Simple Supersymmetric Solution to the Strong *CP* Problem

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It is shown that the minimal supersymmetric left-right model can provide a natural solution to the strong *CP* problem without the need for an axion, nor any additional symmetries beyond supersymmetry and parity.

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Quantum chromodynamics, which is extremely sucessful in describing strongly interacting phenomena in both the low as well as the high energy domain, has the well-known problem that it can lead to an uncontrolled amount of *CP* violation in the flavor conserving hadronic processes. This is the strong *CP* problem [1]. The parameter $\overline{\Theta}$ which characterizes the strength of these *CP*-violating interactions is constrained by present upper limits on the electric dipole moment of the neutron to be less than 10^{-9}–10^{-10}. The presence of such a small number in a theory indicates the existence of new symmetries beyond the standard model of electroweak and strong interactions. Three classes of spontaneously broken symmetries have, in the past, been advocated as solutions to the strong *CP* problem: (i) Peccei-Quinn (PQ) U(1) symmetry [2], (ii) parity (or left-right) symmetry of weak interactions [3], and (iii) softly broken *CP* symmetry [4]. There also exist other solutions which use less transparent symmetries to constrain the form of quark mass matrices into interesting forms thereby suppressing $\overline{\Theta}$ to the desired level [5]. In the absence of any experimental evidence for or against any of these solutions, one can look for theoretical criteria to reduce the number of such possibilities. One criterion discussed in recent years is to use the lore that, unlike local symmetries, all global symmetries are broken by a nonperturbative gravitational effect, such as black holes and wormholes. Since all our solutions involve new global symmetries, one must investigate whether in the presence of these effects the solution to the strong *CP* problem remains viable. In Ref. [6] it was shown that the presently invisible axion models [7] are incompatible with the above nonperturbative effects essentially due to the fact that the PQ symmetry breaking scale in this case must be $\approx 10^{10}$–10^{12} GeV. On the other hand, it was shown in Ref. [8] that as long as the scales of *P* or *CP* violation are less than some intermediate scale, the nonperturbative Planck scale effects do not destabilize the second and third solutions to the $\overline{\Theta}$ problem. In this Letter, we will show that in a class of minimal supersymmetric models recently discussed [9,10] in order to have automatic *R*-parity conservation prior to symmetry breaking, the strong *CP* parameter $\overline{\Theta}$ naturally vanishes at both the tree and the one-loop level, thus providing a solution to the strong *CP* problem. No additional symmetries are need for the purpose. The only difference between earlier supersymmetric (SUSY) left-right models and ours is the inclusion of dimension 4 Planck scale induced terms, which are in general expected to be present [6]. This provides a way to ensure that *R* parity remains an exact symmetry in the theory even after the gauge symmetry is spontaneously broken. This in combination with the constraints of parity invariance on the coupling parameters of the theory lead us to our result that the model provides a solution to the strong *CP* problem without the need for an axion. Since the Yukawa couplings in the model are complex, the observed weak *CP* violation in the kaon system is explained via the usual Cabibbo-Kobayashi-Maskawa phase in the left-handed *W* coupling (as in the standard model). Some additional interesting properties of the model are (i) including Planck scale effects leaves the solution unscathed, as in Ref. [8]; (ii) unlike the minimal supersymmetric model (MSSM) and the model of Ref. [10], *R* parity is naturally conserved to all orders in $1/M_{\mathrm{Pl}}$, so that the lightest supersymmetric particle (LSP) remains absolutely stable in this model and plays the role of cold dark matter (CDM), and (iii) the SUSY contributions to the electric dipole moment of the neutron are automatically suppressed, thereby curing the so-called SUSY *CP* problem.

