Effects of Electron-Electron Interactions on the Integer Quantum Hall Transitions

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We study the effects of the electron-electron interaction on the critical properties of the plateau transitions in the *integer* quantum Hall effect. We find the renormalization group dimension associated with short-range interactions to be -0.66 ± 0.04. Thus the noninteracting fixed point (characterized $z = 2$ and $\nu \approx 2.3$) is stable. For the Coulomb interaction, we find the correlation effect is a marginal perturbation at a Hartree-Fock fixed point ($z = 1$, $\nu \approx 2.3$) by dimension counting. Further calculations are needed to determine its stability upon loop corrections. [S0031-9007(96)00226-8]

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The plateau transitions have been one of the unsolved problems in the quantum Hall effect. The fundamental questions that remain unanswered are: (a) What are the effects of electron-electron interaction on the integer transitions? (b) What are the effects of the quasiparticle statistics on the fractional transitions? Almost all recent works on the integer plateau transitions are based on numerical analyses of models of *noninteracting* electrons [1]. An important outcome of these efforts is the consensus on the approximate values of the localization length exponent (ν), and several others characterizing the participation ratio and its higher moments [1]. In particular, the result $\nu \approx 2.3$ is in excellent agreement with the measured value [2,3]. Nonetheless, such basic issues as the relevance and the effects of the electron-electron interaction have not been addressed. The necessity to understand the interaction effects becomes even more pressing after recent experimental reports of the dynamical exponent $z = 1$, instead of the noninteracting value 2 [4,5]. In this Letter, we focus on the effects of the interactions on the *integer* plateau transitions.

Our strategy is the following. We take the *2 + 1* dimensional noninteracting theory as the starting point. We then ask what the effects are of turning on the interaction. In practice, we calculate the renormalization group (RG) dimension of the interaction Hamiltonian, and also look at the other possible interactions it generates upon renormalization. This is a standard exercise when one analyzes the stability of a known critical point. However, unlike many cases where one has *analytic* knowledge of the critical point in question, in the present case such knowledge is lacking. Thus the results that we report in this Letter are based on *numerical* calculations of various correlation functions at the noninteracting and Hartree-Fock fixed point.

For *short-range* interaction we obtained its RG dimension ≈ -0.65, and found that to the second order in the interaction strength no relevant operators are generated upon renormalization. Thus we conclude that short-range interaction is an irrelevant perturbation at the noninteracting fixed point. Hence for screened electron-electron interactions $z = 2$ and $\nu \approx 2.3$. For Coulomb interaction, we find that it is relevant at the *noninteracting* fixed point. However, by dimension counting, we find that due to a linear suppression in the density of states (DOS) [6] the correlation effect is only a *marginal* perturbation at the Hartree-Fock fixed point. For the latter, we find $z = 1$ and $\nu \approx 2.3$. The Hartree-Fock fixed point provides a concrete example where Coulomb interaction modifies the dynamical exponent and not the static one. The root of such behavior is the *noncritical* suppression of the DOS. Indeed, as was shown in Ref. [6], the Hartree-Fock DOS vanishes linearly with $|E - E_F|$ *regardless of whether the Fermi energy* E_F *coincides with the critical value.* This suppression resulted in a change of z from 2 to 1, and a degradation of the RG dimension of the *residual* Coulomb Hamiltonian from 1 to 0. We do not yet know the effects of the residual Coulomb interaction upon further loop corrections.

We start with noninteracting electrons described by the following Euclidean action (in units $e/c = \hbar = k_B = 1$):

$$S_0 = \int d^2x \sum_{\omega_n} \overline{\psi}_{\omega_n}(x)[-i\omega_n + \Pi^2 + V_{\text{imp}}(x)]\psi_{\omega_n}(x) . \tag{1}$$

