Editor's Note: Related remarks were received from Yu. A. Rylov, Institute for Problems in Mechanics, Moscow 117526, Russia.

Comment on "Quantum Backreaction on 'Classical' Variables"

A recent Letter of Anderson [1] claims to have given a mathematically consistent scheme for coupling (quasi)classical and quantum systems. Although a consistent example of such a dynamical system is already known [2, 3], that proposed by Anderson differs in several ways. He seeks a Heisenberg picture dynamics, and claims no connection between this and the standard semiclassical factorization approximation [4]. However, it seems that there are some errors in his mathematical reasoning which destroy the claimed consistency of his technique. These negate his main result, whatever physical interpretation is contemplated.

In Eq. (3) of his Letter we are invited to consider the mathematical system

$$\dot{q} = -i[q, H] \qquad \dot{p} = -i[p, H], \tag{1}$$

$$\dot{x} = \{x, H\} \qquad \dot{k} = \{k, H\}, \tag{2}$$

where q and p are operators obeying the usual canonical commutation relations $[q, p] = i$, with $[\bullet, \bullet]$ the familiar commutator of quantum mechanics, and $\{\bullet, \bullet\}$ is the bracket

$$\{f, g\} \equiv \partial_x f \partial_k g - \partial_k f \partial_x g, \tag{3}$$

i.e., the classical Poisson bracket. Anderson in inexplicit about the nature of x, and k, but his use of partial derivatives indicates their status as ordinary commuting numbers.

These x and k are his (quasi)classical variables, which are coupled into the quantum q and p once we choose a Hamiltonian $H(q, p, x, k; t)$, containing a mixed functional dependence among both sets of variables (cf. Jones [3]). His Letter is inexplicit about whether H is to be considered an operator, or a pure number. Here lies the avenue towards proving the inconsistency of his mathematics. As we now show, neither assumption is tenable.

Taking the first member of his dynamical system, i.e., Eq. (1), we assume first that H is a pure number, then the commutators vanish, and there is *no quantum dynamics at all*. Taking now the second member, i.e., Eq. (2), we assume that H is an operator. However, then $\{x, H\}$ and $\{k, H\}$ are both operators, in contradiction of their assumed numerical form upon the left-hand side. To escape this contradiction one might assume that x and k are operators, but this would then negate the initial assumption of a Poisson algebra; i.e., the rule (3) fails. Therefore, his scheme is mathematically inconsistent.

The problem, here exposed, lies with ensuring a consistent embedding of two distinct algebras within one system of mathematics. Anderson has simply assumed the *existence* of mixed classical and quantum algebras. The entire problem here is to construct them explicitly and consistently. One cannot simply assume their existence.

In order to obtain a consistent coupling, Jones [3] has previously exploited a remarkable embedding [2] of the Poisson algebra, and commutator algebra (actually homeomorphic copies of them), within the larger algebra of Weinberg [6], defined as an infinite-dimensional Poisson bracket upon Hamiltonians H. These are nonbilinear Hermitian forms, that are constrained by an homogeneity restriction in the manner of Kibble [5], and Weinberg [6]. I refer interested readers to my recent papers for details of the argumentation [2, 3, 7].

In the scheme no such inconsistency arises, and the relevant Schrödinger equation is obtained directly [3]. Further, the physical interpretation of such mixed quantum-classical dynamical systems as a *nonfundamental decorrelated dynamical approximation* is then made transparent. In this role they are fine, but they cannot be employed in fundamental physics, for a most elementary violation of Heisenberg's uncertainty principle would result [3]. The same conclusion holds for the richer structure exhibited by interpolating algebras [7].

In conclusion, the inconsistency of Anderson's scheme is perhaps an invitation to study the uniqueness question in mixed-classical and quantum dynamics. Evidently, it is highly nontrivial, since not all apparently plausible schemes are internally consistent.

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Received 27 February 1995 [S0031-9007(96)00170-6]
PACS numbers: 03.65.Sq, 04.62.+v

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0031-9007/96/76(21)/4087(1)$10.00