Plasma-Based Inverse Free-Electron Laser for High-Gradient Acceleration of Electrons

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It is shown that the acceleration rate and the maximum energy transfer between the pump wave and the electron beam in the plasma beat-wave acceleration can be considerably increased by placing the plasma in the wiggler cavity of an inverse free-electron laser. [S0031-9007(96)00129-9]

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Plasma-based devices for the high-gradient acceleration of electron beams are currently the subject of intense investigations. In particular, the plasma beat-wave acceleration (PBWA), in which the acceleration is provided by a strong electrostatic (ES) wave generated in a cold plasma by the nonlinear resonant beating of two lasers, has been studied in the past years in considerable detail and also tested experimentally [1–4]. Two important quantities in these experiments are the acceleration rate, i.e., the increase in the average energy of the beam in MeV per meter, and the overall maximum increase of the beam energy that the accelerating process produces before saturation.

The basic aim of this Letter is to show that these two quantities can be enhanced considerably by placing the plasma in the wiggler cavity of a free-electron laser (FEL), i.e., by loading with a plasma the wiggler cavity of an inverse FEL, under the basic condition $\eta = n_b/n_p \ll 1$, where n_b and n_p are the average numbers per unit volume of the electrons of the beam and of the plasma, respectively. The same scheme, a plasma-based inverse FEL, was proposed by Bobin [5], but in essentially the opposite situation, namely, $n_b/n_p \gg 1$.

We may consider that the weak electron beam produces only small disturbances in the plasma in equilibrium with the large, static, and spatially periodic magnetic field $\mathbf{B}_w = \nabla \times \mathbf{A}_w$ of the wiggler. We further assume that the equilibrium plasma carries transverse diamagnetic currents that produce, in turn, a diamagnetic field $\mathbf{B}_D = \nabla \times \mathbf{A}_D$, partially opposing that of the wiggler. In the one-dimensional description, if $\mathbf{A}_w(z) = (a_w/\sqrt{2})(\hat{\mathbf{e}}e^{-ik_w z} + \text{c.c.})$, with $\hat{\mathbf{e}} = (1/\sqrt{2})(\mathbf{e}_x + i\mathbf{e}_y)$, is the vector potential of the magnetic field of a helical wiggler and if the usual relation $\mathbf{p}_\perp(z,t) = m\gamma_p \mathbf{u}_\perp(z,t) = (e/c)\mathbf{A}(z,t)$ between the average transverse momentum of the electrons of the plasma and the potential $\mathbf{A}$ is satisfied, we find that the total vector potential in equilibrium is $\mathbf{A}_D + \mathbf{A}_w = \boldsymbol{\gamma}_p \mathbf{A}_w/(\boldsymbol{\gamma}_p + \boldsymbol{\omega}_p^2/c^2k_w^2)$, where $\boldsymbol{\omega}_p = (4\boldsymbol{\pi}e^2n_p/m)^{1/2}$ is the plasma frequency of the electrons of the plasma, $\boldsymbol{\gamma}_p = (1-\boldsymbol{\beta}_p^2)^{-1/2} = (1-u_\perp^2/c^2)^{-1/2}$ is the Lorentz factor of the undisturbed plasma, $\boldsymbol{\beta}_p = a_{w0}/(\boldsymbol{\gamma}_p + \boldsymbol{\omega}_p^2/c^2k_w^2)$, and $a_{w0} = ea_w/mc^2$ is the wiggler parameter.

The large wiggler magnetic field modifies the dispersive properties of the plasma. If we write the normal modes for both transverse vector potential and plasma number density, respectively, as $\delta\mathbf{A} \propto e^{i[(k-k_w)z-\omega(k)t]}$ and $\delta n_p \propto e^{i[kz-\omega(k)t]}$, it can be shown that the dispersion relation of the plasma in the strong wiggler magnetic field is given by the following equation which is of the third degree in the quantity $\theta = \omega^2(k) - \omega_p^2/\gamma_p$:

$$\theta^3 - 2\theta^2[c^2(k^2+k_w^2) - \Gamma] + \theta[c^4(k^2-k_w^2)^2 - 4\Gamma c^2k^2 - 2\Gamma c^2k_w^2] + 2\Gamma c^4k^2(k^2+k_w^2) = 0, \quad (1)$$

where $\Gamma = \omega_p^2\beta_p^2/2\gamma_p$. This equation reduces to the usual dispersion of the unmagnetized plasma, in the limit in which both $\boldsymbol{\beta}_p$ (or a_{w0}) and k_w go to zero. Figure 1 shows the three roots $\boldsymbol{\omega}$ vs k of the dispersion relation (1), as compared with the two branches of the unmagnetized case, for a plasma density $n_p = 10^{14}$ cm^{-3} and a wiggler with wavelength $\lambda_w = 2$ cm and a magnetic field peak value of 17.5 T (the wiggler parameter $a_{w0} \approx 35$).

