Enhancement of the Tunneling Density of States in Tomonaga-Luttinger Liquids

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We have calculated the tunneling density of states (DOS) at the location of a backward scattering defect for quantum wires and for edge state electrons in quantum Hall systems. A singular enhancement of the DOS arises as a result of the combined effect of multiple backward scattering together with a repulsive electron-electron interaction. [S0031-9007(96)00317-1]

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With the rapid advance of the submicron technology [1] the fabrication of one-dimensional (1D) quantum wires has become a reality. The properties of these wires are expected to be unusual. It is known that the electron-electron interaction in a 1D electron gas, when away from the density wave or from the superconductivity instabilities, leads to the Tomonaga-Luttinger (TL) liquid behavior [2]. The spectacular feature of the TL liquid is the vanishing of the single-particle density of states (DOS) at the Fermi energy [3,4]. In this Letter we calculate the tunneling DOS of the TL liquid at the location of a defect center that causes backward scattering of the conduction electrons. By mapping the problem onto a Coulomb gas theory, we show that the DOS diverges at energies close to the Fermi energy when the electron-electron interaction is repulsive and not too strong; i.e., the DOS in the vicinity of a backward scatterer is in clear contrast with the DOS in a clean TL liquid or far away from the scattering center. It has already been noted by us in connection with the study of the Fermi-edge singularity in 1D that the low energy physics of the backward scattering together with the electron-electron repulsion resembles the physics of the Kondo resonance [5]. The singular enhancement of the DOS is a consequence of a many body effect of a similar type.

Recently, considerable efforts have been directed toward the study of the transport properties of the 1D TL liquids [6–14]. For a repulsive electron-electron interaction it has been predicted that at zero temperature even a single weak backward scatterer eventually causes the conductance to vanish. It is widely accepted that the low energy physics of this system can be described by two semi-infinite lines connected by a weak link junction (e.g., in Ref. [6] the vanishing of the conductance has been traced to the fact that the tunneling DOS into the end of a semi-infinite TL liquid vanishes at the Fermi energy). However, as found in the present work, in the vicinity of the backward scatterer the DOS is enhanced for a repulsive electron-electron interaction. We believe that the description of the low energy physics of this problem by two disconnected wires should be exploited with caution.

A measurement of the DOS by tunneling spectroscopy methods will provide, in a direct way, information about non-Fermi-liquid properties of electrons in quantum wires. Such experiments are highly desirable. Besides the quantum wires, the TL-liquid behavior can be displayed by edge state electrons in quantum Hall devices [12,13,15,16]. The tunneling DOS in the quantum Hall systems will be discussed at the end of the paper.

The renormalization group treatment shows that the effective amplitude of the backward scattering increases and becomes strong when the energy of the scattered electrons approaches the Fermi energy [6,17]. Therefore, at low temperatures, one needs to understand the physics of the strong coupling regime. From the experience of the study of local defect problems in metals, it is known that mapping the problem onto a Coulomb gas theory can be instructive (see, e.g., Ref. [18]). The discussed problem has been mapped onto a theory of a neutral gas of positively and negatively charged classical particles interacting via a logarithmic potential [6]. These charges are located on a line, and they describe the time history of the backscattering events. Unlike the Kondo problem, this problem is described by a nonalternating Coulomb gas. The physics of this gas has been well studied. There are two phases separated at a critical temperature, T_{cr}, by a transition of the Kosterlitz-Thouless type [19,20]. The temperature of the Coulomb gas, T_{gas}, is determined by the electron-electron interaction of the original problem. At low temperatures, $T_{\mathrm{gas}} < T_{\mathrm{cr}}$, the particles form dipoles, while in the hot phase, $T_{\mathrm{gas}} > T_{\mathrm{cr}}$, the dipoles dissociate and the gas is in a plasma state. From the renormalization group analysis, it follows that when the electron-electron interaction is repulsive the system is in the hot plasma phase, while the dipole phase corresponds to the attractive electron-electron interaction. In the plasma phase the logarithmic interactions between charged particles are screened off at distances exceeding the radius of screening τ_{scr}. To describe the strong coupling regime of the backward scattering problem in the TL liquid, we will utilize the physics of screening in the plasma phase of the Coulomb gas.

