Theory of High-Mode Phenomena for Stellarators

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It is shown that besides the ion orbit loss mechanism, which occurs in a region $a - \varepsilon_t \rho_p < r < a$, the drift-orbit transport flux driven by the collisionless helically trapped particles can also drive the poloidal $\vec{E} \times \vec{B}$ velocity in a region $r < a - \varepsilon_t \rho_p$ in stellarators. Here, $r(a)$ is the minor (plasma) radius, ε_t is the toroidal amplitude of the magnetic field spectrum, $\vec{E}(\vec{B})$ is the electric (magnetic) field, and ρ_p is the poloidal ion gyroradius. The transport-flux-driven $\vec{E} \times \vec{B}$ velocity can be triggered most efficiently by an increase of the ion temperature gradient. The theory is applied to the high-mode phenomenon observed in stellarators. [S0031-9007(96)00354-7]

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The high-mode (H-mode) phenomenon has been observed in both tokamaks and stellarators [1–6]. The general behavior is similar in both of these devices. It can be understood by a logical sequence proposed in Refs. [7] and [8]. First, the poloidal $\vec{E} \times \vec{B}$ flow velocity driven by the ion orbit loss bifurcates over the local maximum of the plasma viscosity located at $M_p \approx 1$. Here, $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field, and M_p is the poloidal $\vec{E} \times \vec{B}$ Mach number. Second, the turbulence fluctuation is suppressed by the radial gradients of the $\vec{E} \times \vec{B}$ angular velocity and the diamagnetic angular velocity, and the plasma confinement is thus improved. The results of the theory and its extension are in agreement with many H-mode phenomena observed in tokamaks as demonstrated in Refs. [9] and [10]. The absence of unquantifiable parameters from the theory is what makes possible the comparisons with experimental results.

This Letter is directed toward pursuing this quantitative theory to explore the consequences of the differences between tokamak and stellarator transport properties. It is shown that in stellarators the poloidal $\vec{E} \times \vec{B}$ velocity can be driven to bifurcation not only by the ion orbit loss mechanism but also by the low collisionality drift-orbit transport flux. The collisionless drift-orbit transport flux is the radial particle flux driven by the low collisionality helically trapped particles. The ion orbit loss mechanism is important only in the region where $a - \varepsilon_t \rho_p < r < a$, where a is the plasma minor radius, r is the local minor radius, ε_t is the toroidal amplitude of the model magnetic field spectrum $B = B_0[1 - \varepsilon_t \cos\theta - \varepsilon_h \cos(m\theta - n\zeta)]$, B_0 is the magnetic field strength on the axis, ε_h is the helical amplitude, (m, n) is the poloidal and toroidal mode numbers of the helical magnetic field, and ρ_p is the ion poloidal gyroradius. In this region, the bifurcation of the $\vec{E} \times \vec{B}$ velocity in stellarators is similar to that in tokamaks and is not discussed in detail. However, in the region $r < a - \varepsilon_t \rho_p$, the collisionless ion drift-orbit transport flux can also drive the poloidal $\vec{E} \times \vec{B}$ velocity to bifurcation. This bifurcation mechanism does not exist in axisymmetric tokamaks. It can be triggered most efficiently by an increase in the ion temperature gradient. A similar mechanism was employed in Ref. [11] to show that the sign of the radial electric field E_r can be positive [12] in H mode in a rippled tokamak owing to the electron drift-orbit transport flux.

The poloidal and toroidal components of the momentum equation in Hamada coordinates [13] in a toroidal plasma are

$$NM\partial\langle\vec{B}_p \cdot \vec{V}\rangle/\partial t = -\langle\vec{B}_p \cdot \nabla \cdot \pi\rangle - \nu_{\rm eff} NM\langle\vec{B}_p \cdot \vec{V}\rangle - \frac{\psi'\chi'}{c}\langle\vec{J} \cdot \nabla V\rangle, \quad (1)$$

