# Solution to the Strong *CP* Problem: Supersymmetry with Parity

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We find that supersymmetry with parity can solve the strong *CP* problem in many cases including the interesting cases of having the minimal supersymmetric standard model or some of its extensions below the Planck, grand unified theory, and intermediate scales, as well as for the case where we have a low-energy supersymmetric left-right model. Predictions emerge for some of the *CP* violating phases in these supersymmetric models.

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The smallest dimensionless parameter in the standard model is the strong $CP$ phase, $\overline{\theta} = \theta + \operatorname{Arg}\operatorname{Det}(\overline{M})$ where $\theta/32\pi^2$ is the coefficient of the $F\tilde{F}$ term in the QCD Lagrangian and $\overline{M}$ is the quark mass matrix. Experimental bounds on the electric dipole moment of the neutron imply that $\overline{\theta} \leq 10^{-9}$. There is no symmetry by which $\overline{\theta}$ can be made naturally small (or zero) at the level of the standard model, and this has been called the strong $CP$ problem.

Two elegant solutions have been proposed. If the up quark were massless or if there were a $U(1)$ Peccei-Quinn (PQ) symmetry [1], then $\overline{\theta}$ can be rotated away. However, the up quark seems to be massive, and the PQ symmetry leads to an axion which is severely constrained by experiments. Other solutions like spontaneous $CP$ violation or the Nelson-Barr mechanism [2] require heavy quarks. Solutions based on spontaneous $P$ violation have so far required mirror families [3] or $CP$ symmetry as well [4]. While none of the existing solutions is completely ruled out, nevertheless a solution to the strong $CP$ problem continues to occupy our minds.

In the supersymmetric extension of the standard model, not only has no new solution to the strong $CP$ problem been found, but also a new problem gets generated—namely, the small supersymmetric (SUSY) phase problem [5]. Even if the strong $CP$ problem were solved, for example, by the PQ symmetry, direct contributions to the dipole moment of the neutron constrain many other $CP$ violating phases in the theory which could be *a priori* of the order 1. Also the Nelson-Barr mechanism does not seem to generalize to the supersymmetric extension of the standard model even with universal soft SUSY breaking terms at the Planck and grand unified theory (GUT) scales [6]. Thus solutions to the strong $CP$ problem based on spontaneous $CP$ violation or the Nelson-Barr mechanism have not been extended to minimal supersymmetric standard model (MSSM) and this is grounds for serious concern. From an experimental point of view, while the MSSM is very predictive on things such as the Higgs mass, it does very poorly on the important question of additional $CP$ phases. At least two new independent phases in the $A$, $B$, $\mu$, and $m_{1/2}$ terms are expected and so far there is no theoretical prediction on their values [7].

In this Letter we show that if the MSSM [8] is extended to include parity (which can then be broken to MSSM at any scale between $M_{\text{SUSY}}$ and the Planck scale $M_{\text{Pl}}$), a new solution to the strong $CP$ problem is obtained. Further the small SUSY phase problem is also automatically solved. An important prediction emerges that there are no phases in the ratio of the Higgs vacuum expectation values (VEVS), $\mu$, $B$, $A$, or $m_{1/2}$ of MSSM. We also study the solution for the low-energy SUSY left-right symmetric model.

