Quantum Localization Effects on Spin Transport in Semiconductor Quantum Wells with Zinc-Blende Crystal Structure

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We have shown that due to weak localization corrections, in the spin-split conduction band of quantum wells with zinc-blende structure, the electron spin relaxation rate due to the D'yakonov-Perel' mechanism decreases logarithmically with decreasing frequency. The spin diffusion coefficient also decreases. This is quite different from antilocalization behavior at large times of the particle diffusion and conductivity. Possible experimental detection is suggested. [S0031-9007(96)00208-6]

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It is well known [1] that due to the lack of inversion symmetry the spin-orbit interaction (SOI) lifts the spin degeneracy of the conduction band in semiconductors with zinc-blende structure. The corresponding electron energy splitting $h_{\vec{k}}$ depends on the direction of the wave vector $\vec{k}$. At higher electron energies, $h_{\vec{k}}$ increases as $h_{\vec{k}} \propto k^3$ [2]. The spin splitting is deduced from optical orientation measurements [1], and is quite small in bulk semiconductors. However, since $h_{\vec{k}}$ is proportional to k^3, it can be rather large in narrower quantum wells due to the confinement of electrons along the growth direction [3]. For example, a splitting of about 0.4 meV was observed in Raman scattering from the degenerate electron gas in a AlGaAs quantum well [4]. Even higher values of $h_{\vec{k}}$ were obtained with Shubnikov–de Haas measurements on InGaAs/InAlAs heterojunctions [5].

The SOI effect on quantum transport has been extensively studied since Hikami, Larkin, and Nagaoka [6] showed that the SOI between electrons and impurities can change the sign of the localization correction to the *electric* conductivity in metal films. Similar behavior for electric conductivity was studied later in III-V semiconductor quantum wells [7–9], using a model in which impurities are treated as usual potential scatterers, while the spin-orbit effect is associated to the intrinsic spin splitting of the conduction band. This allowed one to explain [8,9] the negative magnetoresistance observed in GaAs quantum wells [10]. The spin splitting can also lead to other quantum phenomena in disordered systems. For example, a new type of level statistics was found in 1D mesoscopic rings [11], and the Aharonov-Casher effect caused by macroscopic electric fields in disordered conductors [12].

More fundamentally, besides its effect on particle diffusion, the SOI has a significant effect on a number of physical processes which involve spin related transport. Many of these effects are associated with the particle flux induced by the spin orientation of electrons such as, for example, in the so called anomalous Hall effect [13]. The spin transport parameters play an important role in determining the shape of low frequency spin-flip electronic Raman spectra and EPR lines. There are two main transport parameters which dominate the spin response of the system: the spin relaxation time and the spin diffusion coefficient. In III-V semiconductors, the electron spin polarization can relax through elastic scattering with impurities. If the elastic scattering time τ is sufficiently short such that $\tau h_{\vec{k}} \ll 1$, the spin relaxation occurs through the D'yakonov-Perel' mechanism [14] with the relaxation rate $1/\tau_s \simeq \tau \langle h_{\vec{k}}^2 \rangle_{\rm dir}$, where $\langle \cdots \rangle_{\rm dir}$ is an average over the direction of $\vec{k}$. Since the spin relaxation rate in the D'yakonov-Perel' mechanism is proportional to the elastic scattering time, the associated spin relaxation belongs to the same class of transport phenomena as conduction or diffusion. With a further extension of this analogy, one would expect that localization effects due to quantum interference of elastically scattered waves must result in a change of the spin relaxation rate, similar to the case of electric conductivity or particle diffusion. Besides spin relaxation, spin diffusion is also modified by weak localization corrections. Within the classical approach, the spin diffusion coefficient, which determines the relaxation of an inhomogeneous spin distribution, is the same as the particle diffusion coefficient. However, in a system with a spin-split conduction band, such as III-V semiconductors, the corrections to these two coefficients due to weak localization effects are expected to be different.

