Hu Replies: In a recent Letter [1], Lin, Chen, and I (HLC) used a histogram Monte Carlo simulation method (HMCSM) to evaluate the existence probability E_p and the percolation probability P of bond and site percolations on a 512×512 square (sq), a 433×500 plane triangular (pt), and a 433×250 honeycomb (hc) lattices with both free (f) and periodic (p) boundary conditions (bc). We found that, in these six models, E_p and $D_3 P/L^{-\beta/\nu}$ as a function of $x = D_1(p - p_c)L^{1/\nu}$ fall on the same universal scaling functions $F(x)$ and $S(x)$, respectively, where D_1 and D_3 are model dependent nonuniversal metric factors [2] and they are independent of bc. We pointed out that our results generalize Privman and Fisher's (PF) original idea of universality of finite-size scaling functions (FSSF) [2] because E_p may not be obtained from the derivative of PF's *free energy*. In the Comment [3] on our Letter [1], Hovi and Aharony (HA) pointed out (1) HLC's results are direct consequences of the renormalization group (RG), as discussed in Ref. [4], (2) E_p may be obtained from the derivative of the free energy, (3) HLC's numerical data for the small-x expansion of F for fbc are in agreement with their results [4], but (4) HLC's data for pbc strongly disagree with their results [3].

I agree that it is possible to arrive at the universality of FSSF from RG arguments. In fact, PF [2] also used RG arguments to support the universality of FSSF. However, RG is basically a theoretical idea. Its consequences should be constantly subject to numerical and experimental tests. The true value of HLC's paper [1] is to provide convincing evidence that so many percolation models indeed may have universal scaling functions, which is missing in previous publications. Furthermore, it is not easy to obtain functional forms of FSSF from RG calculations. It is true that E_p may be obtained from the derivative of the *free energy* with respect to *local fields*. However, local fields in this sense were not considered by PF [2].

In [5], five linear dimensions of sq lattices, from 32 to 512, were considered and the calculated E_p and P on such lattices have very good scaling behavior. We have found similar results for pt and hc lattices. In [1], $433/500$ and $433/250$ were used to approximate the aspect ratios $\sqrt{3}/2$ and $\sqrt{3}$ for pt and hc lattices [6], respectively. Smaller lattices will give worse approximations for $\sqrt{3}/2$ and $\sqrt{3}$. Thus only one size for each lattice is considered in [1]. In [1] and [5], the pbc means periodic in both horizontal and vertical directions; i.e., the top row is also connected to the bottom row by *nearest neighbor* bonds. A cluster is percolating if every horizontal row of the lattice contains at least one site of that cluster. I guessed that HA considered pbc only in one, say horizontal, direction and the other, say vertical, direction has fbc: A cluster is percolating if its sites may be found in the top row and the bottom row of the lattice. To check whether this guess is correct, I have used the HMCSM to calculate E_p for bond percolation on sq lattices with such bc. The calculated E_p's as a function of $x = (p - p_c)L^{1/\nu}$, shown in Fig. 1, have very good scaling behavior. Using HA's convention in which the coefficient of the linear term is normalized to 1, we have $F(\hat{x}) = 0.6(4) + \hat{x} - 0.5(0)\hat{x}^2 - 0.(9)\hat{x}^3 + 0.(7)\hat{x}^4 + \cdots$, which is consistent with HA's result [3].

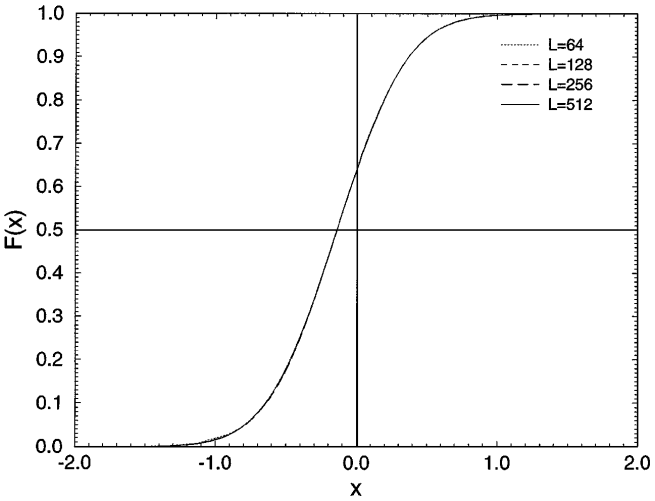

FIG. 1. The calculated E_p as a function of $x = (p - p_c)L^{1/\nu}$ for the bond percolation on $L \times L$ sq lattices with pbc in the horizontal direction and fbc in the vertical direction, where $L = 64$, 128, 256, and 512. The scaling function is $F(x)$.

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Chin-Kun Hu
Institute of Physics, Academia Sinica
Nankang, Taipei 11529, Taiwan

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