*Strong CP problem with parity.*—To be specific we include parity by extending the standard model to the left-right symmetric model $\mathrm{SU}(2)_L \times SU(2)_R \times SU(3) \times \mathrm{U}(1)_{B-L}$ [9]. The matter spectrum consists of the usual quarks and leptons, $Q_i(2,1,3,1/3)$, $Q_i^c(1,2,\overline{3},-1/3)$, $L_i(2,1,1,-1)$, and $L_i^c(1,2,1,1)$ where $i$ is the generation index and runs from 1 to 3. One or more (indexed by $a$) bidoublet Higgs fields $\phi_a(2,2,1,0)$ are introduced to break the theory down to electromagnetism. The $\phi_a$ are each represented by $2\times 2$ complex matrices while the doublet quark and lepton fields by $2\times 1$ column vectors. Under parity $\mathbf{x} \rightarrow -\mathbf{x}$, $Q_i \leftrightarrow Q_i^{c*}$, $L_i \leftrightarrow L_i^{c*}$, and $\phi_a \leftrightarrow \phi_a^\dagger$. Invariance under parity of the Yukawa term $(h_{ij}^a Q_i^T \phi_a Q_j^c + \text{H.c.})$ implies that $h_{ij}^a = h_{ji}^{a*}$, i.e., the Yukawa matrix is Hermitian. The mass matrix is the product of the Yukawa matrix and the VEVs $\langle\phi_a\rangle$. Therefore the quark mass matrix will have a real determinant if we can prove that the matrices $\langle\phi_a\rangle$ are real. This would then lead to a solution of the strong $CP$ problem since the coupling $\theta$ of the parity odd $\theta/32\pi^2\ F\tilde{F}$ term is zero due to parity.

$\langle\phi_a\rangle$ are determined by minimizing the Higgs potential and can be naturally real only if all the coupling constants involving $\phi_a$ in the Higgs potential are real. We begin by making an important observation that term involving only $\phi_a$ are the form $m_{ab}\operatorname{Tr}\phi_a^\dagger\phi_b$, $\mu_{ab}\operatorname{Tr}(\tau_2\phi_a^T\tau_2\phi_b)$, etc. (in general traces of products of $\phi_a$, $\phi_a^T$, $\phi_a^\dagger$, and $\tau_2$). By comparing every term and its Hermitian conjugate, it is easy to see that invariance under $P(\phi_a \leftrightarrow \phi_a^\dagger)$ implies that all the constants $m_{ab}$, $\mu_{ab}$, etc. are real. If we have additional gauge singlet Higgs fields $\sigma$, such that under $P\sigma \leftrightarrow \sigma^\dagger$, then all coupling constants of terms involving $\phi_a$ and $\sigma$ will also be real.

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In order to break $SU(2)_R \times U(1)_{B-1}$ to $U(1)_Y$ at a scale $M_R$ (which can be anywhere between $M_W$ and the Planck scale $M_{Pl}$), we need to introduce Higgs triplet ($2 \times 2$ traceless matrices) or doublet fields namely, either $\Delta(3,1,1,2)$, $\Delta^c(1,3,1,-2)$ or $\chi(2,1,1,1)$, $\chi^c(1,2,1,-1)$ such that under $P\Delta \leftrightarrow \Delta^{c*}$, $\chi \leftrightarrow \chi^{c*}$ and give a VEV to the right-handed fields. There will be coupling terms between $\phi_a$ and $\Delta^c$ or $\chi^c$. Terms of the form $\lambda[\mathrm{Tr}(\Delta^{c\dagger} \times \Delta^c)\,\mathrm{Tr}(\tau_2\phi_a^T\tau_2\phi_a) + \mathrm{Tr}(\Delta^\dagger\Delta)\,\mathrm{Tr}(\tau_2\phi_a^\dagger\tau_2\phi_a^*)]$ are invariant under $P$, and $\lambda$ can be complex. We note that this complex term is the source of the strong $CP$ problem in left-right symmetric theory.

If there is supersymmetry, as we shall see, these terms coupling $\Delta^c$ to $\phi_a$ with complex coupling constants are naturally absent. Therefore no complex numbers appear in the minimization equations that determine the vacuum expectation values of $\phi_a$. Consequently $\langle\phi_a\rangle$ are real and we are led to a solution to the strong $CP$ problem. Thus parity requires that the Yukawa matrix is Hermitian and SUSY makes it possible for the Higgs VEVs to be real, and together they lead to a Hermitian quark mass matrix which has a real determinant. The rest of this Letter analyzes the tree level and loop effects of the solution in SUSY left-right models [10–12] spontaneously breaking to MSSM. We pay particular attention to the interesting case of having the constrained minimal supersymmetric standard model below the GUT scale as well as the case of the low-energy SUSY left-right model. It should also be noted that since a Hermitian Yukawa coupling matrix has complex elements that permit the usual Cabibbo-Kobayashi-Maskawa (CKM) phase, in all the models considered in this Letter the $CP$ violation in the kaon system is explained exactly as in the standard Weinberg-Salam model.