The waves that obey Eq. (1) have a mixed character, in the sense that both axial electric field δE_z (or, equivalently, the plasma number density δn_p) and transverse radiation field $\delta\mathbf{E}_\perp = -(1/c)\partial\delta\mathbf{A}/\partial t$ participate to the

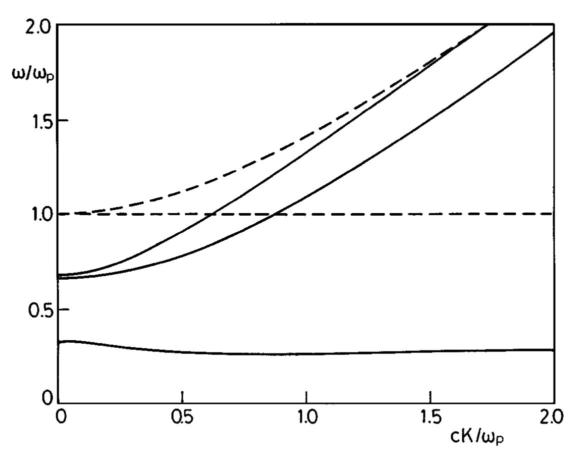

FIG. 1. The three branches of dispersion (1) (solid lines), for $n_p = 10^{14}$ cm^{-3}, $\lambda_w = 2$ cm, $B_w = 17.5$ T, $a_{w0} = 35$. Dashed lines are the usual purely transverse (upper dashed line) and longitudinal (Langmuir) waves in the nonmagnetic case.

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oscillatory motion. If we write

$$\frac{\delta n_p}{n_p} = M_L(t)e^{i[kz-\omega(k)t]} + \text{c.c.} \tag{2}$$

for the longitudinal disturbance, the associated transverse vector potential is given by

$$\frac{e}{mc^2}\,\delta A = M_L(t)Q(k)e^{i[(k-k_w)z-\omega(k)t]} + M_L^\star(t)Q(-k)e^{-i[(k+k_w)z-\omega(k)t]},$$

where the vector $\delta\mathbf{A}$ has been written in the undulator reference system as $\delta\mathbf{A} = \hat{\mathbf{e}}\delta A + \text{c.c.}$, and the polarization ratio $Q(k)$ is

$$Q(k) = \frac{\gamma_p\theta(k)}{\sqrt{2}\beta_p c^2 k^2}\,\frac{\theta(k) - c^2(k + k_w)^2}{\theta(k) - c^2(k^2 + k_w^2)}.$$

There are two circularly polarized transverse waves with the same frequency $\omega(k)$ and wave numbers $k - k_w$ and $k + k_w$.

Since we are analyzing the problem within the framework of temporal rather than spatial problems, we assume that an unbunched beam of electrons and an intense wave with frequency and wavelength satisfying the dispersion relation (1) are present, at $t = 0$, inside the plasma in equilibrium with the wiggler field. For $t > 0$, the beam starts interacting with the longitudinal and transverse components of the strong wave, but on a time scale which is slower than that of the carrier frequency in (2). We admit that the intense accelerating wave inside the plasma does not give rise to any appreciable decay into other unwanted waves and disregard, therefore, all bilinear terms in the fluid equations of the plasma. We assume further that the whole acceleration process takes place over a time scale so short that all nonlinear ponderomotive effects can also be ignored.

