Induced Emission due to the Quantized Motion of Ultracold Atoms Passing through a Micromaser Cavity

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(Received 12 December 1995)

The abrupt change in the atom-field coupling strength which an atom experiences upon passing into and out of a micromaser cavity leads to changes in the atomic center-of-mass motion. For fast (thermal) atoms, small momentum changes give rise to stimulated emission. Very slow (laser-cooled) atoms, however, can be reflected from or tunnel through the cavity, and in the process undergo a new kind of induced emission. This changes the photon statistics of the micromaser completely. Resonances occur for particular values of the interaction length. [S0031-9007(96)00304-3]

PACS numbers: 42.50.Ar, 32.80.–t, 42.50.Dv, 84.40.Ik

Maser action occurs when excited atoms follow classical trajectories through a microwave cavity [1–3]. But with the advent of laser cooling [4] it becomes reasonable to ask what happens when the atoms are cooled to the point that their motion has to be described quantum mechanically? More precisely, how is the physics of induced emission affected when the atomic kinetic energy is smaller than the atom-field interaction energy?

We here show that operation in the limit of ultracold atoms requiring a quantum-mechanical treatment of the center-of-mass (CM) motion [5–7], taken together with a high-Q cavity, leads to a completely new type of induced, but not stimulated, emission. That is, in the ordinary maser, stimulated emission prevails as the mechanism for amplification of radiation, but in the case of ultracold atoms the physics of the induced emission process is intimately associated with the quantization of the CM motion (taken to be in the z direction). For this reason we distinguish between the usual stimulated emission maser physics and that characterized by the present quantized-z-motion induced emission and call the process microwave amplification via z-motion-induced emission of radiation mazer action.

The difference between the classical and the quantum treatment of the CM motion is clearly illustrated by looking at the probability that an excited atom launched into a cavity containing n photons will emit a photon, as depicted in Fig. 1. In making this comparison, we will be considering the cases in which the kinetic energy $(\hbar k)^2/2M$ of the atoms, expressed in terms of the atomic CM wave vector k and mass M, is greater or less than the atom-field coupling energy.

For the case of thermal atoms passing through the micromaser cavity, we find the emission probability associated with maser action

$$P_{\text{maser}} = \sin^2\!\left(\frac{\kappa^2 L}{2k}\sqrt{n+1}\right), \tag{1}$$

where κ is the CM wave vector for which the kinetic energy $(\hbar\kappa)^2/2M$ equals the vacuum coupling energy $\hbar g$, and L is the cavity length. Equation (1) embodies the usual stimulated emission process and the well-known Rabi oscillations since $\kappa^2 L/2k = g\tau$, where the interaction time $\tau = L(\hbar k/M)^{-1}$. As is shown below, Eq. (1) applies only when $k \gg \kappa\sqrt[4]{n+1}$.

For the case of ultracold atoms, such that $k \ll \kappa\sqrt[4]{n+1}$, we find the photon emission probability

$$P_{\text{mazer}} = \frac{\frac{1}{2}[1+\frac{1}{2}\sin(2\kappa L\sqrt[4]{n+1})]}{1+(\kappa\sqrt[4]{n+1}/2k)^2\sin^2(\kappa L\sqrt[4]{n+1})}. \tag{2}$$

Several aspects of Eq. (2), which is only valid for $\kappa L \gg 1$, should be noted. First of all, instead of the usual "Rabi phase" $g\tau\sqrt{n+1} = (\kappa^2 L/2k)\sqrt{n+1}$, now the phase $\kappa L\sqrt[4]{n+1}$ appears, which is independent of k, i.e., independent of the classical interaction time. We further note that Eq. (2) resembles the Airy function of classical optics, $[1+F\sin^2(\Delta/2)]^{-1}$, which gives the transmitted intensity in a Fabry-Pérot interferometer with finesse F and phase difference Δ [8]. In our case, the finesse $F = (\kappa\sqrt[4]{n+1}/2k)^2$ depends on the number of photons in the cavity.

In a Fabry-Pérot configuration, the transmitted intensity reaches a maximum when the phase difference is adjusted

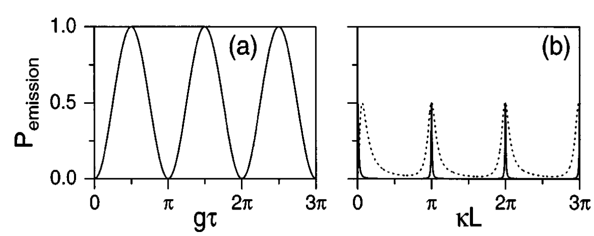

FIG. 1. Emission probability versus the interaction time $g\tau = \kappa^2 L/2k$ (a) for $k/\kappa \gtrsim 10$ and versus the interaction length κL (b) for $k/\kappa = 0.1$ (dotted) and $k/\kappa = 0.01$ (solid), when the cavity field is initially in the vacuum state.