*Tree level solutions: SUSY with parity.*

*Case 1: Minimal left-right model.*—The superpotential for the minimal model is given by [10–12]

$$W = M\,\mathrm{Tr}\Delta^c\overline{\Delta}^c + M^*\,\mathrm{Tr}\Delta\overline{\Delta} + \mu_{ab}\,\mathrm{Tr}\tau_2\phi_a^T\tau_2\phi_b\,. \tag{1}$$

There is no coupling between the $\Delta^c$ and the $\phi_a$. This is the case even for the most general soft SUSY breaking terms since they are given by the most general analytic cubic polynomials in the scalar fields of the theory. Since these have the same form as $W$ (but with arbitrary coefficients), there is no coupling between $\Delta^c$ and $\phi_a$ in these terms either. The $D$ terms only involve real gauge coupling constants. As explained above, $\mu_{ab}$ and coupling constants of the quadratic soft SUSY breaking terms involving $\phi_a$ are all real due to parity. Hence *all* coupling constants in the Higgs potential wherever $\phi_a$ occurs are real. Thus $\langle\phi_a\rangle$ is naturally real and at the tree level there is no strong $CP$ problem.

We would like to preserve this nice feature of the minimal model while extending to nonminimal models. The main reasons to extend the models are that we need to break the left-right symmetric theory to MSSM at a high scale and so we *have* to introduce other fields. Also, as it stands this model will break $Q_{\mathrm{em}}$ spontaneously unless $R$ parity is broken by giving the sneutrino a VEV [10]. We would like to keep $R$ parity unbroken, so as not to introduce the complex phases in the lepton sector and make the problem more complicated. This is another reason to consider nonminimal models. From now on we will interchangeably use $\phi$ for $\phi_a$ since the generalization to more than one doublet is now obvious. Also, in the following we will not explicitly write the squark or slepton fields as their VEVs are zero.

*Case 2: Breaking to MSSM + singlet.*—In order to solve the $\mu$ problem [8], MSSM with a singlet $\sigma$ has been considered previously in the literature. A discrete $Z_3$ symmetry $\phi,\sigma,Q,Q^c,L,L^c \to e^{i2\pi/3}(\phi,\sigma,Q,Q^c,L,L^c)$ prevents the direct $\mu$ term. We will show that SUSY left-right symmetric theory can naturally break to this low-energy theory with zero tree-level strong $CP$ phase. The most general left-right symmetric superpotential is

$$\begin{aligned} W = {} & M\,\mathrm{Tr}\Delta^c\overline{\Delta}^c + M^*\,\mathrm{Tr}\Delta\overline{\Delta} \\ & + \beta(h_\beta\,\mathrm{Tr}\Delta^c\overline{\Delta}^c + h_\beta^*\,\mathrm{Tr}\Delta\overline{\Delta}) \\ & + f(\beta) + h_\sigma\sigma\,\mathrm{Tr}\tau_2\phi^T\tau_2\phi + \lambda\sigma^3\,, \end{aligned} \tag{2}$$

where $f(\beta)$ is any cubic polynomial, and under $Z_3$, $\Delta,\Delta^c \to e^{i2\pi/3}(\Delta,\Delta^c)$, $\overline{\Delta},\overline{\Delta}^c \to e^{-i2\pi/3}(\overline{\Delta},\overline{\Delta}^c)$, and $\beta \to \beta$. Under parity $\sigma \to \sigma^\dagger$ and $\beta \to \beta^\dagger$. The soft SUSY breaking terms can have their most general form consistent with parity and $Z_3$. $F$ terms are obtained by taking the partial derivative of $W$ with respect to each of the fields in the superpotential (denoted here by $A_i$), so that