Using fully relativistic equations of motion for both beam and plasma electrons, the sheet model for the electron beam [6,7] and the slowly varying envelope approximation, we find that the amplitude $M_L(t)$ in (2) varies according to the following equations:

$$\begin{aligned}
\frac{d}{d\tau}M_L(\tau) &= -\,iS_1\frac{1}{N}\sum_{j=1}^{N}e^{-i\theta_j} - iS_2\frac{1}{N}\sum_{j=1}^{N}\frac{1}{\gamma_j}e^{-1\theta_j},\\
\frac{d}{d\tau}\theta_j(\tau) &= \frac{p_j}{\sqrt{\gamma_p^2 + p_j^2}} - \beta_r,\\
\frac{d}{d\tau}p_j(\tau) &= -\,iS_3(\langle b\rangle e^{i\theta_j} - \text{c.c.}) - i\left(S_4 + \frac{S_5}{\gamma_j}\right)(M_Le^{i\theta_j} - \text{c.c.}).
\end{aligned} \tag{3}$$

In these equations, the index j ranges from 1 to N, the total number of electrons of the beam in the wavelength $\lambda = 2\pi/k$ of the longitudinal wave, $\theta_j(t) = kz_j(t) - \omega(k)t$ is the angle of the phase of these same electrons in the field of the wave, $\beta_r = \omega(k)/ck$ is the relativistic factor of the wave, and $\tau = ckt$. Furthermore, $p_j = \beta_j\gamma_j$, where $\gamma_j = (\gamma_p^2 + p_j^2)^{-1/2}$ is the dominant part of the Lorentz factor of the beam electrons and $\beta_j = (1/c)dz_j(t)/dt$, while $\langle b\rangle = N^{-1}\sum_{j=1}^N e^{-i\theta_j}$ is the usual beam bunching factor. Finally, $S_1 = 2\Omega_p^2\Omega_b^2\beta_p/\gamma_pM_0$, $S_2 = 2\Omega_b^2\beta_p\gamma_p(\beta_r^2 - \Omega_p^2/\gamma_p)/M_0$, $S_3 = \frac{1}{2}\Omega_b^2$, $S_4 = \Omega_p^2$, $S_5 = \gamma_p^2(\beta_r^2 - \Omega_p^2/\gamma_p)$, where $\Omega_p = \omega_p/ck$, $\Omega_b = \omega_b/ck$, $\omega_b^2 = 4\pi e^2n_b/m$, and

$$M_0 = \frac{(8\beta_r\gamma_p/\beta_p)(\beta_r^2 - \Omega_p^2/\gamma_p)[\beta_r^2 - (\Omega_p^2/\gamma_p)(1 - \beta_p^2/2) - 1 - k_w^2/k^2] - 4\beta_r\beta_p\Omega_p^2(1 + k_w^2/k^2)}{\beta_r^2 - \Omega_p^2/\gamma_p - 1 - k_w^2/k^2}.$$

Equations (3) have the following constant of the motion:

$$\Omega_b^2\langle p\rangle + \frac{\gamma_pM_0}{2\beta_p}|M_L|^2 = \text{const},$$

where $\langle p\rangle = N^{-1}\sum_{j=1}^N p_j$, which states that the physical system acts as an accelerator when the energy of the wave is transferred to the beam, or as an amplifier when the beam kinetic energy is transferred to the wave (plasma-based FEL). At the same time, it shows that there is a theoretical maximum energy transfer in the acceleration process, i.e., that the energy of the beam saturates at a value not greater than $\gamma_pM_0|M_L(\tau = 0)|^2/2\Omega_b^2\beta_p$.

An important feature of the acceleration process described by Eqs. (3) is its resonant character around the value $\beta_r = 1$. This means that the maximum energy transfer from the accelerating wave to the beam is to be expected when the phase velocity $\omega(k)/k$ of the wave is equal to the velocity of light c. This fact can easily be uncovered by assuming that the acceleration imparted to the electrons of the beam is already so effective that one can safely write $\beta_j \approx 1$ and consequently disregard all terms proportional to $1/\gamma_j$. The second equation in (3) can then be integrated to give $\theta_j = (1 - \beta_r)\tau$, apart from constant values. Taking the average of the third equation over all electrons of the beam then leads to the conclusion that

$$\langle p\rangle \propto S_4\left[\text{const} \times \frac{e^{i(1-\beta_r)\tau}}{1 - \beta_r} + \text{c.c.}\right],$$

which shows that $\langle p\rangle$ grows secularly in time when $\beta_r = 1$.

