Berry's Phase and a Possible New Topological Current Drive in Certain Weak Link Superconducting Systems

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We examine the consequences of Berry's phase for the dynamics of Josephson junctions and junction arrays in which moving vortices are present. For both a large annular Josephson junction and a 2D junction array, Berry's phase produces a new current drive in the superconducting phase dynamics of these weak link systems. This Berry phase effect is shown to be physically inequivalent to a known effect in junction arrays associated with the Aharonov-Casher phase. [S0031-9007(96)00361-4]

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Ao and Thouless [1] recently drew attention to the relevance of Berry's phase [2] for the dynamics of vortices in type-II superconducting films. They argued that Berry's phase would cause the Magnus force to act on a moving vortex. Their work led one of us to examine the nondissipative force which acts on a moving vortex within the context of s-wave BCS superconductivity [3]. It was found (*inter alia*) that vortex motion does generate a Berry phase in the BCS ground state, that this Berry phase enters as a topological term in the condensate effective action known as a Wess-Zumino (WZ) term, and that the WZ term leads to the Magnus force acting on the moving vortex [4]. Reference [3] also remarked that situations could be envisioned in Josephson junctions and 2D Josephson junction arrays in which Berry's phase might also lead to physical effects.

In this Letter we examine the consequences of Berry's phase for two classes of weak link superconducting systems: large annular Josephson junctions (AJJ) and 2D Josephson junction arrays (JJA). We argue that, whenever these systems contain a moving vortex, Berry's phase will modify the electric current density passing through the weak links. This modification, in turn, will cause a new current drive to appear in the superconducting (SC) phase dynamics of these weak link systems which is a generic consequence of the topology and motion of the vortex. In the case of a JJA, we also show that our Berry phase effects are physically inequivalent to known effects associated with the Aharonov-Casher (AC) phase. The structure of this paper follows a progressive development of the Berry phase effects occurring in a type-II film to successively more complicated weak link systems. First, we review the type-II results, then we examine the simplest possible weak link system—a single Josephson junction. We then extend the single JJ analysis to the case of a 2D JJA. This Letter marks the first time Berry's phase has been connected to dynamical effects in weak link systems [5]. A detailed presentation of this work will be reported elsewhere [6].

We begin by summarizing the manner in which Berry's phase enters into the dynamics of vortices in type-II films—for a detailed presentation, see Ref. [3]. Throughout this Letter we assume (1) $T=0$, (2) an s-wave BCS superconductor whose gap phase $\phi=\phi(\mathbf{r}-\mathbf{r}_0(t))$ contains a moving vortex singularity with trajectory $\mathbf{r}_0(t)$, (3) a clean superconductor so that there are no pinning centers for the vortex or scattering centers for the SC electrons, and (4) adiabatic vortex motion so that phase slip voltages are small compared to the energy level spacing of the quasiparticle states bound to the vortex core ($\Delta E \sim \Delta_0^2/eE_f$ in volts).

The SC dynamics is described via the Bogoliubov Hamiltonian $H_b[\mathbf{r}_0]$ which depends parametrically on the vortex position $\mathbf{r}_0$. Vortex motion produces an adiabatic time dependence in the Hamiltonian $H_b[\mathbf{r}_0(t)]$ which can be treated using the quantum adiabatic theorem. Explicit calculation shows that if $|\Psi(t)\rangle$ is the many-body state of the SC electrons which at $t=0$ equals the BCS ground state in which a single vortex is present at $\mathbf{r}_0(0)$, i.e., $|\Psi(0)\rangle = |\mathrm{BCS};\mathbf{r}_0(0)\rangle$, then the many-body state at time t equals the BCS ground state with the vortex at $\mathbf{r}_0(t)$ multiplied by a phase factor which contains a nonintegrable Berry phase Γ:

$$|\Psi(t)\rangle = \exp\left[i\Gamma - (i/\hbar)\int_0^t E_0(\tau)d\tau\right]|\mathrm{BCS};\mathbf{r}_0(t)\rangle$$

and

$$\Gamma = -\int d^2x\, dt\, \rho_s\left(\frac{1}{2}\partial_t\phi + \frac{e}{\hbar}A_0\right). \tag{1}$$

Here $E_0(t)$ is the energy of $|\mathrm{BCS};\mathbf{r}_0(t)\rangle$, and A_0 is the electromagnetic scalar potential induced by the moving vortex.