$$V_F = \Sigma_i \left|\frac{\partial W}{\partial A_i}\right|^2 . \tag{3}$$

It is easy to see that there are solutions for $V_F \approx M_{\mathrm{SUSY}}^4$ such that $\Delta^c,\overline{\Delta}^c \approx M_R$ and $\phi,\sigma$ are less than $M_{\mathrm{SUSY}}$. This implies that we can break the theory down to MSSM + singlet at a high scale $M_R$. Once again since there is no coupling terms between the $\Delta^c$ and $\phi$ fields, the coupling constants in the Higgs potential for *all* the terms which contain $\phi$ are real due to parity. Likewise, all coupling constants for terms involving $\sigma$ are also real. Thus $\langle\phi\rangle$ and $\langle\sigma\rangle$ are naturally real and there is no strong $CP$ phase at the tree level. A point to note is that this model has not been considered in Ref. [10]. Therefore the result of that paper does not apply and there can be $Q_{\mathrm{em}}$ conserving and parity breaking vacuuma without needing $R$-parity breaking. This is because a complex $h_\beta$ leads to a complex VEV for $\beta$, thereby breaking parity. The quartic $F$ term in $\phi$ stabilizes the $Q_{\mathrm{em}}$ conserving vacuum.

*Case 3: Breaking to MSSM.*—We introduce singlet (they could be in general triplet or other fields too) fields $\alpha$, $\beta$, and $\gamma$. Under parity they go to their Hermitian conjugate fields. The most general super potential is given by

$$W = h_\alpha \alpha \,\mathrm{Tr}\Delta^c \overline{\Delta}^c + h_\beta \beta \,\mathrm{Tr}\Delta^c \overline{\Delta}^c + m_\alpha \alpha\gamma + m_\beta \beta\gamma + \lambda_1 \gamma^3 + \lambda_2 \alpha^3 + \lambda_3 \beta^3 + \mu_{ab} \,\mathrm{Tr}\tau_2 \phi_a^T \tau_2 \phi_b + (\Delta\text{terms}), \quad (4)$$

where in order to prevent couplings between $\Delta^c$ and $\phi$ in the $F$ term, we have imposed a discrete symmetry ($D$) such that $\gamma, \overline{\Delta}, \overline{\Delta}^c \to e^{i2\pi/3}(\gamma, \overline{\Delta}, \overline{\Delta}^c)$; $\alpha, \beta \to e^{-i2\pi/3}(\alpha, \beta)$ and the rest of the fields are invariant. The singlets allow us to break the $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry at a high scale ($M_R$) to MSSM. This is important and can be checked by writing out all the $F$ terms obtained by differentiating $W$ with each and every field. The crucial point is that there are solutions for $V_F = 0$, with the singlets and the right-handed fields *alone* picking up VEVs. $m_\alpha$ and $m_\beta$ set the scale for $M_R \gg M_{\mathrm{SUSY}}, M_W$. Once again all coupling constants in terms wherever $\phi$ occurs are real, and hence $\langle\phi\rangle$ is real and there is no strong $CP$ problem. In order to avoid the bound $M_R \leq M_{\mathrm{SUSY}}/f$ of Ref. [13] we can introduce $B - L = 0$ triplet fields $\omega, \omega^c$ such that under $D\omega, \omega_c \to e^{-i2\pi/3}(\omega, \omega^c)$. This does not change the result that $\langle\phi\rangle$ is real.

This case has the advantage over case 2 since the discrete symmetries (parity as well as the discrete symmetry $D$) can be broken at a high scale so that domain walls associated with the breakdown of discrete symmetries can be inflated away.