In the above, ψ is the fermion Grassmann field, $\omega_n = (2n + 1)\pi/\beta$ is the fermion Matsubara frequency, V_{imp} is the disorder potential, and $\Pi^2 \equiv -(1/2m)\sum_k [\partial_k - iA_k(x)]^2$ where $A_k(x)$ is the external vector potential. The action describing the interaction reads

$$S_{\text{int}} = T \sum_{\omega_1,\ldots,\omega_4} \delta_{\omega_1+\omega_2,\omega_3+\omega_4} \int d^2x\, d^2y \times V(|x-y|)\overline{\psi}_{\omega_1}(x)\overline{\psi}_{\omega_2}(y)\psi_{\omega_4}(y)\psi_{w_3}(x) . \tag{2}$$

In this Letter we consider $V(|x - y|) = g/|x - y|^\lambda$. The total action is $S = S_0 + S_{\text{int}}$, in which S_{int} couples the otherwise independent frequency components of S_0 together. To emphasize the symmetry property of S_{int} we rewrite it as

$$S_{\rm int} = \frac{T}{4} \sum_{\omega_1,\ldots,\omega_4} \delta_{\omega_1+\omega_2,\omega_3+\omega_4} \int d^2x\, d^2y \times V(|x-y|)\overline{B}_{(\omega_1,\omega_2)}(x,y) B_{(\omega_3,\omega_4)}(y,x)\,, \quad (3)$$

where $\overline{B}_{(\omega_1,\omega_2)}(x,y) \equiv \overline{\psi}_{\omega_1}(x)\overline{\psi}_{\omega_2}(y) + \overline{\psi}_{\omega_2}(x)\overline{\psi}_{\omega_1}(y)$. In the following we imagine sitting at the fixed point of S_0 and numerically analyze the scaling properties of $\langle S_{\rm int}\rangle$ and $\langle S_{\rm int}^2\rangle$ in finite periodic systems of linear dimension L and imaginary time dimension $1/T$. (Hereafter $\langle\cdots\rangle$ denotes quantum and impurity averages.) Since we are after the *universal* scaling properties of various correlation functions, any representation of S_0 which produces the right universality class suffices. In the following we choose the "quantum percolation" (or the network) model of Chalker and Coddington. For details about this model the readers are referred to Ref. [7]. Moreover, the numerical calculations reported here are done using the $U(2n)|_{n\to 0}$ Hubbard model representation of the network model [8].

In order to evaluate $\langle S_{\rm int}\rangle$, it is necessary to know the correlation function $\langle \overline{B}_{(\omega_1,\omega_2)}(x,y) B_{(\omega_3,\omega_4)}(y,x)\rangle = \delta_{\omega_1,\omega_3}\delta_{\omega_2,\omega_4}\Gamma'^{(4)}(x,y;\omega_1,\omega_2,L)$. To extract the critical piece of $\Gamma'^{(4)}$, it is important to perform the trace decomposition [9,10]. Thus we write

$$\Gamma'^{(4)}(x,y;\omega_1,\omega_2,L) \equiv \Gamma^{(4)}(x,y;\omega_1,\omega_2,L) + \langle \overline{\psi}_{\omega_1}(x)\psi_{\omega_1}(x)\overline{\psi}_{\omega_1}(y)\psi_{\omega_1}(y) + (\omega_1 \to \omega_2)\rangle\,. \quad (4)$$

Each of the last two terms in Eq. (4) involves only a single Matsubara frequency and is noncritical. $\Gamma^{(4)}$ is only critical when $\omega_1\omega_2 < 0$ [11]. Since the scaling dimension of $\overline{\psi}_{\omega_1}(x)\psi_{\omega_2}(x)$ is zero at the noninteracting fixed point (i.e., the density of states has no anomalous dimension) [8], $\Gamma^{(4)}$ obeys the following scaling form:

$$\Gamma^{(4)}(x,y;\omega_1,\omega_2,L) = \mathcal{F}_1(|x-y|/L, \omega_1 L^2, \omega_2 L^2)\,. \quad (5)$$