Previous works on quantum transport in III-V semiconductors focused on weak localization effects on electric conductance or particle diffusion, while the effects on spin transport were not elucidiated. In this Letter we

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investigate weak localization corrections to the spin relaxation and the spin diffusion of a two-dimensional (2D) degenerate electron gas in a spin-split conduction band. The physical system is a doped quantum well with the growth direction along the z axis and the interfaces parallel to the x-y plane. We assume that the amplitude of electron scattering by a random δ-correlated potential is spin independent, and the corresponding elastic mean free path l is large compared to k_f^{-1}, where k_f is the Fermi wave vector. We will show that due to weak localization corrections, with decreasing frequency, both longitudinal and transverse spin relaxation rates as well as the spin diffusion constant decrease. The localization behavior of the spin diffusion constant differs drastically from that of the particle diffusion constant, which, as was found previously [7–9], first decreases and then increases when the frequency is comparable to the spin relaxation rate.

We start with the system Hamiltonian

$$H = \sum_{\vec{k},\sigma} E_{\vec{k}} c^{\dagger}_{\vec{k}\sigma} c_{\vec{k}\sigma} + \sum_{\vec{k},\sigma,\sigma'} \vec{h}_{\vec{k}} \cdot \vec{s}_{\sigma'\sigma} c^{\dagger}_{\vec{k}\sigma'} c_{\vec{k}\sigma} + \sum_{\vec{k},\vec{k}',\sigma} V_{\vec{k},\vec{k}'} c^{\dagger}_{\vec{k}\sigma} c_{\vec{k}'\sigma} . \quad (1)$$

The first term in H is the unperturbed Hamiltonian of electrons in the conduction band $E_{\vec{k}}$. The spin splitting of the conduction band is described by the second term, which has the form of coupling between the electron spin $\vec{s}_{\alpha,\beta} \equiv (\sigma^x_{\alpha,\beta}, \sigma^y_{\alpha,\beta})$ and the "magnetic field" $\vec{h}_{\vec{k}} \equiv (h^x_{\vec{k}}, h^y_{\vec{k}})$, where σ^x and σ^y are Pauli spin matrices. The characteristic functional dependence of $\vec{h}_{\vec{k}}$ on $\vec{k}$ varies with the width of a quantum well. In a sufficiently narrow quantum well, $\vec{h}_{\vec{k}}$ is linear in $\vec{k}$ [3], and has the special form

$$h^x_{\vec{k}} = \alpha k_x, \qquad h^y_{\vec{k}} = -\alpha k_y \quad (2)$$

if the growth direction of the quantum well is [001]. On the other hand, in a wider well or for larger magnitude of $\vec{k}$, the nonlinear terms in $\vec{h}_{\vec{k}}$ can be important [3]. However, in our present work, for simplicity, we will consider a very narrow quantum well for which (2) is valid. Since $E_{(k_x,k_y)} = E_{(k_x,-k_y)}$, with a proper transformation between $c_{(k_x,k_y)\sigma}$ and $c_{(k_x,-k_y)\sigma}$, the second term in (1) can be rewritten in a more convenient form $\alpha \sum_{\vec{k},\sigma,\sigma'} \vec{k} \cdot \vec{s}_{\sigma'\sigma} c^{\dagger}_{\vec{k}\sigma'} c_{\vec{k}\sigma}$.

The third term in (1) represents the scattering of electrons by a random potential, which is produced by impurities in the well and/or imperfections of the well. The electronic mean elastic scattering time is given by the usual form [15] $(2\tau)^{-1} \equiv \Gamma = \pi N(E_f) \langle |V_{\vec{k},\vec{k}'}|^2 \rangle$, where $N(E_f)$ is the density of states at the Fermi level. We assume that the configuration average of the scattering potential does not depend on the directions of $\vec{k}$ and $\vec{k}'$. Therefore, we can set $\langle |V_{\vec{k},\vec{k}'}|^2 \rangle = V^2$.

We will calculate the density-density correlation functions for both the charge density and the spin density. For this purpose, we need to calculate the relevant two-particle Green functions. Within the usual perturbation approach [15], these two-particle Green functions contain both the diffusion propagator (diffuson) which is a sum of ladder diagrams in the particle-hole channel and the Cooperon propagator which is a sum of ladder diagrams in the particle-particle channel. The configuration average $\langle G^a_{\alpha\beta}(\vec{k} - \vec{Q}, \vec{k}' - \vec{Q}, \omega - \Omega) G^r_{\nu\mu}(\vec{k}', \vec{k}, \omega) \rangle$ of the two-particle Green function of interest can be expressed as