For simplicity, we start with the spinless case and will include the spin degrees of freedom later. The Hamiltonian of the LT liquid in 1D can be written in terms of the bosonic field operators ϕ and $\tilde{\phi}$ as [21,22]

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$$H_{TL} = \frac{v_F}{2g} \int dx \left[\left(\frac{d\phi}{dx} \right)^2 + \left(\frac{d\tilde{\phi}}{dx} \right)^2 \right], \qquad (1)$$

where $g = \sqrt{(1-\gamma)/(1+\gamma)}$, $\gamma = V/(2\pi v_F + V)$, v_F is the Fermi velocity, and V describes the density-density interaction with momentum transfers smaller than the Fermi momentum k_F. In Eq. (1) the operator $d\phi(x)/dx$ is proportional to the deviation of the electron density from its average value, and $d\tilde{\phi}(x)/dx$ is proportional to the current density; ϕ and its dual partner $\tilde{\phi}$ are conjugate variables, i.e., $[d\phi(x)/dx, \tilde{\phi}(y)] = i\delta(x-y)$. The Hamiltonian (1) describes the 1D electron liquid when the backward scattering can be ignored in the processes of the electron-electron interaction. The field operators $\psi_{R(L)}(x)$ of electrons with momenta close to $\pm k_F$ can be represented using the bosonization technique [4,17,21,22] as

$$\psi_{R(L)}(x) = \frac{e^{\pm i k_F x}}{\sqrt{2\pi\eta}} \exp\left[-\frac{i}{2} \left(\frac{4\pi}{\beta} \tilde{\phi}(x) \pm \beta\phi(x) \right) \right], \qquad (2)$$

where $\beta^2 = 4\pi g$ and η^{-1} is an ultraviolet cutoff of the order of the conduction bandwidth.

We concentrate below on the $2k_F$ backward scattering only. It will be argued later that the DOS is not influenced by the forward scattering. The backward scattering induced by a defect located at the point $x = 0$ is described by a term

$$H_{bs} = U(2k_F)\psi_R^\dagger(0)\psi_L(0) + U^*(2k_F)\psi_L^\dagger(0)\psi_R(0), \qquad (3)$$

where $U(2k_F)$ is the $2k_F$ Fourier transform amplitude of the scattering potential, and $\psi_{R(L)}(0) = \psi_{R(L)}(x=0)$. In the bosonic representation, H_{bs} can be written as

$$H_{bs} = -\frac{\delta_-}{\pi} \frac{v_F}{\eta} \cos[\beta\phi(0) + \varphi_u], \qquad (4)$$

where $\delta_- = |U(2k_F)|/v_F$ and $\varphi_u = \arg(-U(2k_F))$.

The local tunneling DOS will be found as

$$\varrho(\epsilon, x) = -\frac{1}{\pi} \operatorname{Im} \left\{ \int_0^\infty G(\tau; x, x) e^{i\epsilon_n \tau} d\tau \right\}_{i\epsilon_n \to \epsilon + i\delta}, \qquad (5)$$

where $G(\tau; x, x) = -\langle \mathcal{T}_\tau \psi(\tau, x)\psi^\dagger(0, x)\rangle$ is the Matsubara Green's function of the electrons at the point x and $\psi(\tau, x) = \psi_L(\tau, x) + \psi_R(\tau, x)$. In the absence of scattering, $G(\tau; x, x)$ can be readily obtained using representation (2) and the fact that the Hamiltonian (1) is quadratic. This leads to $G(\tau; x, x) \sim \exp[-\frac{1}{2}(g\mathcal{D}_\phi + g^{-1}\mathcal{D}_{\tilde{\phi}})]$, where $\mathcal{D}_\phi(\tau) = 2\pi\langle\phi(\tau, x)\phi(0, x) - \phi(0, x)^2\rangle = \ln(1 + v_F\tau/\eta g)$ is the Green function of the ϕ operators, and, in a similar way, $\mathcal{D}_{\tilde{\phi}}(\tau) = \ln(1 + v_F\tau/\eta g)$. As a result [3,4],

$$\varrho(\epsilon, x) \epsilon^{(g-1)^2/2g}. \qquad (6)$$

To study the DOS at the location of the backward scatterer, $x = 0$, we treat $G(\tau; 0, 0)$ in the interaction representation with respect to H_{bs}. When the backward scattering is written as in Eq. (4), each term in the perturbation series for $G(\tau; 0, 0)$ can be calculated using the Baker-Haüsdorf formula repeatedly. This procedure gives straightforwardly a representation of the Green function in terms of partition functions of a one-dimensional Coulomb gas of classical particles:

$$G(\tau; 0, 0) \sim \tau^{-1/2g} [Z_e(\tau) - Z_o(\tau)]/Z. \qquad (7)$$

The factor $\tau^{-1/2g}$ originates from the $\tilde{\phi}$-field factors in the bosonic representation of the fermion-field operators [see Eq. (2)]. Since the operators $\psi(\tau, x)$ have left and right components, $Z_e(\tau)$ and $Z_o(\tau)$ contain four contributions each, e.g., $Z_e(\tau) = Z_e^{+-}(\tau) + Z_e^{-+}(\tau) + Z_e^{++}(\tau) + Z_e^{--}(\tau)$. The term Z_e^{+-} is the grand partition function of a neutral Coulomb gas that has a charge $+\frac{1}{2}$ at the point 0, a charge $-\frac{1}{2}$ at the point τ, and an even number of charges ± 1 between them. The other three terms $Z_e^{aa'}(\tau)$ are defined in a similar way, namely, the upper indices correspond to the signs of the $\pm\frac{1}{2}$ charges located at the points 0 and τ. These half-charges originate from the ϕ-field factors of the operators $\psi_{L(R)}$ in the Green function. $Z_o(\tau)$ is analogous to $Z_e(\tau)$, but with an odd number of ± 1 charges inside the interval $(0, \tau)$. The term in the denominator, Z, is the grand partition function of the Coulomb gas without the additional half-charges. The minus sign in front of $Z_0(\tau)$ in Eq. (7) appears because of the anticommutation of the fermion operators. In the discussed Coulomb gases the particles interact via a logarithmic potential $v(\tau - \tau') = \ln(1 + v_F|\tau - \tau'|/\eta g)$, the fugacity is $g\delta_-/2\pi$ and the effective temperature $T_{\text{gas}} = \frac{1}{2g}$. Thus, the calculation of the DOS is reduced to the calculation of correlation functions in the Coulomb gas theory.

To analyze the functions $Z_e^{aa'}(\tau)$ and $Z_o^{aa'}(\tau)$ we integrate out the field $\phi(t, x)$ in the entire space, except the point of the backward scatterer location, and reformulate the problem in terms of a functional integral over $\phi(t) \equiv \phi(t, x = 0)$. The difference $\Delta Z^{aa'} = Z_e^{aa'}(\tau) - Z_o^{aa'}(\tau)$ can be obtained using the effective action

$$S^{aa'}(\phi) = \frac{1}{2} \int dt\, dt'\, \phi(t) v^{-1}(t - t')\phi(t') + \frac{2\delta_-}{\pi\eta} \int dt \cos[\beta\phi(t) + W(t)] + a\frac{i}{2}\beta\phi(0) + a'\frac{i}{2}\beta\phi(\tau). \qquad (8)$$

Here v^{-1} is the inverse of the potential v; the potential $W(t) = \pi[\theta(t - 0^+) - \theta(t - \tau + 0^+)]$ in the cosine term is inserted to weigh the even and odd configurations with opposite signs. In order to estimate $\Delta Z^{aa'}$ we use the mean field approximation. In this approximation the grand partition functions of the system are determined by the saddle point solutions, ϕ_s, of the effective functional $S^{aa'}$. The solutions, ϕ_s, correspond

to the equilibrium electrostatic potential of the plasma gas in the presence of two external half-charges located at points 0 and τ. In this way one obtains $\Delta Z^{++} = \Delta Z^{--} = 0$, while $\Delta Z^{+-}(\tau) = \Delta Z^{-+}(\tau) \to const$ when $\tau \to \infty$. These results are rather natural. The cancellation of Z_e^{++} with Z_o^{++} occurs because the gas configurations with even and odd numbers of charges inside the interval $(0, \tau)$ are equally far away from the optimal configuration. The latter should have, inside the interval, a charge equal to $-\frac{1}{2}$ to screen the two external half-charges of the same sign. (Technically, that cancellation occurs between contributions of different saddle point solutions in the vicinity of consecutive minima of the cosine. The existence of a manifold of minima reflects in a formal way the discreet nature of the charges in the gas.) When the external half-charges have opposite signs, the optimal configuration has an even number of charges in the interval $(0, \tau)$, and such a cancellation does not occur. The value of the action S^{+-} at the optimal configuration ϕ_s determines the screened interaction between the two external charges. For τ exceeding the screening radius $\tau_{\rm scr}$ the bare logarithmic interaction between the external charges is screened off, and therefore $\Delta Z^{+-}(\tau)$ has a finite limit at large τ. In the mean field approximation $\Delta Z^{+-}(\tau)/Z \equiv \Delta_g \sim (\eta/v_F\tau_{\rm scr})^{g/2}$, where $\tau_{\rm scr} = \frac{\eta g}{v_F}(\frac{g\delta_-}{\pi})^{-1/(1-g)}$.

