Comment on "Quantum Backreaction on 'Classical' Variables"

In his recent Letter, Anderson [1] proposed a canonical formalism to couple quantum and (quasi)classical dynamic variables. Although the proposal may promise good physics (cf. Ref. [2]) its mathematical realization seems questionable. It seems the author takes lightly the fact that his quasiclassical bracket [Eq. (2) of [1]] is *not* antisymmetric. In fact, the lack of antisymmetry leads, in due course, to unacceptable consequences for time evolution of dynamic variables.

Consider the equation of motion, Eq. (4) of the Letter [4]. It will violate the Leibniz rule of differentiation as well as hermiticity of the dynamic variable A. In Anderson's first example, the Hamiltonian is $\frac{1}{2}kp^2$ and yields $\dot{q} = kp$ and $\dot{x} = \frac{1}{2}p^2$ for the time derivatives of the canonical coordinates. From them, applying the Leibniz rule first, we can calculate the (initial) time derivative of the dynamic variable $A = xq + qx$ and obtain $\dot{A} = \dot{x}q + x\dot{q} + \dot{q}x + q\dot{x} = \frac{1}{2}p^2q + \frac{1}{2}qp^2 + 2xkp$. If we calculated $\dot{A}$ directly from the equation of motion (4) we would obtain a different expression $\dot{A} = qp^2 + 2xkp$. It is hardly an acceptable result since it is *not* Hermitian and the Leibniz rule fails obviously to hold.

Similar effects will occur quite generally. Consider, e.g., a quantum particle and another (quasi)classical one, interacting via translation invariant potential $V(q - x)$. The Letter's Eq. (4) preserves the total momentum $p + k$ but it leads to an anti-Hermitian time derivative $-i\Delta V(q - x)$ when applied to the *square* $(p + k)^2$ of the total momentum. Anderson himself notices that, e.g., the energy of a conservative system might not be conserved in his theory.

These controversies would not arise at all had we chosen antisymmetric bracket of Aleksandrov [3] and of Boucher and Traschen [4]:

$$[A, B]_{q-c} = [A, B] + \frac{i}{2}\{A, B\} - \frac{i}{2}\{B, A\}$$

instead of the Letter's choice (2). I admit that I have failed to see enough reason of Anderson's departures from the above bracket, especially since the antisymmetric bracket can even be *derived* from quantum mechanics in proper (quasi)classical approximation as shown by Aleksandrov [3]. This should be a maximum justification in favor of the antisymmetric bracket even if the Letter's algebraic construction happened to result in a consistent theory.

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