$$\langle G^a_{\alpha\beta}(\vec{k} - \vec{Q}, \vec{k}' - \vec{Q}, \omega - \Omega) G^r_{\nu\mu}(\vec{k}', \vec{k}, \omega) \rangle = G^a_{\alpha\beta}(\vec{k} - \vec{Q}, \omega - \Omega) G^r_{\nu\mu}(\vec{k}, \omega) \delta_{\vec{k},\vec{k}'} + \sum_{\gamma\gamma'\delta\delta'} G^a_{\alpha\gamma}(\vec{k} - \vec{Q}, \omega - \Omega) G^r_{\nu\delta'}(\vec{k}', \omega) \times U_{\gamma\gamma'\delta'\delta}(\vec{k}, \vec{k}', \vec{Q}, \Omega) G^a_{\gamma'\beta}(\vec{k}' - \vec{Q}, \omega - \Omega) G^r_{\delta\mu}(\vec{k}, \omega) \quad (3)$$

in terms of the matrix vertex function $U(\vec{k}, \vec{k}', \vec{Q}, \Omega)$ and the averaged retarded $G^r(\vec{k}, \omega)$ and advanced $G^a(\vec{k}, \omega)$ one-particle Green functions

$$G^{r,a}(\vec{k}, \omega) = \frac{(\omega - E_{\vec{k}} \pm i\Gamma) + \vec{n} \cdot \vec{s} h}{(\omega - E_{\vec{k}} \pm i\Gamma)^2 - h^2/4} , \quad (4)$$

where $h = \alpha k_f$ and $\vec{n} = \vec{k}/k$. In (3), all subscripts are spin projections onto the z axis.

It is convenient to expand the matrix vertex function U in terms of the complete basis set of matrices: the three Pauli matrices σ^x, σ^y, σ^z, and $\sigma^0 = 1/2$. In this expansion, the matrix elements are defined as $U_{\alpha\beta\nu\mu} = \sum_{i,j} \sigma^i_{\alpha\mu} \sigma^j_{\nu\beta} U^{ij}$. If in the vertex function only diffusons are taken into account, we arrive at the diffusion approximation which is equivalent to the classical approach based on the kinetic equation. In our notation, U_d represents the vertex function calculated in the diffusion approximation. With a summation of the ladder diagrams in the particle-hole channel, under the condition $v_f Q \ll \Gamma$ of our interest, we obtain for the spin-independent propagator

$$U^{00}_d = \mathcal{D}_0 = 4i\Gamma/[\Omega + iDQ^2], \quad (5)$$

where $D = v_f^2/4\Gamma$ is the particle diffusion constant. For the spin-dependent propagator, we have $(i, j = x, y, z)$

$$U^{ij}_d = \frac{V^2}{\mathcal{D}(\vec{Q})} \left[\mathcal{D}_i(\vec{Q}) \delta^{ij} - m^2(\vec{Q}) \mathcal{D}_l(\vec{Q}) \mathcal{D}_t^2(\vec{Q}) \hat{Q}^i \hat{Q}^j + im(\vec{Q}) \mathcal{D}_l(\vec{Q}) \mathcal{D}_t(\vec{Q}) \sum_s \hat{Q}^s (\epsilon^{js} \delta^{iz} - \epsilon^{is} \delta^{jz}) \right], \quad (6)$$

where $\hat{Q}^i = Q^i/Q$, $m(\hat{Q}) = h v_f Q/8\Gamma^2$, and the summation of s runs over x, y, z. Except for $\epsilon^{xy} = -\epsilon^{yx} = 1$, all other ϵ-matrix elements are zero. The various functions in the above equation are defined as

$$\mathcal{D}_\nu(\vec{Q}) = 4i\Gamma/[\Omega + i\Gamma_\nu + iD_\nu Q^2]: \ \nu = t, l, \quad (7)$$

$$\mathcal{D}(\vec{Q}) = 1 - m^2(\vec{Q})\mathcal{D}_l(\vec{Q})\mathcal{D}_t(\vec{Q}), \quad (8)$$

with $\mathcal{D}_x(\vec{Q}) = \mathcal{D}_y(\vec{Q}) = \mathcal{D}_t(\vec{Q})$ and $\mathcal{D}_z(\vec{Q}) = \mathcal{D}_l(\vec{Q})$. Finally, the mixing elements vanish, $U_d^{i0} = U_d^{0i} = 0$ with $i = x, y, z$.