$$NM\partial\langle\vec{B}_t \cdot \vec{V}\rangle/\partial t = -\langle\vec{B}_t \cdot \nabla \cdot \pi\rangle - \nu_{\rm eff} NM\langle\vec{B}_t \cdot \vec{V}\rangle + \frac{\psi'\chi'}{c}\langle\vec{J} \cdot \nabla V\rangle, \quad (2)$$

where the angular brackets denote the flux surface average, N is the plasma density, M is the ion mass, π is the viscous tensor, $\vec{J}$ is the current density, c is the speed of light, $\vec{B}_t = \psi'\nabla V \times \nabla\theta$, $\vec{B}_p = -\chi'\nabla V \times \nabla\zeta$, $\psi' = \vec{B} \cdot \nabla\zeta$, $\chi' = \vec{B} \cdot \nabla\theta$, V is the volume enclosed in the flux surface, and θ and ζ are poloidal and toroidal angles, respectively. The radial current density $\langle\vec{J} \cdot \nabla V\rangle$ in Eqs. (1) and (2) is related to $\partial\langle\vec{E} \cdot \nabla V\rangle/\partial t$ through Ampère's law $\langle\vec{J} \cdot \nabla V\rangle = -(1/4\pi)\partial\langle\vec{E} \cdot \nabla V\rangle/\partial t$. The effective collision frequency of the charge-exchange momentum loss is defined as $\nu_{\rm eff} = N_n\langle\sigma\nu\rangle_{\rm cx}$, where N_n is the neutral density and $\langle\sigma\nu\rangle_{\rm cx}$ is the charge-exchange reaction cross section [14]. Equations (1) and (2) are to be solved for the radial electric field and the parallel plasma flow speed. To accomplish this, we need to know the explicit expressions of $\langle\vec{B}_p \cdot \nabla \cdot \pi\rangle$ and $\langle\vec{B}_t \cdot \nabla \cdot \pi\rangle$.

The plasma viscous forces $\langle\vec{B}_p \cdot \nabla \cdot \pi\rangle$ and $\langle\vec{B}_t \cdot \nabla \cdot \pi\rangle$ contain information about the ion orbit loss, the collisionless drift-orbit transport flux, and the nonlinear collisional velocities. The contributions of electrons to the viscous forces are neglected for being smaller than those of ions in the collisionality regimes in which we are interested. For a given Maxwellian ion distribution function,

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the cold ions are collisional and the hot ions are collisionless. For our purpose, the critical energy that separates these two classes of ions is determined by the existence of the collisionless helically trapped particles. The nonlinear collisional viscosities are from particles in the plateau-fluid Pfirsch-Schlüter regime [15]. The particles in the collisionless regime contribute to either the ion orbit loss or the drift-orbit transport flux, depending on how far they are from the plasma boundary. The relevant collisionality parameter here is ν_*^h for the helically trapped particles, defined as $\nu_*^h = \nu R q / \varepsilon_h^{3/2} v_t |m - nq|$, where ν is the ion-ion collision frequency, R is the major radius, q is the safety factor, and v_t is the ion thermal speed. When $\nu_*^h < 1$, the helically trapped particles become collisionless. We assume that $nq \gg m$, which leads us to conclude that the conventional ion collisionality of the toroidally trapped particles ν_* is larger than ν_*^h if $\varepsilon_t \approx \varepsilon_h$. We are interested in the parameter regime where ν_*^h is of the order of unity. The orbit loss associated with the tokamaklike banana orbit is thus not important if the ion distribution is a simple Maxwellian. This indicates that the most relevant particle orbit topology for the stellarator H mode is that of the helically trapped particles. The radial drift-orbit size Δr of the helically trapped particles in the presence of the poloidal $\vec{E} \times \vec{B}$ drift is of the order of

$$\Delta r \approx v_{\rm dr} \Delta t \,, \tag{3}$$

where $v_{\rm dr}$ is the bounce-averaged radial drift velocity and Δt is the period of the drift orbit. Because the poloidal $\vec{E} \times \vec{B}$ velocity is larger than the $\nabla \vec{B}$ and curvature drifts if $e\Phi/T \approx 1$, Δt is of the order of r/V_E with $V_E = cE_r/B$. Here, Φ is the electrostatic potential, $B = |\vec{B}|$, e is the charge of the ions, and T is the ion temperature. Note that $v_{\rm dr} \approx v^2/2\Omega R$ with v the particle speed, and Ω the ion gyrofrequency, one concludes that

