Moving Glass Phase of Driven Lattices

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We study periodic lattices, such as vortex lattices, driven by an external force in a random pinning potential. We show that effects of static disorder persist even at large velocity. It results in a novel moving glass state which has analogies with the static Bragg glass. The lattice flows through well-defined, elastically coupled, *static* channels. We predict barriers to transverse motion resulting in finite transverse critical current. Experimental tests of the theory are proposed. [S0031-9007(96)00082-8]

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An open question is to understand the effect of static substrate disorder on periodic media such as vortex lattices [1], charge density waves (CDW) [2], Wigner crystals [3], colloids [4], and magnetic bubbles [5]. Numerous experiments study such elastic systems in motion under an applied force produced by a current (vortex lattices), a voltage (CDW), an electric field (colloids), and a magnetic field gradient (magnetic bubbles). It is therefore important to describe the physical properties of both the static and moving lattices. The statics of vortex lattices has been much investigated recently, and it is generally agreed that disorder leads to a glass state with diverging barriers, pinning, and loss of translational order. The precise nature of the glass state, however, has been the subject of much debate [6–8], in particular, concerning the decay of translational order and the presence of topological defects. It was shown recently [9] within an elastic theory that, because of the periodicity of the lattice, the decay of translational order is only algebraic and that the resulting glass phase still exhibits divergent Bragg peaks. We argued that at weak disorder a Bragg glass exists without equilibrium dislocations and also that such a glass will undergo a transition into a strongly disordered vortex glass containing topological defects, or a pinned liquid, upon increase of disorder or field [9]. This is compatible with recent decoration and neutron experiments [10,11], and with the behavior of the critical point [12] in the phase diagram of vortex lattices, where a transition between two different glass states is observed upon raising the field [13].

It is thus crucial to determine how much of the glassy properties of the static system remain once the lattice is set in motion, and how translational and topological order behave. At large velocity v it was expected that, since the pinning force on a given vortex varies rapidly, disorder would produce little effect. Perturbation theories in disorder and $1/v$ were thus developed [14,15] to compute velocity as a function of the external force, and to estimate critical currents. Recently, Koshelev and Vinokur [16] have extended the perturbation theory of [14,15] to compute vortex displacements u. They concluded that, at low T and above a certain velocity, the moving lattice is a crystal at an effective temperature $T' = T + T_{\mathrm{sh}}$. Several experiments indeed suggest that a fast moving lattice is more ordered [10,17]. The effect of pinning can be described [16] by some effective shaking temperature $T_{\mathrm{sh}} \sim 1/v^2$ defined by the relation $\langle |u(q)|^2 \rangle = T_{\mathrm{sh}}/c_{66}q^2$ This would suggest bounded displacements in $d > 2$ and the absence of glassy properties in the moving solid.

In this Letter we reconsider this problem. We show that in the case of a moving lattice the perturbation theory of [16] breaks down, even at large v. The physical reason is that some modes of the disorder are not affected by the motion, and *static* disorder is still present in the moving system. As a result, the moving lattice is, in fact, a *moving glass*. Since translational order in the moving frame decays and relative displacements are not bounded, such a phase cannot be described by simple perturbation theory [18]. As in the statics, *periodicity* is crucial, and the moving lattice has a completely different behavior than other driven systems such as manifolds. The physics of this new phase can be described in terms of *elastic channels*. When the force is applied along a principal lattice direction, the rows of the lattice flow along well-defined, nearly parallel, preferred paths in the pinning potential. The manifold of these optimal channels (lines for 2D lattice and sheets for 3D vortex line lattice), that exhibits a roughness that we estimate, is a purely static and reproducible feature of the disorder configuration. We also predict that the moving glass exhibits barriers to an additional small transverse force and compute the associated transverse critical current. The other modes of the disorder are suppressed by motion and give rise to an additional wiggling motion of the particles around the static channel configuration, which can be treated in perturbation.

We now derive the equation of motion for a lattice submitted to external force F. We denote by $R_i(t)$ the position of an individual vortex in the laboratory frame.