The dynamical significance of Γ follows from its appearance in the condensate effective action, $S_c = S_0 - \hbar\Gamma + S_2$. This effective action is defined via the ground-state–to–ground-state transition amplitude, $\exp[-iS_c/\hbar] = \langle \mathrm{BCS};\mathbf{r}_0(T)|\Psi(T)\rangle$. It is the presence of Γ in the phase of $|\Psi(T)\rangle$ that leads to its appearance in S_c. We will not require the explicit form of S_0 and S_2; they have been calculated in Refs. [3,7], and the interested reader is referred to those papers for further details.

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Berry phase contributions to low energy effective actions are well known [8], and such contributions are known as WZ terms, $S_{\mathrm{WZ}} = -\hbar\Gamma$. WZ terms are topological in origin and occur whenever the line bundle structure inherent in the quantum adiabatic theorem is twisted [9]. S_{WZ} describes a topological coupling between the vortex and the SC electrons. A variation of S_{WZ} with respect to $\mathbf{r}_0(t)$ gives the force $\mathbf{f}_B$ acting on the vortex due to this coupling. Detailed calculation shows that $\mathbf{f}_B$ equals the Magnus force of classical hydrodynamics, $\mathbf{f}_B = -\rho_s h\dot{\mathbf{r}}_0 \times \hat{\mathbf{z}}/2$. Here, $\hat{\mathbf{z}}$ lies along the vortex axis, and h is Planck's constant. We will refer to $\mathbf{f}_B$ as the Berry-Magnus (BM) force. A supercurrent density $\rho_s e\mathbf{v}_T$ causes the Lorentz force, $\mathbf{f}_L = \rho_s h\mathbf{v}_T \times \hat{\mathbf{z}}/2$, to act on the vortex. For extreme type-II films, the total nondissipative force $\mathbf{F}_{nd}$ acting on the vortex is the sum of the Lorentz and BM forces. This completes our review of the type-II film results; we go on to examine their extension to certain weak link systems, beginning with a single Josephson junction.

AJJ.—In a Josephson junction, two superconductors (say, 1,2) are coupled through a weak link via the Josephson effect. Throughout this paper, all weak links are assumed to be tunneling barriers (TB). As is well known [7], the action S_{JJ} governing the SC electrons has the form $S_{\mathrm{JJ}} = S_b(1) + S_b(2) + S_c + S_t$. $S_b(i)$ is the BCS action of an isolated superconductor, and $i = 1, 2$; S_c describes the capacitive coupling of electric charge across the TB; and S_t describes the Josephson coupling of the gap phase of each superconductor ϕ_i across the TB [10]. We restrict ourselves to large JJ's because they allow localized regions of magnetic flux (vortices) to appear inside the TB. We assume unit thickness in the z direction so that the SC dynamics is 2D. Under these assumptions, the TB maps onto a 1D region $P \subset R^2$, and vortex motion is restricted to P. Finally, we assume the SC electrons respond adiabatically to a moving vortex in the TB. This is expected to be true for most kinds of vortex motion since the phase slip voltages produced by a moving vortex are typically of order 0.01–0.1 mV [11], while the smallest spacing between single quasiparticle energy levels for which transitions are not forbidden by the Pauli principle (at $T = 0$) is equal to the BCS gap, $\Delta \sim 1$ mV.

To begin, focus on superconductor i, and assume a moving vortex is present in the TB. Just as with the type-II film discussion above, vortex motion causes the Bogoliubov Hamiltonian of i to develop an adiabatic time dependence. This continues to be true even though the vortex now resides in the TB and *not* in i. This raises the question of whether a Berry phase appears in i's ground state. The crucial issue is whether this Berry phase is nonintegrable. If it is, then it will be dynamically relevant since it cannot be removed by single-valued phase transformations of the instantaneous energy eigenstates [2]. Nonintegrability requires the existence of at least one closed loop C in P for which the associated Berry phase is nonzero. In the usual linear JJ (LJJ), no such closed loop exists. In this case, the TB is topologically equivalent to the unit interval $I = [0, 1]$. All closed loops C in I will originate at some point p_0, move to a point p_1 along the unique contour C_1 which connects these two points, then return to p_0 along $-C_1$. Consequently, however much Berry's phase may twist in traversing C_1, this twist is removed on the return along $-C_1$. Thus, in a LJJ, the ground-state Berry phase $\Gamma(i) = 0$ for all closed loops in P. Consequently, no WZ term appears in $S_b(i)$, and Berry's phase is irrelevant for the dynamics of a LJJ. This conclusion does *not*, however, apply to an AJJ [11]. The essential difference is that the TB of an AJJ is ringlike, and thus topologically equivalent to the unit circle S^1. S^1 allows closed loops C_n which wind n times around the unit circle. It is easy to show using Eq. (1) that $\Gamma(i) = -n\pi\overline{N}_s$ for C_n. Here, $\overline{N}_s$ is the mean number of SC electrons in superconductor i. Thus, in an AJJ, Berry's phase is nonintegrable and physically relevant. As we saw above, Berry's phase causes a WZ term to appear in $S_b(i)$, and it is through the WZ term that it can influence the SC dynamics of the AJJ. Note that, because $\Gamma(i)$ is proportional to $\overline{N}_s$, the WZ term is an extensive quantity. This is quite necessary, given its appearance in the SC effective action.