$$\Delta r \approx \varepsilon_t \rho_p / M_p \,, \tag{4}$$

where M_p is the poloidal $\vec{E} \times \vec{B}$ Mach number defined as $-V_E B / v_t B_p$. Because at the H-mode bifurcation M_p is of the order of unity, the helically trapped drift ion orbits can intersect the plasma boundary if they are within a distance $\varepsilon_t \rho_p$ away from the boundary, that is, if they are in the region $a - \varepsilon_t \rho_p < r < a$. If they are in the region $r < a - \varepsilon_t \rho_p$, they cannot intersect the plasma boundary when $M_p \approx 1$. In that case the physical consequence of their contribution to plasma viscosities is the collisionless transport flux discussed extensively in Refs. [16] and [17]. The relationship between the drift-orbit transport flux and the plasma viscosities is shown to be [18]

$$\langle \vec{\Gamma} \cdot \nabla V \rangle_{\rm non} = -\frac{c}{e\chi'\psi'} \langle \vec{B}_p \cdot \nabla \cdot \pi \rangle = \frac{c}{e\chi'\psi'} \langle \vec{B}_t \cdot \nabla \cdot \pi \rangle \,, \tag{5}$$

where $\langle \vec{\Gamma} \cdot \nabla V \rangle_{\rm non}$ is the radial component of the drift-orbit transport flux. Note that Eq. (5) can also be proven directly from the definitions of the particle flux and the viscosity, and by the fact that the leading order perturbed particle distribution function does not vary along the magnetic field line in the calculations of the drift-orbit transport flux. In general, $\langle \vec{\Gamma} \cdot \nabla V \rangle_{\rm non}$ consists of three regimes as shown in Ref. [16]. Here, we employ only the $1/\nu$ regime flux to demonstrate the bifurcation of M_p for H-mode application. Note that the electron flux in the $1/\nu$ regime is smaller than that of ions and can be neglected.

In the cylindrical coordinates (r, θ, ζ), Eq. (5) can be expressed as

$$\langle \vec{B}_p \cdot \nabla \cdot \pi \rangle = -\langle \vec{B}_t \cdot \nabla \cdot \pi \rangle = -(e/c) B_p B \Gamma_r \,, \tag{6}$$

where $B_p = |\vec{B}_p|$ and Γ_r is the radial component of the particle flux. The particle flux in the $1/\nu$ regime is [17]

$$\Gamma_r = -\frac{64}{9(2\pi)^{3/2}} N \varepsilon_h^{3/2} \varepsilon_t^2 \left(\frac{cT}{eBr} \right)^2 \frac{1}{\nu} \times \left[I_{-1} \left(\frac{P'}{P} + \frac{e\Phi'}{T} \right) + I_{-2} \frac{T'}{T} \right], \tag{7}$$

where $P = NT$ is the ion pressure, $P' = dP/dr$, $T' = dT/dr$, $\Phi' = d\Phi/dr$, and the integrals I_{-1} and I_{-2} are defined as

$$\begin{Bmatrix} I_{-1} \\ I_{-2} \end{Bmatrix} = \int_{\sqrt{\nu_*^h}}^{\infty} dy\, y^4 \begin{Bmatrix} 1 \\ y - \frac{5}{2} \end{Bmatrix} e^{-y}.$$

Note that the energy integration limits in I_{-1} and I_{-2} are between $\sqrt{\nu_*^h}$ and ∞. This is because only those ions with normalized energy $v^2/v_t^2 > \sqrt{\nu_*^h}$ contribute to the $1/\nu$ transport flux.