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to the wavelength of the propagating radiation, e.g., by changing the distance between the mirrors or by placing a medium with a different index of refraction inside the interferometer. In the case of Eq. (2), we can maximize the emission probability of an incident excited atom by adjusting the phase difference (e.g., the cavity length) according to the state of the cavity field.

In the following we develop the quantum theory of the mazer. In terms of the atomic lowering operator σ, the cavity-field creation operator $a^\dagger$, and the CM momentum operator p_z, the atom-field Hamiltonian in the interaction picture reads

$$H = \frac{p_z^2}{2M} + \hbar g u(z)(\sigma a^\dagger + a\sigma^\dagger), \tag{3}$$

where the mesa function $u(z) = 1$ for $0 < z < L$ and zero elsewhere. We assume the atom to be in resonance with the field. In terms of the dressed states $|\gamma_{n+1}^\pm\rangle = (|a,n\rangle \pm |b,n+1\rangle)/\sqrt{2}$, the interaction operator has the values

$$(\sigma a^\dagger + a\sigma^\dagger)|\gamma_{n+1}^\pm\rangle = \pm\sqrt{n+1}\,|\gamma_{n+1}^\pm\rangle. \tag{4}$$

As discussed in Ref. [5], Eqs. (3) and (4) lead to the elementary problem of a particle incident upon a potential $V_n^\pm(z) = \pm\hbar g\sqrt{n+1}\,u(z)$. The cavity field acts as a potential barrier for $|\gamma_{n+1}^+\rangle$ components and as a potential well for $|\gamma_{n+1}^-\rangle$ components. An atom in the excited state $|a\rangle$ and with a CM wave packet $\int dk\,A(k)e^{ikz}$ incident upon a cavity field $\sum_n c_n|n\rangle$ is characterized before the scattering process by the state

$$\langle z|\Psi(0)\rangle = \sum_n c_n \int dk\,A(k)e^{ikz}\theta(-z)\,|a,n\rangle, \tag{5}$$

which after leaving the interaction region evolves into

$$\begin{aligned}\langle z|\Psi(\tau)\rangle = &\sum_n c_n \int dk\,A(k)\exp\!\left(-i\,\frac{\hbar k^2}{2M}\,\tau\right)\\ &\times [R_{an}(k)e^{-ikz}\theta(-z)\,|a,n\rangle + T_{an}(k)e^{ik(z-L)}\theta(z-L)\,|a,n\rangle\\ &+ R_{b,n+1}(k)e^{-ikz}\theta(-z)\,|b,n+1\rangle + T_{b,n+1}(k)e^{ik(z-L)}\theta(z-L)\,|b,n+1\rangle],\end{aligned} \tag{6}$$

where Heaviside's unit step function θ indicates on which side of the cavity the atom can be found.

An excited atom incident upon a cavity that contains n photons is found to be reflected or transmitted while remaining in the excited state $|a\rangle$ with amplitudes

$$R_{an} = \tfrac{1}{2}(\rho_n^+ + \rho_n^-), \qquad T_{an} = \tfrac{1}{2}(\tau_n^+ + \tau_n^-) \tag{7}$$

and is similarly reflected or transmitted while making a transition to the state $|b\rangle$, and emitting a photon, with amplitudes

$$R_{b,n+1} = \tfrac{1}{2}(\rho_n^+ - \rho_n^-), \qquad T_{b,n+1} = \tfrac{1}{2}(\tau_n^+ - \tau_n^-). \tag{8}$$