Substituting these results into Eq. (7) yields in the asymptotic region $\tau \gg \tau_{\rm scr}$

$$G(\tau; 0, 0) \sim \frac{1}{\eta} \exp\left(-\frac{1}{2}\, g^{-1} \mathcal{D}_{\tilde{\phi}}\right) \sim \frac{1}{\eta}\left(\frac{\eta}{v_F \tau}\right)^{1/2g} \Delta_g \,. \tag{9}$$

This result implies that the tunneling DOS, $\varrho(\epsilon, 0)$, diverges in the infrared limit for a moderately repulsive electron-electron interaction, $1/2 < g < 1$:

$$\varrho(\epsilon, 0) \sim \frac{1}{\epsilon\eta}(\epsilon\eta/v_F)^{1/2g} \Delta_g, \qquad \epsilon \ll \tau_{\rm scr}^{-1} \tag{10a}$$

(note that both $\tau_{\rm scr}^{-1}$ and Δ_g go to zero when $g \to 1$).

Up to now the electron-electron interaction was considered a short-range one. However, for quantum wires, the long-range character of the Coulomb interaction may be important. The strength of the interaction depends on the particular electrostatics of the sample. To include the spatial dependence of the interaction amplitude, one should substitute the combination $g^{-1}\mathcal{D}_{\tilde{\phi}}$ by $\int_0^{\eta^{-1}} dp g(p)^{-1}(1 - e^{ipv_F t})/p$ in Eq. (9); here $g(p) = \sqrt{[1 - \gamma(p)][1 + \gamma(p)]}$, $\gamma(p) = V(p)/[2\pi v_F + V(p)]$, and $V(p)$ is the Fourier transform of the electron-electron interaction. For the Coulomb interaction $V(p) = 2e^2/\kappa \ln(1/|p|w)$, where κ is the dielectric constant and w is the width of the wire. In this case, in the asymptotic region $\tau \gg \tau_{\rm scr}$ one gets $G(\tau; 0, 0) \sim \exp[-\frac{1}{2}\frac{1}{3\chi}(1 + 2\chi \times \ln\frac{\tau v_F}{\eta})^{3/2}]$, where $\chi = e^2/\kappa\pi\hbar v_F$. Inserting this expression into Eq. (5) yields that $\varrho(\epsilon, 0)$ is nonmonotonic if $\tau_{\rm scr}^{-1} > \epsilon^* \sim \frac{\eta}{v_F}\exp(-\frac{3}{2}\chi^{-1})$. As ϵ decreases, the DOS increases when $\tau_{\rm scr}^{-1} > \epsilon > \epsilon^*$, and for $\epsilon^* > \epsilon$ the DOS starts to vanish. Since $\tau_{\rm scr}$ is determined by δ_-, while ϵ^* is not, the situation when $\tau_{\rm scr}^{-1}$ is smaller than ϵ^* is possible.

When the spin degrees of freedom of the conduction electrons are included, the above considerations do not change essentially. The calculation of the DOS can be reduced to the calculation of correlation functions in a Coulomb gas. Due to the spin, the plasma contains two types of charged particles. The latter aspect does not alter the physics of screening. There are two fields, ϕ_ρ and ϕ_σ (and, correspondingly, two dual fields $\tilde{\phi}_\rho$ and $\tilde{\phi}_\sigma$), that describe charge and spin density modes. The Green function contains the factor $\exp[-\frac{1}{4}(g_\rho^{-1}\mathcal{D}_{\tilde{\phi}_\rho} + g_\sigma^{-1}\mathcal{D}_{\tilde{\phi}_\sigma})]$, where $g_\sigma = 1$ and $g_\rho = \sqrt{(1 - \gamma_\rho)/(1 + \gamma_\rho)}$, $\gamma_\rho = V/(\pi v_F + V) = 2\gamma/(1 + \gamma)$. This factor is not influenced by the screening, and this yields

$$\varrho(\epsilon, 0) \sim \frac{1}{\epsilon}(\epsilon\eta/v_F)^{1/4g_\rho + 1/4}\Delta_g, \qquad \epsilon \ll \tau_{\rm scr}^{-1}\,. \tag{10b}$$

Therefore, in the spin case the tunneling DOS diverges when $1/3 < g_\rho < 1$.