To bring out the dynamical significance of the WZ terms in S_{JJ}, we consider a current-biased AJJ. Associated with the bias current I is the current density $\mathbf{j}(\alpha) = (\rho_s e v_T)\hat{\mathbf{n}}(\alpha)$. α parametrizes a position along the circular TB, $\hat{\mathbf{n}}(\alpha)$ is the unit vector which is normal to the TB at position α, and v_T is the velocity of the bias supercurrent which, for simplicity, is taken to be space-time independent. The low energy degree of freedom of a JJ is the gauge invariant phase difference $\gamma(\alpha, t) = \phi_2(\alpha, t) - \phi_1(\alpha, t) - (2\pi/\phi_0)\int_1^2 \mathbf{A} \cdot d\mathbf{l}$. The action S_γ governing this low energy dynamics was first derived for a LJJ in Ref. [7]. A similar analysis can be done for an AJJ, though care must be taken to track the consequences of the WZ terms present in $S_b(i)$. A detailed analysis [6] shows that the WZ terms modify the current density in the TB. One can see this by examining the current drive term S_{cd} in S_γ. One finds that the current density which couples to γ contains a modification $\Delta\mathbf{j}_B(t)$ due to Berry's phase. Specifically,

$$S_{cd} = \int dt\, R d\alpha \left(\frac{\hbar}{2e}\right)\hat{\mathbf{n}}(\alpha) \times [\rho_s e v_T \hat{\mathbf{n}}(\alpha) + \Delta\mathbf{j}_B]\,\gamma(\alpha)\,. \tag{2}$$

Here, R is the inner radius of the TB, and $\Delta\mathbf{j}_B(t) = -\rho_s e\dot{\mathbf{r}}_0(t)$. Because of the scalar product, we see that $\Delta\mathbf{j}_B$ enhances the bias current density $\mathbf{j}(\alpha)$ in the half-circle behind the vortex, and reduces it in the half-circle ahead of the vortex. The total current passing through the TB, however, is still equal to I. This Berry phase modification of the charge flow through the TB is a generic consequence of the topology and motion of the vortex.

The equation of motion for γ is found by varying S_γ with respect to γ. One finds [6]

$$\ddot{\gamma} - \nabla_\perp^2 \gamma + \sin\gamma = \beta - \beta\,\hat{\mathbf{n}}(\alpha)\cdot(\dot{\mathbf{r}}_0/v_T)\,, \qquad (3)$$

where length and time intervals are measured in units of the Josephson penetration length and inverse Josephson plasma frequency, respectively, $\beta = I/I_c$, and I_c is the critical current of the junction. As expected, we obtain a biased sine-Gordon equation [10], although the Berry phase modification of the current density produces the second term on the right-hand side of Eq. (3). In analogy with the type-II discussion we will refer to the familiar current drive β in Eq. (3) as the Lorentz drive since it originates from the bias current I. The new current drive will be called the Magnus drive since it is a consequence of Berry's phase. It is clear from Eq. (3) that, when $|\dot{\mathbf{r}}_0| \ll v_T$, the Lorentz drive is the dominant driving force, with perturbative corrections coming from the Magnus drive. However, when $|\dot{\mathbf{r}}_0| \gg v_T$, the Magnus drive is dominant and should lead to noticeable corrections to solutions of Eq. (3) which only include the Lorentz drive. One effect which may be sensitive to the Magnus drive is the critical value of the external bias current β_c at which vortex bunching first occurs [11,12]. When $\beta = \beta_c$, γ develops oscillations in the region behind the vortex. These oscillations lead to an attractive interaction between vortices which causes the bunching. Because Berry's phase increases the current density behind the vortex, one expects that a smaller external bias current $\beta^* < \beta_c$ will be capable of initiating the necessary oscillations in γ. A careful numerical analysis of Eq. (3) is planned which will allow us to test the validity of this conjecture. This concludes our discussion of a single JJ. In the next section we expand our Berry phase analysis to include a 2D JJA.