Here the reflection and transmission coefficients [5] for the CM motion of a particle incident upon the rectangular potential $V_n^\pm(z)$ are

$$\begin{aligned}\rho_n^\pm &= \frac{i}{2}\left(\frac{k_n^\pm}{k} - \frac{k}{k_n^\pm}\right)\sin(k_n^\pm L)\tau_n^\pm,\\ \tau_n^\pm &= \left[\cos(k_n^\pm L) - \frac{i}{2}\left(\frac{k_n^\pm}{k} + \frac{k}{k_n^\pm}\right)\sin(k_n^\pm L)\right]^{-1},\end{aligned} \tag{9}$$

with $k_n^\pm = (k^2 \mp \kappa^2\sqrt{n+1})^{1/2}$. The emission probability is given by

$$P_{\text{emission}} = |R_{b,n+1}|^2 + |T_{b,n+1}|^2. \tag{10}$$

We first consider the situation in which the cavity field is initially in the vacuum state. The kinetic energy of the incident atoms is assumed to be so small that tunneling through the potential barrier is negligible, i.e., $\rho_n^+ \cong -1$ and $\tau_n^+ \cong 0$. An incident excited atom with $k \ll \kappa$ is reflected without emitting a photon ($|R_{a0}|^2 = 1$), unless the cavity length is adjusted so that $\kappa L = m\pi$ with an integer m. Under this resonance condition, as is illustrated in Fig. 2, the atom is only reflected when it hits the repulsive potential and traverses the cavity when it encounters an attractive potential [9]; in both cases, according to Eq. (8), it emits a photon with probability $1/2(|R_{a0}|^2 = |R_{b1}|^2 = |T_{a0}|^2 = |T_{b1}|^2 = 1/4)$. In other words, for very slow atoms, the finesse F is very large so that P_{emission} is strongly peaked at $\kappa L = m\pi$ and practically vanishes in between.

Equation (6) can be used to find the reduced density matrix $\rho(t)$ for the cavity field after the interaction with

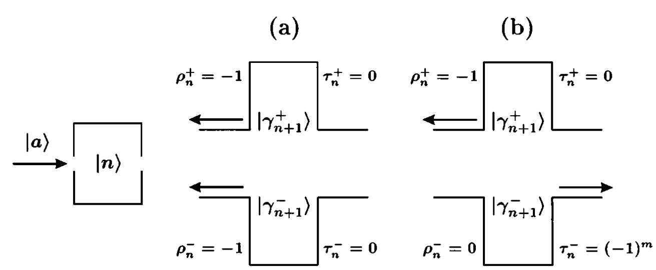

FIG. 2. An excited atom and a cavity field with n photons is described by the state $|a,n\rangle = (|\gamma_{n+1}^+\rangle + |\gamma_{n+1}^-\rangle)/\sqrt{2}$. For very slow incident atoms, the $|\gamma_{n+1}^+\rangle$ component is always reflected by a potential barrier. The $|\gamma_{n+1}^-\rangle$ component, which sees a square-well potential, is reflected for $\kappa L\sqrt[4]{n+1} \neq m\pi$ (a) and is transmitted for $\kappa L\sqrt[4]{n+1} = m\pi$ (b).

the excited atom by forming the atom-field density matrix and tracing over the internal and external atomic degrees of freedom, that is,

$$\rho(t) = \sum_{\alpha=a,b} \int dz \, \langle \alpha, z | \Psi(t) \rangle \langle \Psi(t) | \alpha, z \rangle \,. \tag{11}$$

The course-grained equation of motion for the radiation field is then given by $\dot{\rho}(t) = r[\rho(t + \tau) - \rho(t)]$, where r is the atomic injection rate. Inserting Eq. (6) into (11) and adding the terms describing field damping, we find the master equation for the density-matrix elements

$$\dot{\rho}_{nn'} = r(R_{an}R^*_{an'} + T_{an}T^*_{an'} - 1)\rho_{nn'} + r(R_{bn}R^*_{bn'} + T_{bn}T^*_{bn'})\rho_{n-1,n'-1} - C(n_b + 1) \times [\tfrac{1}{2}(n + n')\rho_{nn'} - \sqrt{(n+1)(n'+1)}\,\rho_{n+1,n'+1}] - Cn_b[\tfrac{1}{2}(n + n' + 2)\rho_{nn'} - \sqrt{nn'}\,\rho_{n-1,n'-1}], \tag{12}$$

where C is the cavity decay rate and n_b is the number of photons in thermal equilibrium.

The equation of motion for the photon-number distribution $P(n) = \rho_{nn}$ reads

$$\begin{aligned}\dot{P}(n) = {} & G_{n-1}P(n-1) - G_nP(n) \\ & - C(n_b + 1)[nP(n) - (n+1)P(n+1)] \\ & - Cn_b[(n+1)P(n) - nP(n-1)],\end{aligned} \tag{13}$$

where the gain coefficient $G_n = rP_{\text{emission}}$ follows for the maser and mazer limits from Eqs. (1) and (2), respectively. The rate of change of the mean photon number

$$\langle \dot{n} \rangle = \langle G_n \rangle - C(\langle n \rangle - n_b) \tag{14}$$

and the steady-state photon distribution

$$P(n) = P(0) \prod_{m=1}^{n} \frac{Cn_b + G_m/m}{C(n_b + 1)} \tag{15}$$

are obtained from Eq. (13). The photon distribution of the mazer pumped by ultracold atoms is completely different from the field in the micromaser operating with a beam of thermal atoms.