Figure 2 gives the results obtained through direct numerical integration of Eqs. (3) and shows how the acceleration process improves by increasing the magnetic field of the wiggler. It gives the time behavior of the average value $\langle\gamma\rangle$ of the electrons of the beam and refers to $\eta = n_b/n_p = 10^{-3}$, a plasma density of $n_p = 10^{14}$ cm^{-3}, and a wiggler wavelength $\lambda_w \approx 2$ cm. The initial value of

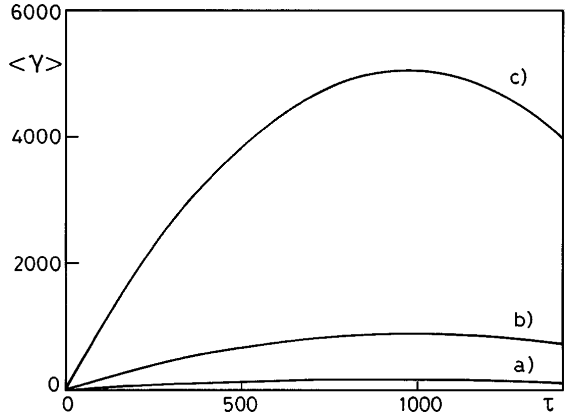

FIG. 2. Time behavior of $\langle\gamma\rangle$ in the case $n_p = 10^{14}$ cm^{-3}, $\eta = n_b/n_p = 10^{-3}$, $\lambda_w = 2$ cm, $\langle\gamma\rangle(\tau = 0) = 4$, $|M_L(\tau = 0)| = 0.3$, $N = 200$, for various values of the wiggler parameter a_{w0}. Curve (a) refers to the nonmagnetic case $a_{w0} = 0$, pump wave wavelength $\lambda_p = 0.33$ cm, and angular frequency $\omega = 5.6 \times 10^{11}$ sec^{-1}; curve (c) $a_{w0} = 38$, $\beta_p = 0.96$, $B_w = 19$ T, $\lambda_p = 1.78$ cm, $\omega = 10^{11}$ sec^{-1}.

$\langle\gamma\rangle$ is 4, corresponding to a 1.5 MeV beam and the initial value of $|M_L|$ is 0.3, corresponding to a peak value of the longitudinal component of the accelerating wave $(\delta n_p/n_p)_{\rm peak} = 0.6$.

Curve (a) with the undulator parameter $a_{w0} = 0$, refers to the acceleration process taking place in a nonmagnetized plasma. In this case, the angular frequency of the accelerating wave is equal to the plasma frequency $\omega_p = 5.64 \times 10^{11}$ sec^{-1}. If we assume as reasonable that there is a relation like $\Delta z = c\Delta t = \Delta\tau/k$ between space and time variables and that the accelerating waves are all exactly resonant, i.e., in all cases, $\omega(k)/k = c$, curve (a) shows that, in the nonmagnetic case, the process saturates at $\Delta z \approx 0.5$ m with an increase in $\langle\gamma\rangle$ of roughly 90 MeV leading to an average acceleration rate of 180 MeV/m. Curve (b) refers to the value $a_{w0} \approx 30$ and to an undulator peak value magnetic field of roughly 15 T ($\beta_p = 0.8$). The process saturates, in this case, in approximately 1 m and leads to a final value of $\langle\gamma\rangle$ of about 900 and to an average acceleration rate of 430 MeV/m. In the extreme case represented by curve (c) in which $a_{w0} \approx 38$ and the undulator magnetic field is of approximately 19 T ($\beta_p = 0.96$), the increase in $\langle\gamma\rangle$ saturates in a distance of 2.5 m and the average acceleration rate is of 980 MeV/m. The acceleration rates are somewhat higher over the first meter of the acceleration length and are of 0.44 and 1.5 GeV/m for curves (b) and (c), respectively. Figure 3 represents the beam electron population in the phase plane $\{\theta_j, \gamma_j\}$ in the case (c) of Fig. 2, at time $\tau = 900$, when $\langle\gamma\rangle$ is near the saturated value of abut 5000. It shows that while a number of electrons are not accelerated and indeed decelerated ($\gamma_j \cong 1$), a considerable fraction of electrons reaches energy values from 3 to 4 GeV. For this case, with the beam initially unbunched ($\langle b\rangle \approx 0$ at $\tau = 0$), the energy spread $\sqrt{|\langle\gamma^2\rangle - \langle\gamma\rangle^2|}/\langle\gamma\rangle$ is rather large, about 60%, for all values of τ. Figure 4 gives the time behavior of both longitudinal and transverse pump wave amplitudes. It shows clearly that there is no dephasing of the electrons of the beam and that the acceleration process saturates solely because of pump depletion.