2D JJA.—A JJA is a lattice whose sites i are occupied by SC grains. Nearest neighbor grains are coupled through a weak link via the Josephson effect. The weak links are assumed to be small TB's, and, for simplicity, we assume a square lattice (lattice constant a_0). The sites of the dual lattice are the equilibrium positions for a vortex, and the TB's provide the paths by which a vortex moves from one dual lattice site to another. Consequently, vortex motion is restricted to the space P which is the union of the dual lattice sites, and the collection of TB's. The action S_{JJA} which governs the dynamics of the SC electrons in the array is a straightforward generalization of the action for a single JJ. S_{JJA} is a sum over the BCS action $S_b(i)$ of the grains i, and the actions $S_c\langle i,j\rangle$ and $S_t\langle i,j\rangle$ which describe the capacitive and Josephson coupling of nearest neighbor grains i and j across the weak link $\langle i,j\rangle$. Based on estimates similar to those given in the AJJ discussion, we assume the SC electrons in a JJA will respond adiabatically to a moving vortex under the conditions usually encountered in the flux-flow regime of the array.

As with an AJJ, vortex motion produces a nonintegrable Berry phase in the BCS ground state of each of the grains. From Eq. (1), it is easy to show that any closed loop $C_n(i)$ which encircles i n times produces a Berry phase $\Gamma(i) = -n\pi\overline{N}_s(i)$. Here, $\overline{N}_s(i)$ is the mean number of SC electrons in i. Thus a WZ term appears in each of the BCS actions $S_b(i)$, and, consequently, they will also appear in the array action S_{JJA}. The low energy degree of freedom of each of the weak links is again the gauge invariant phase difference across the link $\gamma_{i,j} = \phi_i - \phi_j - (2\pi/\phi_0)\int_j^i \mathbf{A}\cdot d\mathbf{l}$. One can apply the analysis of Ref. [7] to S_{JJA} to obtain the action S_γ^{JJA} which governs the phase dynamics of the JJA, though one must again be careful to track the effects of the WZ terms. One finds, assuming only nearest neighbor coupling of the grains, that S_γ^{JJA} is equal to a sum in which each weak link contributes the phase action found in the AJJ discussion S_γ, but with $\gamma \to \gamma_{i,j}$ [6]. Just as in the AJJ discussion, the WZ terms cause a modification of the current density passing through the weak links. This modification causes the Magnus current drive to appear in the equation of motion for $\gamma_{i,j}$,

$$\ddot{\gamma}_{i,j} - \nabla_\perp^2 \gamma_{i,j} + \sin\gamma_{i,j} = \beta - \beta\,\hat{\mathbf{n}}_{i,j}\cdot(\dot{\mathbf{r}}_0/v_T)\,. \quad (4)$$

In Eq. (4) we have assumed a uniform bias current $\beta = I/I_c$ passes through the array. The appearance of the Magnus drive in Eq. (4) is again a consequence of the topology and motion of the vortex. As with the AJJ, the regime $|\dot{\mathbf{r}}_0| \ll v_T$ will be dominated by the Lorentz drive, while the Magnus drive will dominate when $|\dot{\mathbf{r}}_0| \gg v_T$. A numerical analysis of the phase dynamics of a JJA in the presence of the Magnus drive is clearly necessary.

The preceding discussion has considered the lattice limit of the array's SC dynamics; the continuum limit (CL) is also of interest. If l is the length scale over which $\gamma_{i,j}$ varies, the CL of the array's SC dynamics corresponds to $l \gg a_0$. Thus $\gamma_{i,j}$ varies little from grain to grain in this limit. (Note that $a_0 \not\to 0$ in this limit.) In the CL, it is reasonable to coarse grain the array's SC dynamics. One finds [6], perhaps not so surprisingly, that S_{JJA} transforms under coarse graining into the action appropriate for a type-II film: $S_{\mathrm{JJA}} \to \tilde{S}_{\mathrm{JJA}} = \tilde{S}_0 + \tilde{S}_{\mathrm{WZ}} + \tilde{S}_2$, where

$$\tilde{S}_{\mathrm{WZ}} = \int d^2x dt\, \tilde{\rho}_s\left(\frac{\hbar}{2}\,\partial_t\phi + e\tilde{A}_0\right).$$

Quantities with tildes correspond to coarse grained averages. $\tilde{S}_{\mathrm{WZ}}$ arises from the coarse graining of the WZ terms present in S_{JJA}. The presence of the WZ term in $\tilde{S}_{\mathrm{JJA}}$ causes the BM force $\mathbf{f}_B = -\tilde{\rho}_s h(\dot{\mathbf{r}}_0 \times \hat{\mathbf{z}})/2$ to act on a vortex, just as in the type-II film case. We point out that this BM force is physically distinct from the Magnus force introduced in Ref. [13], and which is a consequence of the AC phase [14]. It is important to recognize the physical nonequivalence of these two forces, so we will close by contrasting their origins and consequences.