We have shown three illustrated cases where there is a natural solution to the strong $CP$ problem at the tree level. Other nonminimal models can be easily accommodated, in a similar manner. Now we will study the loop effects.

*The complete solution.*—If $M_R \gg M_{\mathrm{SUSY}}, M_W$ then the effective theory below $M_R$ will by SUSY $SU(2)_L \times SU(3) \times U(1)$ (and in particular in case 3 it will be the MSSM). The MSSM superpotential and the soft SUSY breaking terms are given by [8,14,15]

$$W = \mu H^T \overline{H} + h_{ij}^u Q_i^T H u_j^c + h_{ij}^d Q_i^T \overline{H} d_j^c, \quad (5)$$

$$V_S = m H^T \overline{H} + A_{ij}^u \tilde{Q}_i^T H \tilde{u}_j^c + A_{ij}^d \tilde{Q}_i^T \overline{H} \tilde{d}_j^c + M_3 \tilde{G}\tilde{G} + M_{2L} \tilde{W}_L \tilde{W}_L + M_Y \tilde{Y}\tilde{Y} + \text{quad. scalar masses}, \quad (6)$$

where $\tilde{G}$ and $\tilde{W}_L$ are the gluino and left-handed gaugino ($W$-ino), respectively, and where the standard model Higgs doubles are denoted by $H(2,1,1)$ and $\overline{H}(2,1,-1)$. These doublets are the light elements of the bidoublet fields $\phi_a$. Because of parity and since we have shown that all couplings in terms containing $\phi_a$ are real (in cases 1, 2, and 3 above even after $\Delta^c$ picks up VEV), the following boundary conditions emerge at $M_R$:

$$\mu = \mu^*, \qquad h_{ij}^u = h_{ji}^{u*}, \qquad h_{ij}^d = h_{ji}^{d*},$$
$$A_{ij}^u = A_{ji}^{u*}, \qquad A_{ij}^d = A_{ji}^{d*}, \qquad m = m^*,$$
$$M_3 = M_3^*. \quad (7)$$

We now use the two-loop renormalization group equations [14] to run the above coupling constants to the TeV scale. Our results follow.

*Result 1: Constrained minimal supersymmetric standard model.*—If the soft SUSY breaking terms come from a supergravity sector, there are further constraints that soft SUSY breaking terms can satisfy [15]. These constraints both can be derived from some super-gravity theories and can be motivated by low energy flavor phenomenology. We will first consider the most constrained MSSM that has received the maximum attention with the following universality conditions at the supergravity (SUGRA) breaking (or Planck) scale:

$$A_{ij}^{u,d} = A h_{ij}^{u,d}, \qquad m = B\mu,$$
$$M_3 = M_{2L} = M_Y = m_{1/2}. \quad (8)$$

Now using Eq. (7) it is easy to see that $A$, $B$, and $m_{1/2}$ are real. The quadratic scalar masses are also universal and real. Therefore the only complex phase in the theory is the standard model CKM matrix phase. It is easy to see using the two-loop renormalization group equations (RGE) of Ref. [14] that every coupling constant (coupling matrix) and its Hermitian conjugate evolve according to the same RGE if the above conditions are met. Therefore Hermitian matrices remain Hermitian and real couplings remain real. Thus at the weak scale the Yukawa and squark matrices are Hermitian. The Higgs doublet coupling constants are all real, and the gluino, $B$-ino, and $W$-ino mass terms are real. Hence the expectation value of $H$ and $\overline{H}$ is real, and the quark mass matrix is Hermitian and the strong $CP$ phase is zero. In this case, the loop effects at the weak scale will induce a negligibly small strong $CP$ phase consistent with $\overline{\theta} \ll 10^{-9}$, and we have the solution to the strong $CP$ problem. Note that we have implicitly assumed that $M_R$ is approximately equal to the SUGRA breaking scale (or $M_{\mathrm{Pl}}$) and we will relax this condition later (see result 3).

