|Q_0| \leq 0.5e$  [see the slope of the corresponding curves at small k in Fig. 2(b)]. Thus the SCPT electrometer can reach the ultimate energy resolution  $\epsilon^{\min} = \hbar/2$ , which is the minimum value (in contrast to  $\epsilon_V^{\min} = 0$ ) allowed by quantum mechanics for a phaseinsensitive linear amplifier so that the SCPT electrometer is a *quantum-limited* cryoelectronic device such as the dc SQUID [17].

In experiment the parameters of the SCPT electrometer could be the following. Assuming an Al-junction structure with  $C_{1,2} \sim 0.2$  fF and  $R_{1,2} \sim 3.5$  k $\Omega$ , that corresponds to  $E_{J1} \sim E_{J2} \sim E_C \sim 200 \ \mu \text{eV}$ , and choosing normal-metal shunt with  $R_S \sim 300 \ \Omega$ , we conclude that the characteristic Josephson frequency ( $\omega_c/2\pi$ ) is about 5 GHz ( $\ll E_J/h \sim$  $E_C/h \sim 50$  GHz) and the McCumber parameter  $\beta_c \sim 1$  for a relatively large value of  $C_L \sim 0.2$  pF. This means that the sizes of both the shunting and biasing resistors can be on mm scale (note that if a normal metal strip of resistor  $R_S$  is rather straight and not too narrow, the self-inductance of the shunt can be small, say,  $\omega_c L \sim 30 \ \Omega \ll R_s$ ). Such structure can be easily placed on one standard chip. At an operation temperature of about 10 mK, the maximum charge-to-voltage ratio  $\eta \sim 200 \ \mu V/e$  and the charge sensitivity  $\delta Q_x \sim 5 \times 10^{-7} e/(\text{Hz})^{1/2}$ , which corresponds to  $\epsilon_V \sim \bar{h}$ . For a reliable pickup of the output signal an additional cooled amplification stage might be desirable. In this case, in contrast to the case with an SET transistor,

no serious problems of impedance matching are expected because of the low output resistance  $R_S$ .

In conclusion, we have calculated the dynamic and noise characteristics of the SCPT electrometer whose performance takes advantage of the electrostatic control of the Josephson supercurrent. We have shown that these characteristics are almost similar to those of the SET counterpart. However, the coherent Cooper pair tunneling radically differs from electron (quasiparticle) tunneling because the former occurs without dissipation of energy in junctions. The small power ( $\sim \Phi_0 I_c \sim 1 \text{ pW}$ ) dissipates in the large (mm size) resistor which can therefore be easily cooled down to the mixing chamber temperature of 10-20 mK. In contrast to this the self-heating effect in SET devices is essential [18], and the enhanced electron temperature of the electrometer island usually imposes a limit to sensitivity already at  $T \sim 50-100$  mK. Hence, the SCPT electrometer can be superior to the SET counterpart in high precision charge measurements at lower temperature.

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- T. A. Fulton and G. J. Dolan, Phys. Rev. Lett. 59, 109 (1987).
- [2] P. Lafarge et al., Z. Phys. B 85, 327 (1991).
- [3] D. V. Averin and K. K. Likharev, in *Mesoscopic Phenom*ena in Solids, edited by B. L. Alshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, 1991).
- [4] K. K. Likharev and A. B. Zorin, Jpn. J. Appl. Phys. 26, Suppl. 3, 1407 (1987).
- [5] P. Joyez et al., Phys. Rev. Lett. 72, 2458 (1994).
- [6] T. M. Eiles and J. M. Martinis, Phys. Rev. B 50, 627 (1994).
- [7] K.K. Likharev, Dynamics of Josephson Junctions and Circuits (Gordon and Breach, New York, 1986), Chap. 4.
- [8] R. L. Kautz and J. M. Martinis, Phys. Rev. B 42, 9903 (1990).
- [9] A. N. Korotkov et al., in Single Electron Tunneling and Mesoscopic Devices, edited by H. Koch and H. Lübbig (Springer-Verlag, Berlin, 1992), p. 45.
- [10] D. V. Averin and Yu. V. Nazarov, Phys. Rev. Lett. 69, 1993 (1992).
- [11] M.T. Tuominen et al., Phys. Rev. Lett. 69, 1997 (1992).
- [12] A. O. Caldeira and A. J. Leggett, Ann. Phys. (N.Y.) 149, 374 (1983).
- [13] R. H. Koch, D. J. Van Harlingen, and J. Clarke, Phys. Rev. Lett. 45, 2132 (1980).
- [14] K. K. Likharev and A. B. Zorin, J. Low Temp. Phys. 59, 347 (1985).
- [15] K. K. Likharev, Moscow State University Report No. 29 1986; see also K. A. Matveev *et al.*, Phys. Rev. Lett. **70**, 2940 (1993).
- [16] H. B. Callen and T. A. Welton, Phys. Rev. 83, 34 (1951).
- [17] V. V. Danilov, K. K. Likharev, and A. B. Zorin, IEEE Trans. Magn. **19**, 572 (1983).
- [18] R.L. Kautz, G. Zimmerli, and J.M. Martinis, J. Appl. Phys. 73, 2386 (1993).