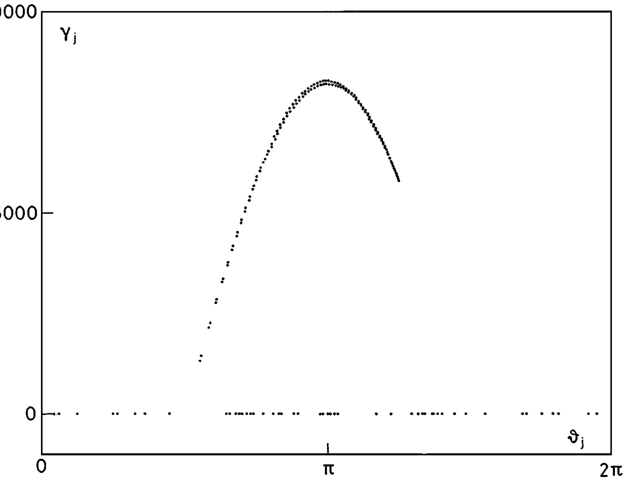

FIG. 3. Phase-plane γ_j vs θ_j representation for the case (c) of Fig. 2 at time $\tau = 900$.

The density of the electron beam submitted to the acceleration is severely limited by the minimum possible wavelengths and/or maximum magnetic field strengths in conventional undulators. The preceding example referred to a beam with $n_b = 10^{11}$ electrons/cm^3. Much higher values of the beam density, e.g., $n_b \approx 10^{15}$ cm^{-3}, would require correspondingly higher values of n_p, i.e., $n_p \approx 10^{18}$ cm^{-3}, very short magnetic field wavelengths, of the order of 0.1 cm or less and, above all, very high values, $B_w \approx 10^6$ G or more, of the strength of the magnetic field structure inside the plasma.

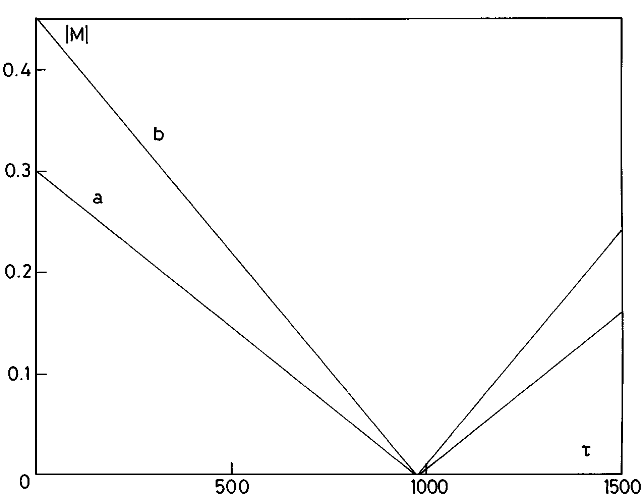

FIG. 4. Time behavior of longitudinal (a) and transverse (b) pump wave amplitudes showing nearly complete depletion at $\tau \approx 1000$.

In conclusion, we have shown that a static, rippled magnetic field structure perpendicular to the beam is able to increase considerably the acceleration rate in experiments of the PBWA type, along the line suggested in Ref. [8], where "surfing" of the electrons of the beam over and along the crests of the Langmuir wave has been proposed as a means of improving the quality of the acceleration process. All two- and three-dimensional phenomena, like transverse instabilities of the electron beam, beam erosion, as well as radiative losses due to the more complex transverse motions of the electrons of the beam, are obviously absent in the present, one-dimensional treatment. A proof-of-principle experiment could be done with rather low density electron beams, very limited plasma lengths, pump waves in the microwave region, and using conventional undulators. The pump wave is partially a transverse wave and, therefore, it could even be excited in the wiggler cavity by direct outside injection. For instance, with $n_p = 10^{12}$ cm^{-3}, $\lambda_w = 12$ cm, $B_w = 1.3$ T, and an input frequency $\omega = 1.3 \times 10^{10}$ sec^{-1}, a beam with density $n_b = 10^9$ cm^{-3} and initial energy 1.5 MeV can be accelerated in 1 m, up to the value $\langle\gamma\rangle \approx 270$, with an acceleration rate of about 140 MeV/m. Without magnetic field, the same experiment would give an acceleration rate of only 37 MeV/m.

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