In the limit $Q \to 0$, $m(\vec{Q})$ approaches zero, and so the matrix U^{ij} becomes diagonal. Its components $\mathcal{D}_l$ and $\mathcal{D}_t$ describe relaxation of a homogeneous spin polarization via the D'yakonov-Perel' mechanism with the longitudinal and transverse spin relaxation rates $\Gamma_l = h^2/2\Gamma$ and $\Gamma_t = h^2/4\Gamma$, respectively. At finite Q the longitudinal and transverse spin polarization are coupled due to the finite mixing parameter $m(\vec{Q})$. In this case of inhomogeneous spin distribution, besides the D'yakonov-Perel' spin relaxation mechanism, there occurs an additional spin relaxation process. This new process is the spin diffusion with the corresponding bare diffusion coefficients $D_l = D_t = D = v_f^2/4\Gamma$, which appear in (7). The relaxation of the total electron density, including both up and down spins, is spin independent as described by the diffusion pole of U_d^{00} given by (5), in which D is the particle diffusion coefficient. Let us remind ourselves the transformation previously made in the Hamiltonian with $k_x \to k_x$ and $k_y \to -k_y$. To get proper matrix elements U^{ij} we must return to the initial basis transforming $\vec{Q}$ as $Q_x \to Q_x$ and $Q_y \to -Q_y$. This will change the sign of U^{xy}, U^{xz}, and U^{yz}.

To find the quantum corrections to the above results of classic theory, we need to calculate the contribution of the Cooperon propagator to the matrix U. The dominating contribution comes from the region of small $\vec{S} = \vec{k} + \vec{k}'$ [16]. In the region $v_f S \ll \Gamma$ of our interest, we find $(i, j = x, y, z, 0)$

$$U_c^{ij} = \frac{V^2}{\mathcal{D}(\vec{S})}[C_i(\vec{S})\delta^{ij} + m^2(\vec{S})\mathcal{D}_l(\vec{S})\mathcal{D}_t^2(\vec{S})\hat{S}^i\hat{S}^j - m(\vec{S})\mathcal{D}_l(\vec{S})\mathcal{D}_t(\vec{S})(\hat{S}^i\delta^{j0} + \hat{S}^j\delta^{i0})], \quad (9)$$

where

$$\begin{aligned} C_x(\vec{S}) &= C_y(\vec{S}) \\ &= [\mathcal{D}(\vec{S})\mathcal{D}_t(\vec{S}) + \mathcal{D}(\vec{S})\mathcal{D}_0(\vec{S}) - \mathcal{D}_t(\vec{S}) + \mathcal{D}_l(\vec{S})]/2, \\ C_z(\vec{S}) &= [\mathcal{D}(\vec{S})\mathcal{D}_t(\vec{S}) - \mathcal{D}(\vec{S})\mathcal{D}_l(\vec{S}) + \mathcal{D}_t(\vec{S}) + \mathcal{D}_0(\vec{S})]/2, \\ C_0(\vec{S}) &= [\mathcal{D}(\vec{S})\mathcal{D}_t(\vec{S}) - \mathcal{D}(\vec{S})\mathcal{D}_0(\vec{S}) + \mathcal{D}_t(\vec{S}) + \mathcal{D}_l(\vec{S})]/2. \end{aligned} \quad (10)$$

Now we can derive the localization corrections following the standard perturbation theory for systems with weak disorder [15,16]. In order to ensure the particle number conservation, besides diffusons and Cooperons, the vertex function also includes diagrams with an additional impurity line embracing a Cooperon. We then pick up all diagrams irreducible with respect to the diffuson ladder and finally obtain the diffusion propagator of the same form as (6), but with the renormalized parameters $\Gamma'_{l,t}$, $D'_{l,t}$, D', and m'. Under the conditions $\Omega \ll \Gamma_t$ and $\Gamma_t \ll \Gamma$, these parameters are given by

$$D'/D - 1 = [\ln(\Gamma/\Omega) - 3\ln(\Gamma/\Gamma_t)]/4\pi E_f\tau, \quad (11)$$

$$D'_{l,t}/D_{l,t} - 1 = \Gamma'_{l,t}/\Gamma_{l,t} - 1 = m'/m - 1 = -[\ln(\Gamma/\Omega) + \ln(\Gamma/\Gamma_t)]/4\pi E_f\tau. \quad (12)$$

Since in the main logarithmic approximation $\ln(\Gamma/\Gamma_t)$ is assumed to be much larger than $\ln(\Gamma_l/\Gamma_t) = \ln 2$, in the above equations we have neglected the terms of the order $(4\pi E_f\tau)^{-1}\ln(\Gamma_l/\Gamma_t)$, which is small compared to $\ln(\Gamma/\Gamma_t)$.

