Comment on "Nature of Crossover between Ising-Like and Mean-Field Critical Behavior in Fluids and Fluid Mixtures"

In Ref. [1] the expression "*mean-field behavior*" is utilized ambiguously to refer to two different kinds of behavior: (I) The *regular* (i.e., not critical) behavior that occurs in any system outside the critical region. (II) The *classical critical* behavior that is characterized (on a theoretical ground) by the proximity of the Gaussian fixed point.

Behavior I is not describable purely by the renormalization group theory (RGT) contrary to behavior II which is well controlled within the RGT. In real solutions, such as some nonaqueous solutions of ionic fluids which are thought to belong to the Ising universality class, the effect of the Coulombic interaction may first drive the effective Wilson Hamiltonian close to the Gaussian fixed point before it approaches the Ising fixed point. In such cases, an actual classical-to-Ising crossover occurs, and is at least partially, *observable within the critical domain*.

In reading the conclusion of Ref. [1], one could understand that complex systems may display, *within the observable critical domain*, both the transitory almost classical power law behavior at some distance from T_c and a subsequent approach *from above* of the Ising exponent $\gamma \simeq 1.24$ (see Fig. 1 of Ref. [1]). We would like to indicate here that such a nonmonotonic crossover (occurring within the observable critical domain) is most likely impossible because it does not correspond to the conditions of applicability of the RGT.

For the sake of clarity, we first recall the following two points: (1) The RGT is based on a simplification of real systems and aims at describing them in a limited range of the physical parameters, namely, in the range where the correlation length ξ is larger than some (microscopic) length (say, Λ^{-1}). (2) A classical-to-Ising crossover may actually occur within the critical domain of a system (that belongs to the Ising universality class) as a consequence of a phase transition driven by a long range interaction. In such a situation, the asymptotic critical behavior is quite controlled by the (stable) Ising fixed point but before reaching this ultimate behavior, the critical phenomenon is first influenced by the (unstable) Gaussian fixed point. This means that, in the whole range of t where this crossover occurs, the system is actually critical, the condition $\xi > \Lambda^{-1}$ is fulfilled, and the RGT applies.

Roughly speaking, the RGT transfers the physical variable ξ into a fictitious variable $u_R(\xi/\Lambda^{-1})$ such that $u_R(\infty) = u^*$ and $u_R(1) = u$. The parameter u is nonuniversal; it is theoretically associated with Λ^{-1}. As it is clearly said in Ref. [1], u can take on any value in the range $[0, \infty[$. Specifically, one may have $u < u^*$, in which case one has $u < u_R(\xi/\Lambda^{-1}) < u^*$ and the system belongs to type B of Ref. [1], or $u > u^*$ with then $u^* < u_R(\xi/\Lambda^{-1}) < u$ (type A). Now, in the RGT, critical behavior governed by classical power laws is controlled by the Gaussian fixed point, the approach of which corresponds to $u_R(\xi/\Lambda^{-1}) \to 0$. Hence in order to allow this approach while preserving the condition $\xi > \Lambda^{-1}$ when ξ decreases from infinity, it is necessary that u be effectively small and Λ large. As it is mentioned in Ref. [1], these conditions correspond precisely to the approach of the continuum limit of the scalar field theory in three dimensions ($\Lambda \to \infty$ implying $u \to 0$). Since, in that case, u is much smaller than u^*, the system belongs necessarily to type B and the crossover is monotonic (as shown in Fig. 1 of Ref. [1]). On the contrary, since for systems of type A one has $u > u^*$, the classical power laws *controlled by the Gaussian fixed point* cannot be approached. Then a way to recover a classical behavior from the RG equations is to assume $\xi \ll \Lambda^{-1}$, a condition for which the RGT does not physically apply (to our knowledge) and, in any way, which would make it unlikely to further (when ξ increases) *observe also* the asymptotic approach (allowing us to distinguish between type A and type B) to the universal Ising behavior that implies $\xi \gg \Lambda^{-1}$.

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