We consider again a cavity field initially in the vacuum state and ultracold atoms. If $\kappa L = m\pi$, the first incident atom may emit a photon. This changes the cavity field, and therefore the potential $V_n^{\pm}(z)$, which determines the resonance condition; hence the next incident atom is reflected with certainty without emitting a photon (if no photon decays out of the cavity in the meantime). Therefore, in the limit of very slow atoms and in the absence of thermal photons, at most one photon is in the cavity at a time. The average photon number has to be between zero and one and is determined by the ratio r/C between the injection rate and the cavity decay rate.

With increasing atomic momentum, the finesse F of the emission probability decreases, so that there is a nonvanishing probability of depositing a photon in the cavity even when the resonance condition is not fulfilled. As a consequence, more resonances (corresponding to larger photon numbers) become accessible and can be excited, as shown in Fig. 3. The resonances may occur for particular values of the interaction length, namely, for $\kappa L\sqrt[4]{N} = m\pi$ ($N = 1, 2, 3, \ldots$). Under this resonance condition, incident atoms emit a photon with maximal probability if they find $N - 1$ photons in the cavity.

For very slow atoms and zero temperature of the cavity, only the vacuum resonance (with $N = 1$) comes into play. Initial field states with larger photon numbers will be damped until there is at most one photon in the cavity in steady state. In the presence of thermal photons, however, other resonances may be excited even for very slow atoms. The thermal photons ensure that there is a nonvanishing probability for having different numbers of photons in the cavity, which give rise to different potentials and different resonances; we are not constrained to the resonances of Fig. 3. This is shown in Fig. 4(a) for the resonance sequence corresponding to $m = 1$ and the parameters $n_b = 1$, $k/\kappa = 10^{-3}$, and $r/C = 10^3$. The peaks in the mean photon number $\langle n \rangle$ are accompanied by resonances in the normalized standard deviation $\sigma = [(\langle n^2 \rangle - \langle n \rangle^2)/\langle n \rangle]^{1/2}$.

When we choose κL such that the resonance condition $\kappa L\sqrt[4]{N} = m\pi$ is fulfilled for exactly one pair of integers N and m, we obtain from Eqs. (15) and (2) for the

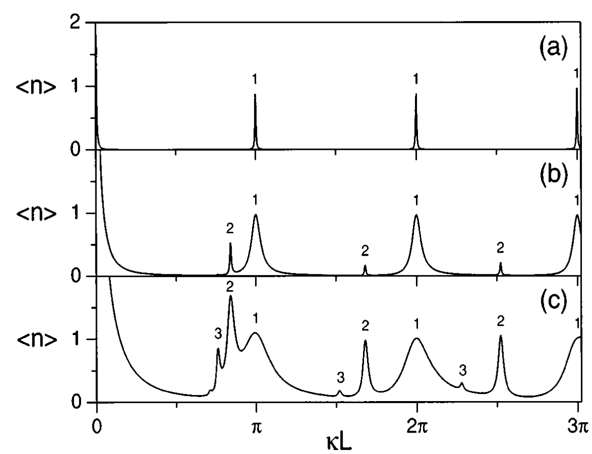

FIG. 3. Mean photon number $\langle n \rangle$ versus κL for $n_b = 0$, $r/C = 50$, and (a) $k/\kappa = 0.001$, (b) $k/\kappa = 0.01$, and (c) $k/\kappa = 0.03$. The peaks are labeled by the integer N, which appears in the resonance condition $\kappa L\sqrt[4]{N} = m\pi$.