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The lattice as a whole moves with a velocity v. We thus introduce the displacements $R_i(t) = R_i^0 + vt + u_i(t)$, where the R_i^0 denote the equilibrium positions in the perfect lattice with no disorder. u_i represent the displacements in the moving frame. We consider in the following the elastic limit in the absence of topological defects, thus assuming $|u_i - u_{i+1}| \ll a$, where a is the lattice spacing, an assumption which may be checked self-consistently. One then takes the continuum limit $u_i(t) \to u(r,t)$, where $u(r,t)$ is a smoothly varying n-component vector field, which components we denote by $u_\alpha(r,t)$. It is convenient here to express the displacement field $u_\alpha(r,t)$ in terms of the coordinates (r,t) of the laboratory frame. The equation of motion in the laboratory frame is then

$$\eta \partial_t u_\alpha + \eta v \cdot \nabla u_\alpha = c\nabla^2 u_\alpha + F_\alpha^{\mathrm{pin}} + F_\alpha - \eta v_\alpha + \zeta_\alpha , \qquad (1)$$

where η is the friction coefficient, $F_\alpha^{\mathrm{pin}}(r,t) = -\delta\mathcal{E}/\delta u_\alpha(r,t)$ is the pinning force, $\mathcal{E}[u(r,t)]$ is the pinning energy, and the thermal noise satisfies $\overline{\zeta_\alpha(r,t)\zeta_\beta(r',t')} = 2T\delta_{\alpha\beta}\delta^d(r-r')\delta(t-t')$. For clarity we use here an isotropic elastic constant c. The realistic case, discussed at the end, has the same large distance physics. The term $\eta v \cdot \nabla u_\alpha$ comes from expressing the displacement field in the laboratory frame, and $-\eta v_\alpha$ is the average friction. v is determined by the condition that the average of u is zero. Equation (1) is exact up to higher powers of derivatives of u, negligible in the elastic limit. The pinning energy can be expressed in terms of the vortex density $\rho(r,t) = \sum_i \delta(r - R_i^0 - vt - u_i(t))$. One has $\mathcal{E}[u(r,t)] = \int d^d r \rho(r,t) V(r)$, where the random potential has correlations $\overline{V(r)V(r')} = \Delta(r-r')$ of range r_f. Since even for smooth displacement fields the density is a series of delta peaks, the continuum limit for $\mathcal{E}[u]$ should be performed by distinguishing [9] the various Fourier components of the density $\rho(r,t) = \rho_0(1 - \nabla \cdot u + \sum_{K\neq 0} \exp\{iK \cdot [r - vt - u(r,t)]\})$, where K spans the reciprocal lattice and ρ_0 is the average density. Using this decomposition in (1) the force due to disorder naturally splits into a *static* and a time-dependent part:

$$\begin{aligned}\eta \partial_t u_\alpha + \eta v \cdot \nabla u_\alpha = c\nabla^2 u_\alpha &+ F_\alpha^{\mathrm{stat}}(r, u(r,t)) \\ &+ F_\alpha^{\mathrm{dyn}}(r,t,u(r,t)) \\ &+ F_\alpha - \eta v_\alpha + \zeta_\alpha(r,t), \end{aligned}$$

$$\begin{aligned} F_\alpha^{\mathrm{stat}}(r,u) &= V(r)\rho_0 \sum_{K\cdot v = 0} iK_\alpha e^{iK\cdot(r-u)} - \rho_0 \nabla_\alpha V(r), \\ F_\alpha^{\mathrm{dyn}}(r,t,u) &= V(r)\rho_0 \sum_{K\cdot v\neq 0} iK_\alpha e^{iK\cdot(r-vt-u)}. \end{aligned} \qquad (2)$$

The static part of the random force comes from the modes such that $K \cdot v = 0$ which exist for any direction of the velocity commensurate with the lattice. The maximum effect is obtained for v parallel to one principal lattice direction, the situation we study now. Reflection symmetry then imposes that v and F are aligned along direction x, the $d-1$ transverse directions being denoted by y. F_α^{stat} gives the dominant contribution to the lattice deformations. In first approximation we drop F_α^{dyn} and solve the remaining static problem (leading to a reference ground state at $T=0$). F_α^{dyn} gives additional fluctuations on top of this ground state, estimated below. The static term $\rho_0\nabla_\alpha V$, that comes from the Fourier components $k \ll 1/a$ of the disorder, produces alone only bounded displacements for $d>1$. Thus, as for the nonmoving lattice [9] for $d>2$, it does not change the large scale physics and we drop it. Since F_α^{stat} is now along y, and depends only on u_y, $u_x = 0$ in the ground state.