*Result 2: With universality only for gauginos.*—There are supergravity models where only some but not necessarily all universality conditions are predicted. Also in string theory we may have nonuniversal terms [16]. The only universality condition that is really needed for us is that of gaugino phases, namely,

$$\mathrm{Arg}M_3 = \mathrm{Arg}M_{2L} = \mathrm{Arg}M_{2R} = \mathrm{Arg}M_{B-L}. \quad (9)$$

We will not assume any other universality condition and so the rest of the soft SUSY breaking terms can be general. Even in this case, and using Eq. (7), it is easy to see that the RGE preserve the Hermitian and real nature of the respective coupling constants and therefore, just as in result 1 it follows that there is no strong $CP$ problem. In addition to supergravity models already included in the first result, such a universality condition can be obtained from models where the gaugino mass term ratios depend only on real numbers like the structure constants of the gauge groups [16]. It can also happen due to an underlying grand-unified group.

*Result 3: Bound on W-ino phase, accessible strong CP.*—Parity relates only the left and the right $W$-ino phases but does not set them to zero. If instead of at $M_{\mathrm{Pl}}, SU(2)_R$ is broken at an intermediate scale $M_R$ then even if $W$-ino phases are real at $M_{\mathrm{Pl}}$ they will pick up a complex value from the complex terms in the $\Delta$ sector, due to the renormalization group running from $M_{\mathrm{Pl}}$ to $M_R$. This phase will in turn give rise to a gluino phase because while the left-handed $W$-ino contributes to the renormalization group running from $M_R$ to $M_W$, the right-handed $W$-ino does not. Both effects are at the two-loop level [14]. Hence the gluino mass term picks up a phase of the order $(1/16\pi^2)^2(1/16\pi^2)^2\delta$, which is about $10^{-9}\delta$ where $\delta$ are typical phases in the $\Delta$ terms. This resultant strong $CP$ phase is consistent with current experimental bounds, and at the same time is reasonably exciting for the neutron electric dipole moment (edm) searches.

Even if the Planck scale universality conditions on the gaugino phases are not exact, what the above estimate implies is that the $W$-ino phases must be within $(1/16\pi^2)^2$ at that scale or they will induce too large a strong $CP$ phase.

*Result 4: Without universality, low-energy SUSY left-right model.*—Even if no supergravity universality constraints are imposed (and the soft SUSY breaking terms have their most general form with no constraints even on the gaugino mass terms), the solution to the strong $CP$ problem with left-right symmetry preserved all the way up to the low-energy scale $M_R \leq M_{\mathrm{SUSY}}$ works. This is because both left and right $W$-inos are present at low energies if $M_R \leq M_{\mathrm{SUSY}}$, and their effects will cancel each other giving no strong $CP$ phase. Note that the illustrative models of case 1, 2, and 3 can have $M_R \leq M_{\mathrm{SUSY}}$.

Another very interesting possibility for low-energy left-right symmetry is with $\chi, \chi^c, \overline{\chi}, \overline{\chi}^c$ fields instead of the triplets, and with universal $A$ and $B$ terms. Once we impose $R$ parity, and noting that terms like $\chi^T \phi \chi^c$ must have a real coupling constant (due to parity), we can see that the phase in the $\chi^{cT}\overline{\chi}^c$ can be rotated away. Universal $A$ and $B$ terms must be real due to parity. Hence we once again have real vacuum expectation values for the Higgs fields. There is no problem with having to worry about $Q_{\mathrm{em}}$ breaking minima as the result of Ref. [10] does not apply to doublet fields.