Fundamentally, the AC phase Γ_{AC} is a consequence of electrodynamics and topology, and it appears in the wave function of a quantum vortex [14]. In the JJA scenario of Ref. [13], a nonvanishing Γ_{AC} requires a nonfluctuating electric charge Q_f to be present on each grain. This requires the grain self-capacitance C_0 to be sufficiently small, and the application of a gate voltage $V_g(i)$ to each grain. On the other hand, Berry's phase Γ_B is a consequence of adiabatic quantum dynamics and topology. For the systems we consider, Γ_B appears in the BCS ground state wave function of each grain. A moving vortex is sufficient to produce a nonzero Γ_B in large AJJ's and JJA's. Furthermore, the vortex in the Berry phase scenario is classical, not quantum. Thus, for an array, the physical independence of these two phases is clear: (1) One is inherently an electromagnetic effect, the other is not; (2) Γ_{AC} appears in the wave function of a quantum vortex, while Γ_B appears in the many-body wave function of a SC grain; and (3) the experimental circumstances necessary to produce Γ_{AC} in an array are physically inequivalent, and more restrictive than those necessary to generate Γ_B. These substantial fundamental differences manifest also in quantitative differences, as we now show by considering the Hall effect in a perfectly ordered array in the CL.

The forces acting on an array vortex will be the coarse grained (1) Lorentz force ($\tilde{\rho}_s h \mathbf{v}_T \times \hat{\mathbf{z}}/2$), (2) Magnus force ($-\alpha \dot{\mathbf{r}}_0 \times \hat{\mathbf{z}}$), and (3) dissipative force ($-\eta \dot{\mathbf{r}}_0$). For the BM force, $\alpha_B = \tilde{\rho}_s h/2$. The AC-Magnus (ACM) force has the form [13] $\mathbf{f}_{\mathrm{AC}} = -(Q_f/eV)h\dot{\mathbf{r}}_0 \times \hat{\mathbf{z}}/2$, where V is the grain volume and $Q_f = C_0 V_g$ is the electric charge induced on the grain by the applied voltage V_g. Thus, $\alpha_{\mathrm{AC}} = C_0 V_g h/2eV$. For steady state vortex motion, the total force on the vortex is zero: $\tilde{\rho}_s h \mathbf{v}_T \times \hat{\mathbf{z}}/2 - \alpha \dot{\mathbf{r}}_0 \times \hat{\mathbf{z}} - \eta \dot{\mathbf{r}}_0 = 0$. Solving this equation for $\dot{\mathbf{r}}_0$ determines the Hall angle Θ_H, $\tan \Theta_H = -\dot{x}_0/\dot{y}_0 = \alpha/\eta$. From this expression we can compare the size of the Hall angles produced by the BM and ACM forces. To obtain a comparison that is independent of a model for η, we evaluate the ratio $R = \tan \Theta_H^B / \tan \Theta_H^{\mathrm{AC}} = \tilde{\rho}_s eV/C_0 V_g$. For typical array values ($\tilde{\rho}_s \sim 10^{27}\ \mathrm{m}^{-3}$, $V \sim 10^{-21}\ \mathrm{m}^3$, $C_0 \sim 10$ fF, $V_g \sim 1$ mV), one finds $R \sim 10^4$. Thus the BM force overwhelms the ACM force in a Hall effect experiment. Clearly the two forces cannot be the same. In the Hall experiment of Chen *et al.* [15], $V_g \equiv 0$ so that $\mathbf{f}_{\mathrm{AC}} \equiv 0$. Thus the observed nonzero Hall angles are clearly due to the BM force ($\alpha_B \neq 0$, $\alpha_{\mathrm{AC}} = 0$). If one uses the Kim-Bardeen-Stephen empirical expression for η [16], it is possible to reproduce the Chen *et al.* results ($\Theta_H \sim 1°$) using reasonable values for the subgap conductivity [6]. Finally, since the BM force dominates the ACM force, one expects the Hall angle will be (effectively) insensitive to variations of $V_g \sim 1$ mV, when $C_0 \sim 1$ fF.

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Note added. —Since this work was submitted, a preprint has appeared [17] which also argues that Berry phase effects will occur in JJA's, though it concludes that the BM force will not be active in arrays.

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