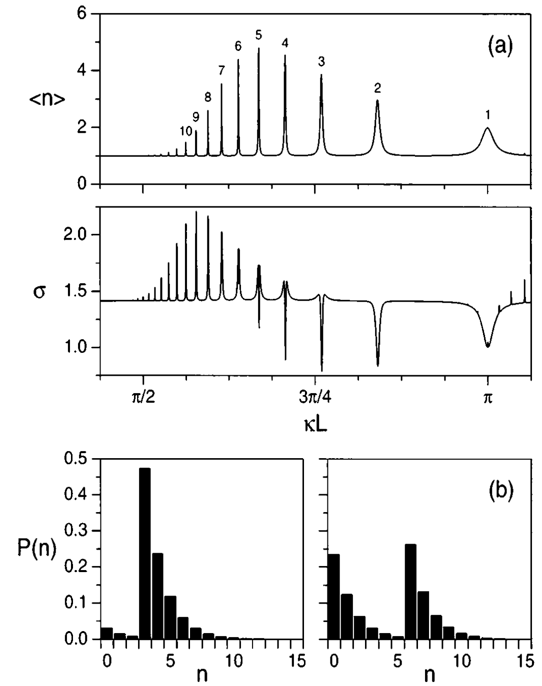

FIG. 4. (a) With thermal photons present, $\langle n\rangle$ and σ show resonances at $\kappa L = m\pi/\sqrt[4]{N}$. The peaks are labeled by the integer N. (b) The distribution $P(n)$ looks like a pair of thermal distributions for $\kappa L = 10^3\pi/\sqrt[4]{N}$ with $N = 3$ (left plot) and $N = 6$ (right plot). The parameters are $n_b = 1$, $r/C = 10^3$, and $k/\kappa = 10^{-3}$.

steady-state photon distribution of the mazer with $k \ll \kappa$

$$P(n) = \begin{cases} P(0)\left(\frac{n_b}{n_b+1}\right)^n & \text{for } n < N\,, \\ P(0)\,\frac{n_b + r/2CN}{n_b+1}\left(\frac{n_b}{n_b+1}\right)^{n-1} & \text{for } n \geq N\,, \end{cases}$$

and the normalization condition $\sum_n P(n) = 1$ implies $P(0) = [n_b + 1 + rn_b/2CN(n_b + 1)]^{-1}$. In Fig. 4(b), we plot the distributions for the resonances at $\kappa L = 10^3\pi/\sqrt[4]{N}$ with $N = 3$ and $N = 6$. Each distribution looks like a pair of thermal distributions, one of which is shifted by N photons towards larger photon numbers.

In order to understand the shape of this steady-state distribution, we consider an initial thermal distribution $P_0(n)$ with $\langle n\rangle = n_b$ which is the steady-state solution without injected atoms. A very slow atom will only emit a photon (with probability 1/2) if it encounters $N - 1$ photons in the cavity. In all other cases, P_{emission} is negligible since $\kappa L\sqrt[4]{n+1}$ is an integer multiple of π only for $n = N - 1$. Thus, whenever there are $N - 1$ photons in the cavity, there is a large probability for an incident atom to deposit an additional photon, thereby increasing the probability for having N photons in the cavity and decreasing $P(N - 1)$. Without the interaction with the thermal reservoir (or for a very small cavity decay rate), the photon distribution after the passage of several atoms would be $P(N - 1) \cong 0$, $P(N) \cong P_0(N - 1) + P_0(N)$, and $P(n) = P_0(n)$ for all $n \neq N - 1$ or N. However, the interaction with the bath leads to cavity damping and provides thermal photons. This ensures that the ratio $P(n + 1)/P(n)$ approaches its thermal-equilibrium value $n_b/(n_b + 1)$ for all n except for $n = N - 1$ where the ratio is increased due to the pumping by the atoms.

Moreover, as seen from the expression for $P(0)$, the probability for finding less than N photons in the cavity can be suppressed by increasing r/C so that the resulting photon distribution is a shifted thermal distribution. Shifting a distribution to larger photon numbers does not change its variance $v = \langle(n - \langle n\rangle)^2\rangle$. The normalized standard deviation $\sigma = \sqrt{v/\langle n\rangle}$, however, is decreased since $\langle n\rangle$ is increased. Thus, in Fig. 4(a), the resonances for $1 \leq N \leq 5$ show reduced photon-number fluctuations as compared to the thermal level $\sigma = \sqrt{1 + n_b}$. The photon distribution can even be sub-Poissonian ($\sigma < 1$) as in the left plot of Fig. 4(b), where $\sigma \cong 0.81$.

In this Letter, we have focused on the case of very slow atoms. However, the influence of the potential barrier on the photon statistics can already be observed with faster atoms. For example, the well-known trapping resonances [10] of the conventional micromaser, which occur at very low temperatures, i.e., in the absence of thermal photons, begin to disappear when the atoms are cooled down so that $k = 10\kappa$ (for $r/C = 50$).

This work has been supported by the Office of Naval Research, the Welch Foundation, and the Texas Advanced Research Program.

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