To see how parity symmetry really suppresses the $\overline{\Theta}$, let us start by noting that in an electroweak theory there are two contributions to $\overline{\Theta}$ at the tree level: $\overline{\Theta} = \Theta + \operatorname{Arg}\det(M_u M_d)$, where Θ is the coefficient of the $G\tilde{G}$ term in the QCD Lagrangian induced by instanton effects, and the second term is self-explanatory with M_u and M_d denoting the up and down quak mass matrices. Since $G\tilde{G}$ is odd under parity, if the theory is required to be parity invariant, we must have $\Theta = 0$. The vanishing of the second term is, however, more tricky. In the nonsupersymmetric left-right models based on the gauge group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ [11], the quark masses arise from the following gauge invariant Lagrangian:

$$\mathcal{L}_Y = \mathbf{h}^i_{ab} \overline{Q}_{L,a} \Phi_i Q_{R,b} + \mathrm{H.c.}, \tag{1}$$

where $Q_a = (u_a, d_a)$ ($a = 1, 2, 3$ for three generations) and Φ_i are bidoublets (2,2,0). In the minimal nonsupersymmetric model, one usually considers one Φ so that

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there exists another bidoublet $\tilde{\Phi} \equiv \tau_2 \Phi^* \tau_2$ leading to two Yukawa matrices $\mathbf{h}^{(1)}$ and $\mathbf{h}^{(2)}$. Under left-right (P) symmetry, one assumes that $Q_{L,a} \leftrightarrow Q_{R,a}$ adn $\Phi_i \leftrightarrow \Phi_i^\dagger$. It is then easy to show that parity invariance demands that $\mathbf{h}^{(i)} = \mathbf{h}^{(i)\dagger}$. Now, if the ground state had the property that $\langle \Phi_i \rangle$ is real (i.e., the ground state is *CP* conserving) then one would have Hermitian mass matrices implying that the second term in $\overline{\Theta}$ above is zero. One would then have obtained $\overline{\Theta}_{\text{tree}} = 0$. Unfortunately, without extra symmetries, the most general Higgs potential in a nonsupersymmetric left-right model has complex couplings, and therefore the vacuum state is necessarily *CP* violating. As an example consider the Higgs system Φ, (Δ_L, Δ_R) [12], where Δ_L and Δ_R are left and right SU(2) triplets, respectively, with $B - L = 2$. In this model, all but one scalar coupling in the Higgs potential are real, but the complex one corresponds to $|\lambda| \det \Phi (e^{i\alpha} \Delta_L^\dagger \Delta_L + e^{-i\alpha} \Delta_R^\dagger \Delta_R)$ + H.c. which induces a complex vacuum expectation value (VEV). Note now that in the presence of complex VEVs $\langle \Phi \rangle$, the mass matrix is not Hermitian and at the tree level $\overline{\Theta} \neq 0$ despite the theory being parity invariant. One therefore needs new symmetries that forbid the above term [3].

The supersymmetric model.—As already mentioned, the gauge group of the theory is $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with quarks and leptons transforming as doublets under $SU(2)_{L,R}$ depending on their chirality as follows: Q $(2, 1, +\frac{1}{3})$; $Q^c (1, 2, -\frac{1}{3})$; L $(2, 1, -1)$; and L^c $(1, 2, +1)$. The Higgs fields and their transformation properties are $\Phi_{1,2}(2, 2, 0)$; $\Delta(3, 1, +2)$; $\overline{\Delta}(3, 1, -2)$; $\Delta^c(1, 3, -2)$; and $\overline{\Delta}^c(1, 3, +2)$. The superpotential for this theory is given by (we have suppressed the generation index)

$$W = \mathbf{h}_q^{(i)} Q^T \tau_2 \Phi_i \tau_2 Q^c + \mathbf{h}_l^{(i)} L^T \tau_2 \Phi_i \tau_2 L^c + i(\mathbf{f} L^T \tau_2 \Delta L + \mathbf{f}_c L^{cT} \tau_2 \Delta^c L^c) + \mu_\Delta \mathrm{Tr}(\Delta \overline{\Delta}) + \mu_{\Delta^c} \mathrm{Tr}(\Delta^c \overline{\Delta}^c) + \mu_{ij} \mathrm{Tr}(\tau_2 \Phi_i^T \tau_2 \Phi_j) + W_{\text{NR}}\,, \tag{2}$$