From Eqs. (6) and (7), we obtain the explicit expressions of $\langle \vec{B}_p \cdot \nabla \cdot \pi \rangle$ and $\langle \vec{B}_t \cdot \nabla \cdot \pi \rangle$ for those particles with normalized energy $v^2/v_t^2 > \sqrt{\nu_*^h}$. For collisional particles with normalized energy $v^2/v_t^2 < \sqrt{\nu_*^h}$ the explicit expressions for $\langle \vec{B}_p \cdot \nabla \cdot \pi \rangle$ and $\langle \vec{B}_t \cdot \nabla \cdot \pi \rangle$ are already calculated in Ref. [15]. Thus, we have all the necessary information of plasma viscosity, which is the sum of the contributions from these two classes of particles, to solve Eqs. (1) and (2). At the steady state, Eqs. (1) and (2) in cylindrical coordinates become

$$-\frac{32}{9(2\pi)^{3/2}} \frac{\varepsilon_t^2}{|m - nq|} \frac{1}{\nu_*^h} [I_{-1}(M_p - V_{p,p}) - I_{-2} V_{p,T}] = \frac{\sqrt{\pi}}{4} \sum_{mn} \varepsilon_{mn}^2 m(m - nq) \times \left\{ I_{mn} \left[\frac{V_\parallel}{v_t} + \frac{m}{m - nq} (M_p - V_{p,p}) \right] - L_{mn} \frac{m}{m - nq} V_{p,T} \right\} + \varepsilon_t^2 \frac{\nu_{\rm eff}}{v_t/Rq} \left[\frac{V_\parallel}{v_t} + \frac{1 + 2q^2}{q^2} (M_p - V_{p,p}) \right], \tag{8}$$

$$\frac{32}{9(2\pi)^{3/2}}\frac{\varepsilon_t^2}{|m-nq|}\frac{1}{\nu_*^h}[I_{-1}(M_p - V_{p,p}) - I_{-2}V_{p,T}] = \frac{\sqrt{\pi}}{4}\sum_{mn}\varepsilon_{mn}^2 nq(nq-m) \times\left\{I_{mn}\left[\frac{V_\|}{v_t} + \frac{m}{m-nq}(M_p - V_{p,p})\right] - L_{mn}\frac{m}{m-nq}V_{p,T}\right\} + \frac{\nu_{\text{eff}}}{v_t/Rq}\frac{V_\|}{v_t}, \quad (9)$$

where $V_\|$ is the parallel (to $\vec{B}$) flow speed, $V_{p,p} = -cP'/Nev_tB_p$, $V_{p,T} = -cT'/ev_tB_p$, and ε_{mn} is the Fourier amplitude of the magnetic field spectrum $B = B_0[1 + \sum_{mn}\varepsilon_{mn}\cos(m\theta - n\zeta)]$. The integrals I_{mn} and L_{mn} in Eqs. (8) and (9) are defined as

$$\begin{Bmatrix} I_{mn} \\ L_{mn} \end{Bmatrix} = \frac{1}{\pi}\int_0^{\sqrt{\nu_*^h}} dx\, x^2 e^{-x}\begin{Bmatrix} 1 \\ x - \frac{5}{2} \end{Bmatrix} \times \int_{-1}^{1} dy(1-3y^2)^2\left(\frac{v\chi'}{B}\right)R_{mn},$$

