# Dynamical Current Redistribution and Non-Gaussian $1/f$ Noise

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Dynamical current redistribution (DCR) in noisy resistors is discussed. Contrary to a common assumption in many theories and in the interpretation of many experiments, DCR can cause large-amplitude non-Gaussian (NG) noise for statistically independent fluctuators. NG effects are predicted to be large near the percolation threshold and near the metal-insulator transition. We report the first observations of NG noise in semiconductor superlattices and attribute this to DCR. We also show that DCR is the most likely explanation of NG noise in several other systems. [S0031-9007(96)00027-0]

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Physical systems with a large number of degrees of freedom ubiquitously demonstrate Gaussian fluctuations (noise) as a consequence of the central limit theorem [1]. All correlation functions of Gaussian noise of an order greater than 2 can be expressed in terms of first- and second-order correlation functions [1]. Non-Gaussianity is therefore naturally quantified by the deviation of high-order statistics (order greater than 2) from the expectation for Gaussian noise.

High-order statistics of resistance noise have been measured in such diverse systems as epitaxial Si [2], $a$-Si:H [3–5], GaAs [6], amorphous metal alloys [7], metallic spin glasses [8], and type II superconductors [9]. Often, non-Gaussianity has been taken as a signature of fluctuator interactions [5–8]. For classical conduction, coarse graining of the resistor allows discussion of the noise in the context of classical percolation resistor networks. Extensive theoretical treatment exists of noise in fixed-topology resistor networks, in which the elements are statistically independent fluctuators [10]. Surprisingly, all such studies have focused on the limit of small fluctuations of the elements, where the noise is necessarily Gaussian.

The present Letter addresses the opposite limit of large local resistivity fluctuations. The key distinction between the two cases rests in the importance of dynamical current redistribution (DCR): When one element of a resistor network fluctuates to a larger resistance, it will carry lower current while other elements in parallel current paths must carry higher average current. Systems with large local resistivity fluctuations will in general exhibit non-Gaussian (NG) noise due to DCR *even for noninteracting statistically independent fluctuators*. Although the consequences of DCR will be maximal at the percolation threshold, we report experimental evidence for DCR and NG noise in several materials away from the percolation threshold.

Cohn's theorem for resistor networks states that [11]

$$\frac{\partial R}{\partial r_\alpha} = \frac{i_\alpha^2}{I^2}, \tag{1}$$

where $R$ is the resistance between two chosen nodes, $I$ is the measurement current, $r_\alpha$ are the resistances of the network elements, and $i_\alpha$ are the corresponding currents through each element. Whether explicitly or implicitly, most conventional analyses of the statistics of experimentally observed resistance noise [5–8,12] and many theories of noise in resistor networks [10] assume that the local resistance fluctuations are small. Consequently, they invoke a generalized Cohn's theorem (GCT) which is first order in the local fluctuations,

$$\delta R(t) = \frac{1}{I^2} \sum_{\alpha=1}^{N} \langle i_\alpha(t) \rangle^2 \delta r_\alpha(t), \tag{2}$$

where $t$ is time, $\langle \cdots \rangle$ indicates a time average, and $\delta r_\alpha(t) = r_\alpha(t) - \langle r_\alpha(t) \rangle$. If the sample is relatively homogeneous, the central limit theorem requires that $\delta R(t)$ in Eq. (2) be Gaussian for statistically independent fluctuators in the limit of large $N$. Often an ensemble of statistically independent, thermally activated fluctuators with a distribution of activation energy barriers $\mathcal{D}(E)$ has been postulated [13–16]; let $\underline{\mathbf{S}}^{(1)}(f, E, \Theta)$ be the power spectrum of the statistical variable of a model fluctuator [for example, a two-level system (TLS)] with $E$ being the activation energy barrier and $\Theta$ the temperature. If we assume sample homogeneity, the power spectrum $\mathbf{S}^{(1)}$ of the sample resistance is

$$S^{(1)}(f) \propto \int_0^\infty \underline{S}^{(1)}(f, E, \Theta) \mathcal{D}(E)\, dE \tag{3}$$

because the GCT requires that the autocorrelation function of $\delta R$ be a sum of the autocorrelation functions of the $\delta r_\alpha$ weighted by $\langle i_\alpha(t) \rangle^2 / I^2$. The detailed behavior of $\mathbf{S}^{(1)}$ in simple metal films has been explained semiquantitatively by assuming a broad $\mathcal{D}(E)$ in Eq. (3) [12,14,15]. The GCT presented as Eq. (2) is the bridge between the microscopic phenomenon $\underline{\mathbf{S}}^{(1)}$ and the macroscopic measured $\mathbf{S}^{(1)}$ of Eq. (3).

Given the direct observation of physical interactions between discrete fluctuators [17], high-order statistics have been employed to study fluctuator interactions when the noise signal from individual fluctuators cannot be resolved [2,5–8,12,18]. The modulation of the activation energy of one fluctuator by other fluctuators results in "spectral wandering" [14]; that is, the integrated power in some band $f_L \leq f \leq f_H$ will have a large variance and will itself have a power spectrum reflecting the frequency modulation of fluctuators by their neighbors [12]. This power spectrum of the spectral wandering has been named the "second spectrum" and is denoted by $\mathbf{S}^{(2)}$ with a frequency variable $f_2$ [2,12]. For noninteracting TLS's, the GCT leads to a white $\mathbf{S}^{(2)}$ at low $f_2$ [12,18]. Spectral wandering with a nonwhite $\mathbf{S}^{(2)}$ has therefore been taken as a fingerprint of interacting fluctuators [5–8,12].