The most important terms in (1) thus lead to the following equation of motion in the laboratory frame which involves only the *transverse* displacements u_y:

$$\eta\partial_t u_y + \eta v \partial_x u_y = c\nabla^2 u_y + F^{\mathrm{stat}}(r, u_y(r,t)) + \zeta_y(r,t),$$

$$F^{\mathrm{stat}}(x,y,u_y) = V(x,y)\rho_0 \sum_{K_y\neq 0} K_y \sin K_y(u_y - y). \qquad (3)$$

This is now a nontrivial static disordered model, and one expects a glass phase at low temperature, with pinning of the field $u(r,t)$ into preferred configurations. Thus, the moving vortex configurations can be described in terms of *static channels* which are the easiest paths where particles follow each other in their motion. Channels in the elastic flow regime behave differently than the one introduced to describe slow plastic motion between pinned islands [19]. In the topologically ordered moving glass they form a manifold of elastically coupled, almost parallel lines or sheets (for vortex lines in $d=3$) directed along x and characterized by transverse wandering u_y. In the laboratory frame they are determined by the static disorder and do not fluctuate with time. In the moving frame, since each particle is tied to a given channel which is now moving, it indeed wiggles and dissipates but remains highly correlated with the neighbors. To obtain the roughness of the manifold of channels, we compute the correlator of relative displacements $B(x,y) = \langle\overline{[u(x,y) - u(0,0)]^2}\rangle$. A detailed analysis will be presented elsewhere [20]. One defines two characteristic lengths for the decay of translational order, R_x^a and R_y^a along the longitudinal and transverse direction by $B(R) \sim a^2$. One expects three regimes.

Short scale regime: At very short scales one can expand the pinning force to lowest order in u. This gives a simple model where pinning is described by a random force $F^{\mathrm{stat}}(x)$ independent of u whose correlator is $\overline{F^{\mathrm{stat}}(r)F^{\mathrm{stat}}(r')} = \Delta\delta^d(r-r')$ with $\Delta = \rho_0^2 \sum_{K_y} K_y^2 \Delta_K$. This is the dynamic equivalent of the Larkin random force model and $B = B_{\mathrm{rf}} + \langle u^2\rangle_{\mathrm{th}}$:

$$B_{\mathrm{rf}}(x,y) = \int \frac{dq_x d^{d-1}q_y}{(2\pi)^d} \frac{\Delta[1 - \cos(q_x x + q_y y)]}{(\eta v q_x)^2 + c^2(q_x^2 + q_y^2)^2} \qquad (4)$$

and $\langle u^2\rangle_{\mathrm{th}}$ is the thermal displacement. One finds for $x > c/\eta v$ $B(x,y) \sim \Delta \frac{y^{3-d}}{c\eta v} H(cx/\eta v y^2)$, where $H(0) = \mathrm{const}$

and $H(z) \sim z^{(3-d)/2}$ at large z. x scales as y^2, and the displacements are very anisotropic. For $x < c/\eta v$ one has the usual isotropic result. If one could extrapolate this behavior to larger scales, it would result in an algebraic decay of translational order in $d = 3$ $[B(x,y) \sim \ln|y|]$ and exponential decay in $d = 2$. However, since the Larkin model rests on the expansion in powers of u, it is valid only as long as $K_{\max} u \ll 1$, where $K_{\max} \sim 1/r_f$ is the highest Fourier component of the random potential [9]. This defines two lengths R_x^c and R_y^c such that $B(R) \sim r_f^2$, below which the Larkin model is valid. At large velocity $R_y^c = (r_f^2 v c/\Delta)^{1/(3-d)}$, $R_x^c = v(R_y^c)^2/c$. For smaller velocities $v < v^* \sim c(\Delta/c^2 r_f^2)^{1/(4-d)}$, the elastic term dominates and $R_x^c \sim R_y^c \sim R_{\text{iso}}^c$ where R_{iso}^c is the static pinning length. These lengths are renormalized by temperature and by the dynamical part u^d. Note that this Larkin random force corresponds formally to the so-called "random mobility term" considered in [21,22], and by keeping only this term one misses all the physics of the moving glass, e.g., the channels and the transverse barriers. As for the static case [9], the pinning force in (3) should be treated to all orders in u. Above this length scale, pinning and metastability appear.