where W_{NR} denotes nonrenormalizable terms arising from Planck scale physics. Typically, $W_{\text{NR}} = (\lambda/M)[\mathrm{Tr}(\Delta^c \tau_m \overline{\Delta}^c)]^2$ + other terms. Being a Planck scale effect, it can violate parity symmetry, and we assume it does. At this stage all couplings $\mathbf{h}_{q,l}^{(i)}, \mu_{ij}, \mu_\Delta, \mu_{\Delta^c}, \mathbf{f}, \mathbf{f}_c$ are complex with μ_{ij}, $\mathbf{f}$, and $\mathbf{f}_c$ being symmetric matrices. The terms that break supersymmetry softly to make the theory realistic can be written as

$$\mathcal{L}_{\text{soft}} = \int d^4\theta \sum_i m_i^2 \phi_i^\dagger \phi_i + \int d^2\theta\, \theta^2 \sum_i A_i W_i + \int d^2\overline{\theta}\, \overline{\theta}^2 \sum_i A_i^* W_i^\dagger + \int d^2\theta\, \theta^2 \sum_p m_{\lambda_p} \tilde{W}_p \tilde{W}_p + \int d^2\overline{\theta}\, \overline{\theta}^2 \sum_p m_{\lambda_p}^* \tilde{W}_p^* \tilde{W}_p^*\,. \tag{3}$$

In Eq. (3), $\tilde{W}_p$ denotes the gauge-covariant chiral superfield that contains the $F_{\mu\nu}$-type terms with the subscript going over the gauge groups of the theory including $SU(3)_c$. W_i denotes the various terms in the superpotential, with all superfields replaced by their scalar components and with coupling matrices which are not identical to those in W. Equation (3) gives the most general set of soft breaking terms for this model.

To see how $\overline{\Theta} = 0$ in the model, let us choose the following definition of left-right transformations on the fields and the supersymmetric variable θ: $Q \leftrightarrow Q^{c\dagger}$; $L \leftrightarrow L^{c\dagger}$; $\Phi_i \leftrightarrow \Phi_i^\dagger$; $\Delta \leftrightarrow \Delta^{c\dagger}$; $\overline{\Delta} \leftrightarrow \overline{\Delta}^{c\dagger}$; $\theta \leftrightarrow \overline{\theta}$; $\tilde{W}_{SU(2)_L} \leftrightarrow \tilde{W}^*_{SU(2)_R}$; and $\tilde{W}_{B-L, SU(3)_C} \leftrightarrow \tilde{W}^*_{B-L, SU(3)_C}$. With this definition of L-R symmetry, it is easy to check that $\mathbf{h}_{q,l}^{(i)} = \mathbf{h}_{q,l}^{(i)\dagger}$; $\mu_{ij} = \mu_{ij}^*$; $\mu_\Delta = \mu_\Delta^*$; $\mathbf{f} = \mathbf{f}_c^*$; $m_{\lambda_{SU(2)_L}} = m^*_{\lambda_{SU(2)_R}}$; and $m_{\lambda_{B-L, SU(3)_C}} = m^*_{\lambda_{B-L, SU(3)_C}}$. From these constraints, we see that Yukawa couplings still remain complex, whereas all couplings involving only bidoublet Higgs fields are real. This is the first step in our proof that $\overline{\Theta} = 0$.

Now we are ready to look for minima of the Higgs potential to see whether $\langle \Phi_i \rangle$ have phases or not. In discussing this, we must first recall the relevant result of Ref. [10] which showed that in order for the ground state to respect electromagnetic gauge invariance, one must break R parity, i.e., $\langle \tilde{\nu}^c \rangle \neq 0$ for at least one generation. This is not desirable for our purpose since the $\langle \tilde{\nu}^c \rangle$ VEV will always induce the VEV of $\langle \tilde{\nu} \rangle$ via the leptonic Yukawa couplings. Because of these sneutrino VEVs the minimum equations generate a small phase in the bidoublet VEVs, which will upset the Hermiticity of the quark mass matrices leading to nonzero $\overline{\Theta}$. Thus in order to solve the strong *CP* problem we need to work with the minimum where $\langle \tilde{\nu}^c \rangle = 0$. So how does one evade the theorem of Ref. [10]? Let us recall that the result of Ref. [10] is valid for the most general renormalizable superpotential of the model. However, if one assumes that nonperturbative Planck scale effects can induce operators with dimension 4 or higher, the result of Ref. [10] is easily avoided leading to the charge conserving minimum with $\langle \tilde{\nu}^c \rangle = 0$. The simplest operator that is helpful is $(\lambda/M_{\text{Pl}})[\mathrm{Tr}(\Delta^c \tau_m \overline{\Delta}^c)]^2$. The main point is that in the absence of the dimension 4 terms in the superpotential, the global minimum of the theory not only conserves parity but also violates electric charge conservation as soon as it breaks the gauge symmetry (i.e.,