Note that near the noninteracting fixed point, it is sufficient to consider any pair of a positive and a negative frequency. The latter scales with L^{-2} in Eq. (5) which reflects the fact that $z = 2$ at the noninteracting fixed point. We have calculated $\Gamma^{(4)}$ using the Monte Carlo method of Ref. [8] for $\omega_{1,2}L^2 = {\rm const}_{1,2}$. The details of the calculation will be reported elsewhere [11]. The scaling behavior of $\Gamma^{(4)}$ vs $|x-y|/L$ is shown in Fig. 1. The results are consistent with $\mathcal{F}_1(|x-y|/L, \omega_1 L^2, \omega_2 L^2) \sim (|x-y|/L)^{x_{4s}}$ for $|x-y|/L \ll 1, |(\omega_{1,2}L^2)|^{-1/2}$, and $x_{4s} \approx 0.65$. Thus, in terms of the properly scaled variables, the first order correction to the *singular part* of the quench-

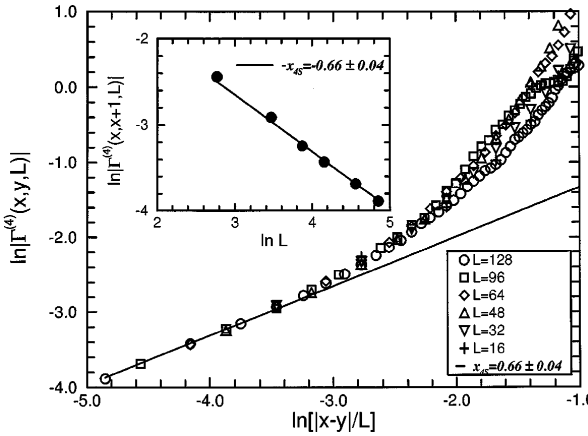

FIG. 1. The scaling plot of $\Gamma^{(4)}$. Inset shows the L dependence of $\Gamma^{(4)}(x, x+1)$.

averaged action is

$$\Delta S_{\rm sing}^{(1)} = \frac{TL^2}{4}\left(\frac{g}{L^{\lambda-2}}\right) \sum_{n_1,n_2}{}' \int d^2\left(\frac{x}{L}\right) \int d^2\left(\frac{y}{L}\right) \times \left|\frac{L}{x-y}\right|^{\lambda} \mathcal{F}_1\left(\frac{|x-y|}{L}; \pi(2n_{1,2}+1)TL^2\right). \quad (6)$$

In the above $\sum'$ denotes the restricted sum satisfying $n_1 n_2 < 0$. Let us change the integration variables $d^2(x/L)d^2(y/L)$ to $d^2(x+y)d^2(x-y)/L^2$. The part that depends on the relative coordinate reads

$$\int d^2\left(\frac{x-y}{L}\right)\left(\frac{L}{|x-y|}\right)^{\lambda} \mathcal{F}_1\left(\frac{|x-y|}{L}, \omega_1 L^2, \omega_2 L^2\right), \quad (7)$$

where the upper limit of the integral is 1 and the lower one is a/L (a is the lattice spacing). Naively, one would deduce from Eq. (6) that the RG dimension of g is $2-\lambda$ as the result of dimensional analyses. This conclusion can be modified if the integral in Eq. (7) depends on a/L, i.e., if it diverges at the lower limit. Since $\mathcal{F}_1 \sim (|x-y|/L)^{x_{4s}}$ for $|x-y|/L \ll 1$, the integral diverges (we will henceforth refer to this case as that of short-range interaction) when $\lambda \geq x_{4s} + 2$, and converges (long-range interaction) otherwise.