Intermediate regime: At intermediate scales $R_y^c < y < R_y^a$ and $R_x^c < x < R_x^a$, the analogous random manifold regime [6,9] exists for which $u_y \sim y^{\zeta}$. The channels are determined by optimization of elasticity (cq^2 term), dissipation ($i\eta v q_x$ term), and the random potential seen independently by each channel in its vicinity. One expects many metastable nearly optimal configurations in that regime and glassy behavior. Flory-type arguments suggest that the scaling properties of this glass are related to the static Bragg glass by $d \to d+1$ and $n \to n-1$. The former comes from assuming $q_x \sim q_y^2$ and the latter from $u \to u_y$. The Flory estimate is then $\zeta^F = [(3-d)/(n+3)]$.

Asymptotic regime: At large distances $x > R_x^a, y > R_y^a$, in $d = 3$ the displacements have a slower, logarithmic growth. Estimates *à la* Fukuyama and Lee then give

$$R_y^a \sim (a^2 v c/\Delta)^{1/(3-d)}, \qquad R_x^a = v(R_y^a)^2/c\,. \quad (5)$$

The moving glass is highly anisotropic since R_x^a/R_y^a diverges as $v \to \infty$. Its upper critical dimension is $d = 3$, instead of $d = 4$ for the static one. For $d > 3$ the moving system is not a glass but a perfect crystal at weak disorder or large v. For $d \le 3$ weak disorder destroys long-range order and results in a moving glass.

As an important consequence of the existence of the moving glass, barriers for transverse motion exist once the pattern of channels is established. Thus the response to an additional small transverse force F_y is very nonlinear with activated behavior. At $T = 0$, neglecting the dynamic part of the disorder, a true transverse critical current J_y^c should exist. This can be seen by adding a transverse force in (3). For $\eta v < \eta v^* \sim (R_{\text{iso}}^c/r_f)F_c$, where F_c is the isotropic critical force, the Larkin domains remain isotropic, and one expects J_y^c to decrease slowly from J_{iso}^c to a fraction of the longitudinal critical current J_{iso}^c (since only the K_y modes contribute). For $\eta v > \eta v^*$ one expects a much faster decay, and a naive estimate for J_y^c is obtained by balancing the pinning force with the transverse force acting on a Larkin domain:

$$J_y^c = \frac{c_\rho}{\phi_0 r_f}\Delta^{1/2}(R_y^c)^{-(d-1)/2}(R_x^c)^{-1/2} \sim \tilde{\Delta}^{2/(3-d)},$$

where $\tilde{\Delta} = \Delta/v$ is an effective velocity-dependent disorder and c_ρ is the speed of light. In $d = 3$ it yields exponential decay with v. In practice, ηv^* can correspond to a large driving compared to F_c. The above regimes correspond to collective pinning with $R_y^c > a$. For $R_y^c < a$, i.e., $v < v_0 = \Delta a^{3-d}/c r_f^2$, single channel pinning leads to different estimates for J_y^c. Since motion is not modified below J_y^c, dv_x/df_y also vanishes below the transverse critical force. One can expect nonlinear effects in the flow along x since channel configuration is modified when F_x is increased. A simpler example of transverse barriers is a lattice driven in a commensurate washboard potential $V(x,y) = U_0 \cos(K_0 y) - F_x x$. There it is clear that even in the moving frame the problem is static and that the transverse critical force is $F_y^c \sim U_0 K_0$. Even a single particle in the 2D potential $V(x,y) = f_c \cos(x) + f_c \cos(y) - F_x x$ has finite F_y^c. Its velocity can be computed as in [23] and becomes, for $T \to 0$, $V_y = (F_y^2 - f_c^2)^{1/2}\theta(F_y - f_c)$, independent of F_x.