The correction factor to the particle diffusion given by (11) is the same as that obtained in Refs. [7–9], and at very low frequency becomes positive, while the correction factor in (12) is negative. As the frequency increases to the region $\Gamma_t \ll \Omega \ll \Gamma$, in the poles of diffusons given by (7), the frequency Ω becomes the largest parameter and so determines the cutoff frequency when the integration over Q is performed in the Cooperon propagator (9). As a result, on the right hand side of (11) and (12), Γ_t is replaced by Ω. In this case the corrections to all transport parameters are given by the same negative factor $-(\ln\frac{\Gamma}{\Omega})/(2\pi E_f\tau)$. Therefore, at this frequency region the transport parameters are renormalized with the same factors as those of the usual diffusion of spinless particles [16].

We have shown that due to weak localization both the longitudinal and transverse spin relaxation rates as well as spin diffusion decrease with lowering frequency. The behavior of spin diffusion is very different from that of the particle diffusion which, as the frequency decreases, first decreases and then increases with the turning point around the spin relaxation rate. The change of sign on the right hand side of (11) when Ω varies from $\Omega \ll \Gamma_t$ to $\Omega \gg \Gamma_t$ is due to the spin dependence of the quantum amplitude of a particle passing along closed trajectories. In the high frequency region $\Omega \gg \Gamma_t$, the lengths of these trajectories are of the order of $(\Omega/D)^{-1/2}$, which are not sufficiently long. Therefore, when a particle passes along one such trajectory, the probability of flipping its spin

is small. As a result, the localization correction to spin diffusion has the same form as that found previously for a spinless particle [6,15]. In the other region of $\Omega \ll \Gamma_t$, when a particle passes through a closed trajectory, the amplitude of its wave function depends strongly on its spin. This dependence leads to the spin-dependent interference of these amplitudes, and consequently to the corresponding change of the sign of localization correction to the particle diffusion, as well as to the different behaviors of the spin and the particle diffusion coefficient.

The antilocalization behavior of particle diffusion is quite similar to that discovered in systems with spin-orbit scattering (SOS) of particles from randomly distributed impurities [6], where the increase of diffusion at low frequency was also found. However, there is an essential difference between these two cases. For the SOS, the logarithmic increase of the diffusion coefficient was found in films with thickness $d \ll l$ and $d > k_f^{-1}$, but not in pure 2D system where $d \ll k_f^{-1}$. On the other hand, for III-V semiconductor quantum wells where the electron motion is two dimensional, the electron mobility increases up to inelastic dephasing times. Because of this difference, it remains uncertain whether the antilocalization behavior predicted by the perturbation theory to the first logarithmic order implies a similar universal behavior of films with SOS and of 2D systems with a spin-split conduction band. Nevertheless, this is certainly not the case in 1D mesoscopic rings with spin-split electron energy where a new type of electron level statistics was found [11].

The scaling behavior of films with the SOS is dominated by the singlet component of the Cooperon [6], while the triplet component (total spin 1) in the two-particle propagator is unimportant at large scales because this triplet channel decays due to the spin relaxation. This singlet gives rise to destructive interference between the clockwise and the counterclockwise closed path, and so results in the antilocalization. If particle diffusion is the only scaling parameter in the renormalization group analysis [17], the conductance increases with the scale continuously, which seems unphysical. One might expect the same scaling dependence in 2D systems with a spin-split conduction band. However, we found that the spin diffusion and spin relaxation rate decrease at large time, and hence at large scales. Therefore, the contribution of the triplet component of Cooperon to the renormalization of transport parameters increases with increasing scale. Such competition between the singlet and the triplet channel can show up in higher order terms of the perturbation expansion, which were not considered in the present work.

The quantum localization effects on spin transport investigated in this Letter can be observed when the D'yakonov-Perel' mechanism of spin relaxation is effective. This will be the situation if (1) the spin splitting is much less than the electronic elastic relaxation rate, (2) the dephasing time is long, and (3) other spin relaxation effects are weak compared to the D'yakonov-Perel' relaxation. We then expect these effects to show up in transport of spin oriented electrons, in EPR measurements under a weak magnetic field, in low frequency spin-flip Raman scattering, and in far infrared optics.

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