Let us now concentrate on the case $\lambda > x_{4s} + 2$ (i.e., short-range interaction). Simple analyses of Eq. (6) show that

$$\Delta S_{\rm sing}^{(1)} = (g/L^{\lambda-2})[A + B(a/L)^{2+x_{4s}-\lambda}], \quad (8)$$

where A and B are nonuniversal functions of TL^2. Since $\lambda - 2 > x_{4s}$, the asymptotic scaling behavior of $\Delta S_{\rm sing}^{(1)}$ is controlled by $\Delta S_{\rm sing}^{(1)} = Bu/L^{x_{4s}}$ where $u \equiv ga^{2+x_{4s}-\lambda}$. In the language of the renormalization group, the density operators at nearby points have fused together to form a new operator with a RG dimension $-x_{4s}$. Thus for screened Coulomb interactions, we conclude that the RG dimension for T is 2 (thus $z = 2$), and that for u is $-x_{4s} \approx -0.65$. Therefore to this order the interaction is *irrelevant.* Here we note that if similar analyses are done for the weak-field (i.e., the "singlet only") metal-insulator transition in $2 + \epsilon$ dimensions, one obtains $x_{4s} = \sqrt{2\epsilon}$ agreeing with the results obtained in, e.g., Ref. [10].

In order to perform a self-consistency check on the RG dimension of u, and to study the fusion products [12] of two interaction operators, we next calculate $\langle S^2_{\rm int}\rangle$. For that purpose we need to consider $\langle \overline{B}_{(\omega_1,\omega_2)}(x,y)B_{(\omega_3,\omega_4)}(y,x)\overline{B}_{(\omega_5,\omega_6)}(x',y')B_{(\omega_7,\omega_8)}(y',x')\rangle_c = \delta_{\omega_1,\omega_7}\delta_{\omega_2,\omega_8}\delta_{\omega_3,\omega_5}\delta_{\omega_4,\omega_6}\Gamma^{(8)}(x,y,x',y';\omega_1,\omega_2,\omega_3,\omega_4;L)$. For short-range interactions we need to concentrate only on the limit $|x-y|,|x'-y'| \ll |R-R'|$, where $R = (x+y)/2$ and $R' = (x'+y')/2$. In that limit and for $\omega_1\omega_2 < 0, \omega_3\omega_4 < 0$ (other combinations give noncritical contributions [11]), the result is

$$\Gamma^{(8)}(x,y,x',y';\{\omega_i\};L) = \mathcal{F}_2\left(\left|\frac{x-x'}{R-R'}\right|,\left|\frac{y-y'}{R-R'}\right|,\frac{|R-R'|}{L},\{\omega_i L^2\}\right), \tag{9}$$

In the limit $|x(y) - x'(y')|/|R-R'| \ll 1, \omega_i L^2|^{-1/2}$, $\mathcal{F}_2$ reduces to

$$\mathcal{F}_2 \sim \left|\frac{x-y}{R-R'}\right|^{x_{4s}}\left|\frac{x'-y'}{R-R'}\right|^{x_{4s}} \mathcal{F}_3(|R-R'|/L,\{\omega_i L^2\})\,. \tag{10}$$

The result for $|R-R'|^{2x_{4s}}\Gamma^{(8)}$ vs $|R-R'|/L$ for small, typical, fixed $|x-y|,|x'-y'|$, $\omega_i = \mathcal{O}(1/L^2)$, and $x_{4s} = 0.65$, is shown in Fig. 2. This result indicates that the previously obtained $x_{4s} \approx 0.65$ is the consistent scaling dimension of the short-range interaction. Going through similar manipulations one can show that the second order correlation correction to the singular part of the quench-averaged action, $\Delta S^{(2)}_{\rm sing}$, is

$$\begin{aligned}\Delta S^{(2)}_{\rm sing} = &-(TL^2)^2\frac{u^2}{L^{2x_{4s}}}\sum_{n_1,\dots,n_4}{}' \int d^2 \\ &\times \left(\frac{R}{L}\right) d^2\left(\frac{R'}{L}\right)\left|\frac{L}{R-R'}\right|^{2x_{4s}} \\ &\times \mathcal{F}_4\left[\frac{|R-R'|}{L},\pi(2n_{1\to 4}+1)TL^2\right], \end{aligned}\tag{11}$$