where $R_{mn} = \nu_k/[(m\omega_\theta - n\omega_\zeta)^2 + \nu_k^2]$, $v_\|$ is the parallel particle speed, $\nu_k = 3\nu_D + \nu_E + \nu_{\text{eff}}$, ν_D is the deflection frequency [19], and ν_E is the energy exchange frequency [19]. Note that we have employed in Eqs. (8) and (9) the poloidal and toroidal viscosities calculated in Hamada coordinates in Ref. [15] from the solution of the drift kinetic equation and the definitions given in Ref. [20] and converted the results approximately to the cylindrical coordinates based on the formula derived in Ref. [21]. Note also that the energy integrals I_{mn} and L_{mn} are truncated. This indicates that we are assuming that the particle distribution function is a Maxwellian. The hot particles contribute to low collisonality drift-orbit transport flux, and the cold particles contribute to the nonlinear collisional viscosities. We are interested in the case where ν_*^h is of the order of unit [22]. There is a bifurcated state in this parameter range which is relevant to H mode. For simplicity, we use the simple model field $B = B_0[1 - \varepsilon_t\cos\theta - \varepsilon_h\cos(m\theta - n\zeta)]$. We also assume that the toroidal flow speed $V_t \approx V_\|$ is damped by toroidal viscous force and charge-exchange momentum loss so that $V_t/v_t \approx V_\|/v_t \approx 0$. However, we would like to note that this assumption is not necessary. The bifurcation solutions for the coupled nonlinear equations (8) and (9) will be presented in a separate article. With this simplification Eq. (8) becomes a nonlinear equation of M_p for the set of plasma parameters employed. This equation is solved by plotting the left and right sides of Eq. (8) as functions of M_p. The solution is the intersection of these two curves. For the plasma parameters, similar to those of Wendelstein 7-AS, $\varepsilon_t = 0.053$, $\varepsilon_h = 0.025$, $q = 1.92$, $m = 2$, $n = 5$, and $\nu_{\text{eff}} = 0.01$, the results are shown in Figs. 1–3.

In Fig. 1, the collisionality is high, $\nu_* = 15$, and $V_{p,p} = V_{p,T} = 0.25$. The $\vec{E}\times\vec{B}$ Mach number is subsonic. This is the low-mode (L-mode) solution. When the collisionality decreases from the Pfirsch-Schlüter regime to the plateau regime, the neoclassical ion energy confinement improves, which leads to higher values of ion temperature and ion temperature gradient dT/dr if ion energy confinement is not dominated by the anomalous process. Thus, if ν_* decreases, $V_{p,p} = V_{p,T}$ increases. (The exact relation between ν_* and $V_{p,p} = V_{p,T}$ can be determined from the transport modeling which is beyond the scope of this paper.) In that case, the value of M_p increases. If the ion collisionality keeps decreasing, there can be three solutions for M_p, as shown in Fig. 2, where $V_{p,p} = V_{p,T} = 0.50$ and $\nu_* = 7.5$. The one with the smallest value is the continuation of the L-mode solution. The one with the largest value is the new H-mode solution. Both of these solutions are stable. The one in the middle is unstable and is not relevant. If the ion collisionality

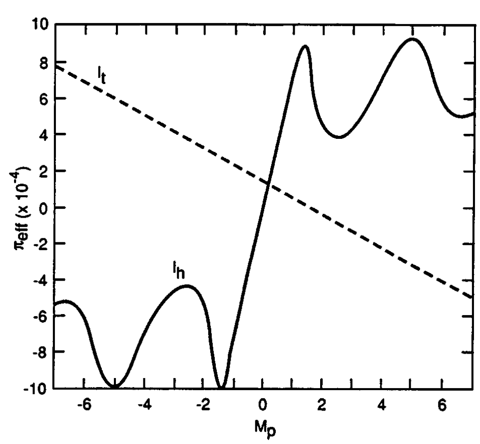

FIG. 1. The left side (I_t) and the right side (I_h) of Eq. (8) versus M_p for $\nu_* = 15$, $\varepsilon_t = 0.053$, $\varepsilon_h = 0.025$, $q = 1.92$, $m = 2$, $n = 5$, $\nu_{\text{eff}} = 0.01$, and $V_{p,p} = V_{p,T} = 0.25$. There is only one solution of M_p.