$\langle\Delta^c\rangle \neq 0$) and is given by $\langle\Delta\rangle = \langle\Delta^c\rangle = (1/\sqrt{2})v\tau_1$ and similarly for $\langle\overline{\Delta}\rangle = \langle\overline{\Delta}^c\rangle = (1/\sqrt{2})v'\tau_1$. This happens because the D term vanishes for this charge violating minimum [10], whereas it is nonzero for the charge conserving one for which $\langle\Delta^c\rangle = v(\tau_1 - i\tau_2)/2$ and $\langle\overline{\Delta}^c\rangle = v'(\tau_1 + i\tau_2)/2$. As soon as the Planck scale terms are included, it lifts the charge violating minimum higher than the charge conserving one for a large range of parameters. In typical singlet hidden sector Polonyi type models, we estimate $v^2 - v'^2 \approx f^2 M_{\rm SUSY}^2/16\pi^2$ so that the charge conserving minimum occurs for $f \leq 4\pi(4\lambda\mu_\Delta v^4/M_{\rm Pl}M_{\rm SUSY}^4)^{1/4}$. Here f is one of the leptonic Yukawa couplings defined in Eq. (2). For $\lambda \approx 1, \mu_\Delta \approx v \approx M_{\rm SUSY} \approx 1$ TeV, we get $f \leq 10^{-3}$. The parity asymmetric nature of this operator is also crucial for obtaining a parity violating minimum. We also note that λ can be chosen complex, and yet the phase it induces in the VEVs being of order $v^2/M_{\rm Pl}^2$ is negligible.

Having chosen the VEV with $\langle\tilde{\nu}^c\rangle = 0$, let us now see whether the VEVs of the Φ field are real as is needed to solve the strong CP problem. We have carried out a detailed analysis of the Higgs potential and find that, at the minimum of the potential, it is indeed true. It is clear that the two-bidoublet SUSY left-right model being discussed is a special case of the four Higgs extension of the minimal supersymmetric standard model as far as the doublet Higgs sector is concerned. The question of spontaneous CP violation in the latter case has been recently studied in Ref. [13], where it is shown that if a general supersymmetric model with two pairs of Higgs doublets has no complex parameters in the doublet Higgs sector, it cannot break CP spontaneously for any range of values of the parameters of the Higgs potential. Since our model in the Higgs sector is a special case of this, it follows that the VEVs of Φ_i must be real. This then implies that the quark mass matrices are Hermitian and therefore $\overline{\Theta} = 0$ naturally at the tree level in our model.

Let us now turn to the one-loop contribution to the quark mass matrices to see if they make any contributions to $\overline{\Theta}$. Because if the quark mass matrices lose their Hermiticity at the one loop they will induce too large a value for $\overline{\Theta}$. There are both Higgs and gaugino mediated diagrams (Figs. 1 and 2, respectively). The Higgs mediated graph contributes as follows:

$$\delta M_q^H = [A_{ij}\mathbf{h}^{(i)}M_q^{(0)}\mathbf{h}^{(j)}]\,. \tag{4}$$

Here $M_q^{(0)}$ denotes the tree level contribution. Because of the symmetry property $\mu_{12} = \mu_{21}$ and the reality of μ_{ij}, it follows that δM_q^H is Hermitian. As far as the gauge mediated contribution is concerned, $\delta M_q^G \propto M_q^{(0)}$. Turning to gaugino contributions, since m_λ for the $\mathrm{SU}(2)_{L,R}$ can be complex, a careful analysis is needed to see what their contribution to $\overline{\Theta}$ is. We find these contributions come always in pairs for both left and right gauginos, and because of the constraint $m_{\lambda_{\mathrm{SU}(2)_L}} = m^*_{\lambda_{\mathrm{SU}(2)_R}}$ derived earlier, their complex parts cancel out when the diagrams are summed up. Two typical graphs are shown in Fig. 2. Therefore the gauge mediated contribution is also automatically Hermitian. Thus, the total one-loop contribution to $\overline{\Theta}$ vanishes.

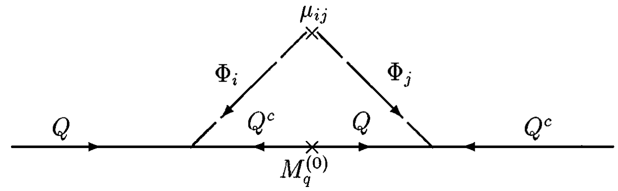

FIG. 1. Higgs contibution to one-loop calculation of $\overline{\Theta}$.

From the above discussion, we conclude that the lowest order contribution to $\overline{\Theta}$, if any, can arise only at the two-loop level. Its contribution to $\overline{\Theta}$ can be crudely estimated to be

$$\overline{\Theta} \simeq \left(\frac{m_t m_b}{V_{WK}^2}\right)\frac{1}{(16\pi^2)^2}\left(\frac{\mu_{ij}^2}{M^2}\right)I\,. \tag{5}$$

For $\mu_{ij} \simeq 10^{-1}M$, this "primitive" estimate gives $\overline{\Theta} \simeq 4 \times 10^{-9}I$, where I denotes the value of the two-loop integral. A more careful estimate will also bring in small mixing angles, which will further suppress $\overline{\Theta}$.

An interesting point to note is that since in our model the $B - L$ gaugino and gluino mass terms are CP conserving, the problem of a large neutron electric dipole moment does not exist, and one has a simple resolution of the SUSY CP problem encountered in the MSSM.

In summary, we have shown that minimal models that combine supersymmetry and parity invariance provide a simple solution to the strong CP problem without the need to invoke any additional symmetries. The key elements in our proof are (i) the transformation of supersymmetry coordinate $\theta \leftrightarrow \overline{\theta}$ under parity, and (ii) the inclusion of the nonperturbative Planck scale suppressed operators in the superpotential. The latter ensures that the ground state

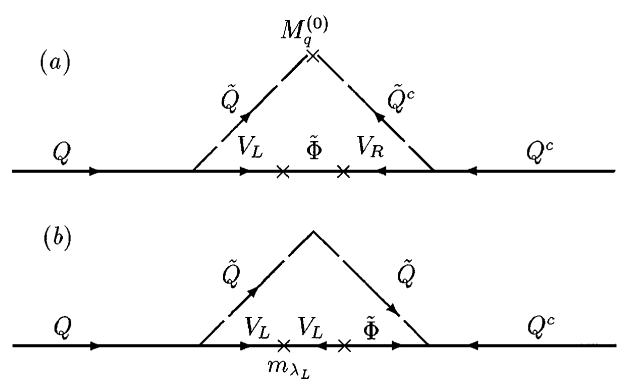

FIG. 2. Examples of gaugino contributions to one-loop calculation of $\overline{\Theta}$. $V_{L,R}$ are left and right gauginos, respectively. The gaugino mass m_{λ_L} is in general complex. There is an analogous graph to (b) that involves right-handed gauginos.

of the theory conserves R parity, which in turn leads to real vacuum expectation values of the bidoublet fields for arbitrary values of the parameters in the theory. This together with the Hermiticity of the Yukawa couplings generic to left-right models leads to our solution to the strong CP problem.

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Note added.—After this work was completed, we came across a paper by R. Kuchimanchi [14] which also arrives at the same result, under the assumption that all gaugino masses are the same at the Planck scale.

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