where $\sum'$ denotes the restricted sum satisfying $n_1 n_2 < 0$ and $n_3 n_4 < 0$, and $F_4 \propto F_3$. In Eq. (11) let us convert d^2Rd^2R' to $d^2(R+R')d^2(R-R')$. In the integral over the relative coordinates, the short-distance cutoff is again a/L. As before, new dependence on L could emerge if the integral over $R-R'$ diverges at the lower limit. In general, if $\mathcal{F}_4(|R-R'|/L,\{\omega_i L^2\}) \sim |(R-R')/L|^{\alpha}$, and if $2x_{4s} - \alpha - 2 > 0$ then

$$\Delta S^{(2)}_{\rm sing} = -[C(u/L^{x_{4s}})^2 + Dv/L^{2+\alpha}], \tag{12}$$

where C,D are nonuniversal functions of TL^2, whereas $v \equiv g^2 a^{2(2-\lambda)+2+\alpha}$. In this case a new scaling operator, fused from two interaction operators, would emerge with a RG dimension $-(2+\alpha)$. Moreover, since $2+\alpha < 2x_{4s}$ this operator would control the asymptotic scaling of $\Delta S^{(2)}_{\rm sing}$. On the other hand, if $2x_{4s} - \alpha - 2 < 0$ the integral over the relative coordinates converges, and $\Delta S^{(2)}_{\rm sing} = -C(u/L^{x_{4s}})^2$; thus no new scaling variable needs to be introduced. Our results shown in Fig. 2 indicate that $\alpha = 0$; thus $2x_{4s} - 2 - \alpha < 0$ and hence we do not need to introduce any new scaling operator at this order.

Now we summarize our results for short-range interaction. For interaction $V(r) = g/|r|^{\lambda}$, we find that the noninteracting fixed point is *stable* (thus $z = 2$ and $\nu \approx 2.3$) if $\lambda > 2 + x_{4s}$ (here $-x_{4s}$ is the RG dimension of short-range interactions). Our numerical results give $x_{4s} \approx 0.65$. Although the above analyses do not form a "proof" that strong short-ranged interactions are irrelevant, we believe that the evidence is sufficiently strong.

Next, we consider the long-range Coulomb interaction, i.e., $\lambda = 1$. In that case $\lambda < x_{4s} + 2$; therefore Eq. (8) is asymptotically controlled by $\Delta S^{(1)}_{\rm sing} = Ag/L^{\lambda-2}$, which implies a relevant RG dimension for g of $2 - \lambda = 1$. Thus the *noninteracting fixed point* is *unstable* upon turning on the Coulomb interaction. This result is not surprising given the fact that the measured value for z is 1 instead of the noninteracting value 2. But if so, why should the static exponent ν remain unchanged?

In two recent papers, MacDonald and co-workers studied the integer plateau transition under a Hartree-Fock treatment of the Coulomb interaction [6]. They found

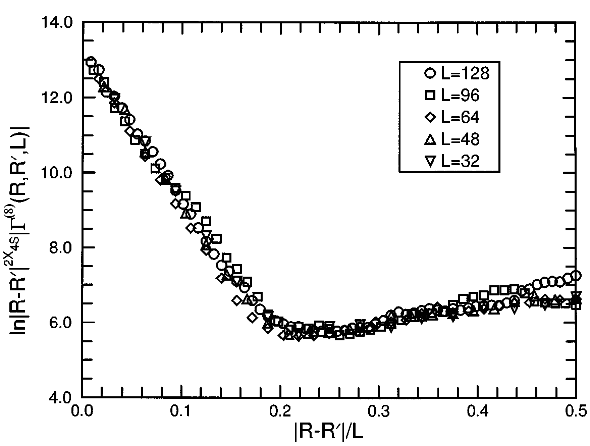

FIG. 2. The scaling plot of $|R-R'|^{2x_{4s}}\Gamma^{(8)}$ obtained with $x_{4s} \approx 0.65$.

that (a) the DOS shows the Coulomb gap behavior [i.e., $\rho(E_F) \sim L^{-1}$ in samples of linear dimension L [13]] *regardless of whether the system is at criticality or not;* (b) despite the dramatic *noncritical* DOS suppression, the localization length exponent and the fractal dimension of the critical eigen wave functions remain unchanged. In addition, the conductivities *did not* show any qualitative change. We take these results as indicating that the Hartree-Fock theory is in the same universality class as the noninteracting one. Thus the field theory should be the same nonlinear σ model [14] in which the *bare* parameters do not have nontrivial scale dependence except that the DOS in the symmetry-breaking term should be replaced by the appropriate Coulomb gap form. Since the combination $\pi T \rho$ should have dimension 2, and $\rho \sim 1/L$, it implies $z = 1$. Thus, z is modified while ν is not, and the change in z is caused by a *noncritical* modification of the DOS.