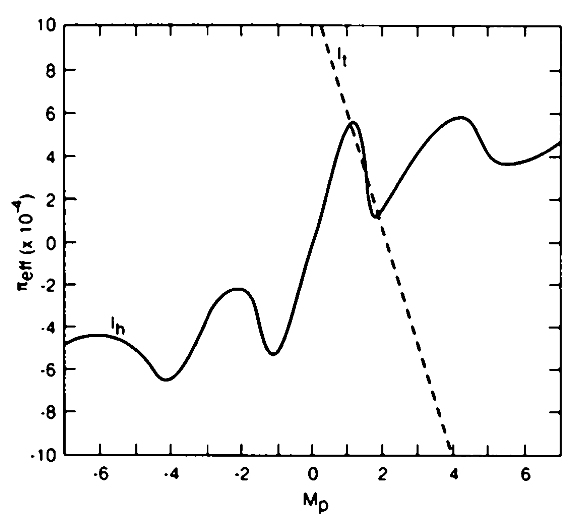

FIG. 2. I_t and I_h versus M_p for the same parameters as in Fig. 1 except $\nu_* = 7.5$ and $V_{p,p} = V_{p,T} = 0.50$. There are three solutions of M_p. The one in the middle is unstable.

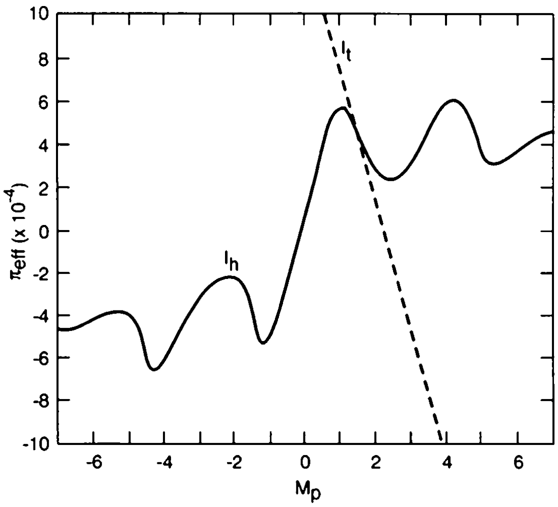

FIG. 3. I_t and I_h versus M_p for the same parameters as in Fig. 1 except $\nu_* = 7.0$ and $V_{p,p} = V_{p,T} = 0.55$. There is only one solution of M_p.

decreases further to $\nu_* = 7.0$, only the H-mode solution exists, as shown in Fig. 3, where $V_{p,p} = V_{p,T} = 0.55$. Note that the H-mode solution has $M_p \approx 1.5$. This value of M_p may be a factor of 2 larger than the earlier experimental results in Wendelstein 7-AS [5].

One can conclude that increasing the ion temperature gradient is more efficient than increasing the density gradient in driving M_p to the H-mode value. The reason is that the magnitude of I_{-2} is about a factor of 2.5 larger than I_{-1}. Thus, the increase in dT/dr needed to push the zero of the left side of Eq. (8) to a higher value, to facilitate bifurcation, is smaller than the necessary increase in dN/dr. Furthermore, the ion energy confinement could be close to be neoclassical. In that case, lower ion collisionality can improve the ion energy confinement and increase dT/dr. However, particle confinement in the edge region is likely to be anomalous. Lower ion collisionality may not increase dN/dr.

We emphasize that the process described here is applicable in the region $r < a - \varepsilon_t \rho_p$. Because $\varepsilon_t \rho_p$ is much less than 1 cm for typical edge parameters, this is the region that is most likely to be measured with limited spatial resolution. Also, because this mechanism is not restricted to the edge region, one could, in principle, have a much broader bifurcated region to achieve better confinement by tailoring the density and the temperature profiles. Only in the region $a - \varepsilon_t \rho_p < r < a$ can the ion orbit loss process be observed. As noted earlier, the drift-orbit-transport-driven $\vec{E} \times \vec{B}$ flow only exists in nonaxisymmetric toroidal plasmas such as stellarators and not in axisymmetric tokamaks.

We summarize the H-mode transition sequence in the region $r < a - \varepsilon_t \rho_p$ in stellarators as follows: (a) The ion collisionality decreases due to plasma heating, which leads to higher values of T and dT/dr. (2) Lower collisionality and larger dT/dr drive the poloidal $\vec{E} \times \vec{B}$ flow to bifurcation. (3) Plasma confinement is improved because of the increase of the gradients of the poloidal $\vec{E} \times \vec{B}$ angular velocity and the diamagnetic angular velocity which suppresses turbulence.

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