To estimate the effects of the time-dependent pinning force in (2), we split $u = u^s + u^d$ into a static u^s and a dynamics u^d part. A reasonable estimate is

$$\langle u^d \cdot u^d \rangle_{q,\omega} = \sum_{K \cdot v \neq 0} \frac{\rho_0^2 K^2 \Delta_K \delta(\omega - K \cdot v)}{\eta^2 v^2 (K_x + q_x)^2 + c^2 q^4}\,. \quad (6)$$

The dynamic correlations are bounded due to the presence of mass term $K_x v$ in the denominator. u^d saturates at large distances, even in $d = 2$ if $T = 0$. u^d is smaller by a factor $a/R_x \ll 1$ compared to u^s at the length scales R_x. u^d thus represents a small additional wiggling motion around the ground state. The massive propagator in (6) is very different from a thermal one $1/q^2$.

Extension to realistic elastic energy, e.g., a triangular lattice in $d = 2$, is straightforward. The static displacements, within the random force model, are now $u_\alpha(q) = F_y(q)[P_{\alpha y}^T(i\eta v q_x + c_{66}q^2)^{-1} + P_{\alpha y}^L(i\eta v q_x + c_{11}q^2)^{-1}]$, where c_{11} and c_{66} are (dispersionless) bulk and shear moduli, respectively. Thus the mean square displacement $B(x,y)$ is again given, for $y > y^*$, by (4) with c replaced by c_{11}. Note that only shear modes were considered in [16], an approximation which may hold for $y \ll y^*$ but misses the physics of the glass. Indeed, only the *compression* modes are responsible for the glass (and lead to unbounded displacements for $d > 2$) since both displacements and force have to be considered along y. In $d = 3$, tilt modes would also be relevant for flux lines and transverse shear modes for a solid. One finds $y^* = (c_{11}/\eta v)[(c_{11}^2 - c_{66}^2)/(c_{11}^2 + c_{66}^2)]^{1/2}$.

The predictions of glassy structure, topological order, channel motion, and transverse critical force can be tested in experiments. For vortex lattices, J_c^y can be measured in the presence of a longitudinal current. Magnetic noise experiments and NMR probe $vt + u(x,t)$ and the phase at the washboard frequency should contain a static component with slow and anisotropic decay. Other experimental systems such as colloids, magnetic bubbles, and double or triple incommensurate CDW should exhibit similar behavior. A transverse critical force may explain recent Hall voltage experiments in 2D Wigner crystals [24]. The predictions of channel motion and transverse critical current can be directly tested in numerical simulations [25].

Equation (2) shows the importance of the relative orientation of the lattice and the applied force. It has been argued [14] that to minimize power dissipation the lattice aligns with the force. This process may be slow, and other orientations can be studied by applying a transverse field. We expect commensuration effects with a devil's staircase type structure in response to additional force. At higher commensuration vectors the channel structure may become unstable due to the stronger effect of u^d. Conversely, the larger the static part, the more stable the glass with fewer topological defects. The size of the plastic flow regime should thus depend on the lattice orientation. It is then possible that in $d = 3$ for weak disorder the glass remains topologically ordered at all v and that the intermediate plastic regime disappears at low T. One would thus go smoothly from the moving to the static Bragg glass. At large v, channels are nearly straight, and out of equilibrium dislocations are thus suppressed. This allows us to understand why in [10] the plastic regime disappears when J is slowly decreased.

Previous descriptions of moving systems, such as manifolds driven in periodic [26] or disordered potentials, focused on the generation under the renormalization group of dissipative Kardar-Parisi-Zhang (KPZ) $(\nabla u)^2$ nonlinearities. They do occur, due to lattice cutoff, in driven random sine-Gordon models [21,27]. They are not important here because of the statistical symmetry $u_y \to -u_y$ in (3) and the fact that dynamic modes (6) are massive. Thus, because of periodicity, this problem belongs to a new universality class. KPZ terms may play a role for incommensurate motion.

In conclusion, we studied a lattice moving in a random potential. Static disorder dominates motion along symmetry directions, and the moving system is a *glass* with a large amount of topological order. It is continuously related to the static Bragg glass, and, although the decay of translational order is slow, it has genuine glass properties different from a usual solid. We predict experimental signatures such as elastic channels and transverse critical current.

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