A direct consequence of the DOS suppression is that the dimension of $\overline{\psi}_{\omega_1}\psi_{\omega_2}$ is changed from 0 to 1. Indeed, it can be shown [11] that the two-particle spectral function that is consistent with the results in Ref. [6] and the scaling arguments in Ref. [15] are given by

$$S_2(E_1, E_2, \vec{q}) = \frac{\rho^2 \sigma q^2}{\rho^2 (E_1 - E_2)^2 + (\sigma q^2)^2}. \tag{13}$$

In the above ρ depends on $E \equiv (E_1 + E_2)/2$ and σ, a quantity with the dimension of conductivity, depends on $\omega \equiv (E_1 - E_2)/2$ and the wave vector $\vec{q}$. At the critical point, $\rho(E) \sim 1/L$ and $\sigma(\omega, \vec{q}) =$ const for $|\rho\omega| \gg q^2$; and const $\times (q^2/|\rho\omega|)^{x_2/2}$ for $|\rho\omega| \ll q^2$. Here $x_2 \approx -0.5$ is the exponent characterizing the anomalous diffusive behavior in the critical regime [8,15]. Note that the new exponents x_{4s} are independent of x_2. They are, respectively, the scaling dimensions of the operators associated with the fusion products of four fermion operators, or two SU($2n$) spin operators that are symmetric and antisymmetric under permutations [9]. If one uses Eq. (13) to compute the two-particle Green's function, one can show that both z and the scaling dimension of $\overline{\psi}_{\omega_1}\psi_{\omega_2}$ are unity [11].

To support the predictions of the Hartree-Fock theory, one has to analyze the stability of the Hartree-Fock fixed point when the residual Coulomb interaction is taken into account. Because of the normal ordering with respect to the Hartree-Fock ground state, there is no contribution to $\Delta S^{(1)}_{\rm sing}$ due to the residual Coulomb interaction [16]. The lowest order effects now come in via $\Delta S^{(2)}_{\rm sing}$. The new scaling form for $\Gamma^{(8)}$ is $\Gamma^{(8)}(r_1, r_2, r_3, r_4, \{\omega_i\}, L) = L^{-4}\mathcal{F}_5(r_{ij}L^{-1}, \{\omega_i L\})$. Inserting this result into

$$\Delta S^{(2)}_{\rm sing} = -\frac{1}{32}(gT)^2 \sum_{n_1,\ldots,n_4}{}' \int d^2x\, d^2y\, d^2x'\, d^2y' \times \frac{\Gamma^{(8)}(x, y, x', y'; \pi(2n_{1\to 4} + 1)T, L)}{|x - y|\,|x' - y'|} \tag{14}$$

and *ignoring the possible short-distance divergence* we obtain $\Delta S^{(2)}_{\rm sing} \sim g^2(TL)^2$, thus g is marginal. In order to go beyond this analysis (i.e., to determine the outcome of short-distance fusion) we need to know the behaviors of $\Gamma^{(8)}$ in a number of limits, information that we do not have at present. Finally, we would like to emphasize that the Hartree-Fock theory presents a concrete example where, due to a *noncritical* suppression of the DOS, z is modified while ν is not.

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Note added.—In an interesting recent work [17], the effects of interactions are studied via the nonlinear σ model [14] where the topological term is handled by the dilute instanton gas approximation. Since the latter *has not* produced the correct critical properties even for the noninteracting transition, it is difficult for us to judge the reliability of the results on the effects of interactions.

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