*Non supersymmetric left-right model.*—It is worth noting that the term $\lambda[\Delta^{c\dagger}\Delta^c \operatorname{Tr}(\tau_2\phi^T\tau_2\phi)]$ that was the source of the strong $CP$ problem in nonsupersymmetric left-right models can be eliminated by discrete symmetries like $\phi_1 \to i\phi_1$, $Q_3^c \to iQ_3^c$, etc. We need to introduce enough bidoublets and have enough nontrivial symmetries that transform the quark fields in a family number dependent way, so as to prevent all such troublesome terms, while at the same time obtaining a consistent quark mass matrix. In this case we can have solutions to the strong $CP$ problem in the nonsupersymmetric version, without having to introduce $CP$ as a good symmetry of the Lagrangian.

In *conclusion,* we have shown that supersymmetry with parity can solve the strong $CP$ problem in many cases including the interesting cases of having the MSSM or some of its extensions below the Plank, GUT, and intermediate scales, as well as for the case where we have a low-energy SUSY left-right model. The small SUSY phase problem is also solved in these models.

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[1] For review, see R. Peccei, in *CP Violation,* edited by C. Jarlskog (World Scientific, Singapore, 1989).
[2] A.E. Nelson, Phys. Lett. B **136**, 387 (1984); S.M. Barr, Phys. Rev. D **30**, 1805 (1984).
[3] S.M. Barr, D. Chang, and G. Senjanovic, Phys. Rev. Lett. **67**, 2765 (1991); K.S. Babu and R.N. Mohapatra, Phys. Rev. D **41**, 1286 (1990).
[4] M.A.B. Beg and H.S. Tsao, Phys. Rev. Lett. **41**, 278 (1978); R.N. Mohapatra and G. Senjanovic, Phys. Lett. **79B**, 283 (1978); H. Georgi and J. Hadronic, **1**, 155 (1978).
[5] K.S. Babu and S.M. Barr, Phys. Rev. Lett. **72**, 2831 (1994).
[6] S.M. Barr and G. Segre, Phys. Rev. D **48**, 302 (1993); M. Dine, R.G. Leigh, and A. Kagan, Phys. Rev. D **48**, 2214 (1993).
[7] R. Garisto, Nucl. Phys. **B419**, 279 (1994), and references therein.
[8] For reviews of supersymmetry, see H. Haber and G. Kane, Phys. Rep. **117**, 76 (1984); H.P. Nilles, Phys. Rep. **110**, 1 (1984); A. Chamseddin, P. Nath, and A. Arnowitt, *Applied $N = 1$ Supergravity* (World Scientific, Singapore, 1984); M. Drees and S.P. Martin, Report No. MAD-PH-95–879, hep-ph 9504324, 1995.
[9] J.C. Pati and A. Salem, Phys. Rev. D **10**, 275 (1974); R.N. Mohapatra and J.C. Pati, Phys. Rev. D **11**, 566, 2558 (1975); G. Senjanović and R.N. Mohapatra, Phys. Rev. D **12**, 1502 (1975); R.E. Marshak and R.N. Mohapatra, Phys. Lett. **91B**, 222 (1980).
[10] R. Kuchimanchi and R.N. Mohapatra, Phys. Rev. D **48**, 4352 (1993).
[11] M. Cvetic and J.C. Pati, Phys. Lett. **135B**, 57 (1984).
[12] R.M. Francis, M. Frank, and C.S. Kalman, Phys. Rev. D **43**, 2369 (1991); Y.J. Ahn, Phys. Lett. **149B**, 337 (1984); K. Huitu, J. Maalampi, and M. Raidala, Nucl. Phys. **B420**, 449 (1994); R.M. Francis, C.S. Kalman, and H.N. Saif, Z. Phys. C **59**, 655 (1993).
[13] R. Kuchimanchi and R.N. Mohapatra, Phys. Rev. Lett. **75**, 3989 (1995).
[14] S.P. Martin and M.T. Vaughn, Phys. Rev. D **50**, 2282 (1994).
[15] G.L. Kane, C. Kolda, L. Roszkowski, and J.D. Wells, Phys. Rev. D **49**, 6173 (1994).
[16] Ibanez and Lust, Nucl. Phys. **B382**, 305 (1992).