Let us now discuss why the forward scattering does not influence the DOS. In terms of the ϕ-field operators the forward scattering can be written as $\frac{\beta\delta_+}{\pi}\frac{d\phi}{dx}|_{x=0}$, where $\delta_+ = U(k = 0)/v_F$, and the amplitude $U(k = 0)$ is the Fourier component of the scattering potential with $k = 0$. For the linearized spectrum the forward scattering can be absorbed in a phase factor which finally disappears. To see this in a formal way, one can apply the canonical transformation $\mathcal{U} = \exp[i\frac{\delta_+}{2\pi} g\beta\tilde{\phi}(x = 0)]$ which removes the forward scattering term from the Hamiltonian, but produces a phase factor in the backward scattering term and in the electron operators $\psi_{R(L)}(x = 0)$. However, due to the charge neutrality of the Coulomb gas (including the external half-charges), these phase factors cancel each other out.

In a two-dimensional electron gas under the conditions of the quantum Hall effect (QHE) the edge excitations are described by TL-like theories [15]. In the case of the integer QHE, it is the interedge interaction that leads to the TL-liquid behavior [16]. The above treatment of the backward scattering is not altered considerably. However, since the particles which are moving on opposite edges are spatially separated, the amplitudes which describe the interedge ($V_{\rm er}$) and the intraedge ($V_{\rm ra}$) electron-electron interactions are not equal. For that reason the expression for γ should be modified. For the symmetrical case, when the velocities of the excitations moving in opposite directions are the same, $\gamma = V_{\rm er}/(2\pi v_F + V_{\rm ra})$. Since the electron liquid in the case of a fully occupied

Landau level is incompressible, it cannot screen off the long-range Coulomb interaction between the edge electrons. In order to consider this effect the dependence of $V_{\rm er}(p)$ and $V_{\rm ra}(p)$ on the momentum should be included, as discussed above.

Because of the nontrivial character of the electron liquid in the fractional QHE state, electrons close to the edges exhibit an abnormal TL-type behavior even in the absence of interedge interaction [15]. The backward scattering term describes now the scattering of fractionally charged quasiparticles from one edge to the other. To find the asymptotic behavior of the Green function of edge electrons for a filling factor $\nu = 1/n$, where n is an odd integer, one should replace g with ν in expression (9). This leads to $\varrho(\epsilon, 0) \sim \epsilon^{n/2-1}$ near the backward scattering center. This is a considerable enhancement compared to $\varrho(\epsilon, 0) \sim \epsilon^{n-1}$ in the absence of the backward scattering. When the interedge electron-electron interaction is relevant, one gets $\varrho(\epsilon) \sim \epsilon^{1/2(\nu g_\nu)^{-1}-1}$ where $g_\nu = \sqrt{(1-\gamma_\nu)/(1+\gamma_\nu)}, \gamma_\nu = \nu V_{\rm er}/(2\pi v_F + \nu V_{\rm ra})$. The long-range Coulomb interaction can be treated as in the case of the 1D quantum wire.

Let us discuss now the mechanism of the enhancement of the DOS. Following the interpretation of the low energy physics of the backward scattering in a 1D wire as a weak link junction, one would not expect an enhancement but a vanishing of the DOS such as $\epsilon^{g^{-1}-1}$ for $g < 1$ [6]. To understand the enhancement, note that the bosonic representation (2) makes evident the affinity of the DOS with the "Debye-Waller factor" of the ϕ mode. As a result of the backward scattering, together with the repulsion between electrons, the propagator of small oscillations of the field $\phi(t, x = 0)$ acquires a mass and becomes $(|\omega_n| + m)^{-1}$, where ω_n is a Matsubara frequency (see, e.g., Ref. [14]). The zero mode oscillations of the ϕ mode becomes less effective and the Debye-Waller factor does not vanish. Because of such pinning of the ϕ mode, the amplitude of an electron created at the location of the backward scattering center falls down slower than in the case of free electrons [see Eq. (9)]. Thus, because of the multiple backward scattering, the escape rate of an electron from the defect center slows down. The enhancement of the DOS is a consequence of this effect. In the study of the Fermi-edge singularity in the TL liquid, it has been concluded [5] that the infrared physics of the backward scattering problem resembles the physics of a Kondo resonance. We believe that the enhancement of the tunneling DOS is reminiscent of a resonance of a similar type. We emphasize, however, that the treatment above is not related directly to the analysis of the transport properties of TL liquids.

In summary, we have calculated the tunneling DOS at the location of a backward scatterer in a 1D quantum wire and for edge state electrons under the conditions of the QHE. A singular enhancement of the DOS was obtained. The enhancement of the DOS in the TL liquid may be observed not only when the backward scattering is due to an internal defect. When the counterelectrode in a tunneling experiment has the shape of a sharp tip, then the tip itself may cause a backward scattering and lead to the enhancement of the DOS..

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