The above conclusion is unambiguous in the limit of small local fluctuations and relatively homogeneous transport, such as for the disordered metal and metallic spin-glass samples studied by Weissman and co-workers [7,8]. However, DCR can generate strongly NG noise for large local resistivity fluctuations or strong transport inhomogeneity. Consider a hypothetical resistor with only two fluctuators with characteristic frequencies separated by more than an order of magnitude. Take the fluctuators to be statistically independent TLS's. The relative positions of the two fluctuators determines whether the resistor is more accurately modeled by a series or a parallel circuit of two TLS resistors. Let $r_1$ denote the faster fluctuating network element. For a series circuit the GCT is exact, and the simulation in Fig. 1(a) shows no spectral wandering, i.e., the contribution to the noise power from $r_1$ is independent of the state of $r_2$. However, for a parallel circuit [Fig. 1(b)], the amplitude of the fluctuations in $R_{\mathrm{parallel}}$ due to the switching of $r_1$ is explicitly dependent on the state of $r_2$. The spectral wandering due to DCR is unaccounted for by Eq. (2).

Monte Carlo simulations of the normalized second spectra $\mathbf{s}^{(2)}$ of the series and parallel networks are shown in Fig. 2 for several values of $\Delta r/\langle r\rangle$ for $r_1$ and $r_2$. The second spectrum $\mathbf{S}^{(2)}$ is normalized by the square of the integrated power in the band $(f_L, F_H)$ to yield $\mathbf{s}^{(2)}$ [19]. As $\Delta r/\langle r\rangle$ and hence the DCR increase, the non-Gaussianity of the parallel circuit monotonically increases. In contrast, $\mathbf{s}^{(2)}$ for the series circuit (dashed curve) is independent of $\Delta r/\langle r\rangle$ and is less than 25% above the Gaussian background (not shown).

It is convenient to write $i_\alpha(t)$, $r_\alpha(t)$, and $v_\alpha(t)$ for a network as the sum of time-averaged and time-varying parts: $i_\alpha(t) = \langle i_\alpha(t)\rangle + \delta i_\alpha(t)$, etc. Hence,

$$\langle v_\alpha(t)\rangle = \langle i_\alpha(t) r_\alpha(t)\rangle = \langle i_\alpha(t)\rangle\langle r_\alpha(t)\rangle + \langle \delta i_\alpha(t)\delta r_\alpha(t)\rangle\,,$$
$$\delta v_\alpha(t) = \langle i_\alpha(t)\rangle\delta r_\alpha(t) + \langle r_\alpha(t)\rangle\delta i_\alpha(t) + \delta i_\alpha(t)\delta r_\alpha(t) - \langle \delta i_\alpha(t)\delta r_\alpha(t)\rangle\,.$$

Tellegen's theorem [18,20] provides the exact relations

$$I\delta V(t) = \sum_{\alpha=1}^{N}\langle i_\alpha(t)\rangle\delta v_\alpha(t)\,,$$
$$0 = \sum_{\alpha=1}^{N}\langle v_\alpha(t)\rangle\delta i_\alpha(t)$$

for a constant-current experiment. It can then be shown that

$$\delta R(t) = \frac{1}{I^2}\sum_{\alpha=1}^{N} i_\alpha(t)\{\langle i_\alpha(t)\rangle\delta r_\alpha(t) - \langle \delta i_\alpha(t)\delta r_\alpha(t)\rangle\}\,. \tag{4}$$

Equation (4) is exact to the extent that the fluctuators are stationary and the time averages consequently well defined. The DCR contributes to the moments of $\delta R(t)$ calculated from Eq. (4) through correlation functions of the form $\langle i_{\alpha 1}(t),\ldots,i_{\alpha n}(t)\delta r_{\beta 1}(t),\ldots,\delta r_{\beta m}(t)\rangle$; such terms are absent if the GCT is used.

The question immediately arises as to the size of the contribution to either $\mathbf{S}^{(1)}$ or $\mathbf{S}^{(2)}$ by DCR in real samples. To first nonvanishing order $\langle \delta i_\alpha(t)\delta r_\alpha(t)\rangle$ scales as $(\Delta r_\alpha/\langle r_\alpha\rangle)^2$ [21]. The GCT is therefore an excellent approximation to Eq. (4) for materials in which $\Delta\rho/\rho \ll 1$ [7,8,14]. On the other hand, near the percolation threshold the correlation length for the pair-connectedness correlation function diverges [22], and long-range DCR is necessarily present. Non-Gaussian fluctuations were recently reported in simulations of a dynamical resistor network at the percolation threshold [23]. Although noise near the percolation threshold has also been studied by several experimental groups [24], we are unaware of any experimental investigation of high-order statistics of the noise in this regime. Existing theory for noise near the percolation threshold requires that the local fluctuations be small and the noise be Gaussian [10]. We conjecture that the absence of universality and the often poor agreement between theory [10] and experiment [23] is due in part to DCR.

Farther from the percolation threshold the situation is less clear; however, the absolute value of a correlation function such as $\langle \delta i_\alpha(t)\delta r_\beta(t)\rangle$ will decrease only as a dipole $|\mathbf{x}(\alpha,\beta)|^{-d}$ at long distances $\mathbf{x}(\alpha,\beta)$ for dimension $d$ [25]. We note that many experiments have failed to find long-range spatial correlation of $1/f$ noise, as was discussed in Ref. [12]. However, all such experiments measured the cross correlation of the noise signal between pairs of electrodes in series along thin wires; this electrode configuration is insensitive to DCR.

Three examples of materials with NG resistance noise are discussed below. In each case the NG noise is most naturally explained by DCR, rather than by the direct interaction of fluctuators.

First, NG resistance noise has been reported for $a$-Si:H films [3–5]. Transport by an interlocking network of filamentary conducting paths is required by the observation of telegraph fluctuations with $\delta R(t)/\langle R(t)\rangle \sim 0.01$

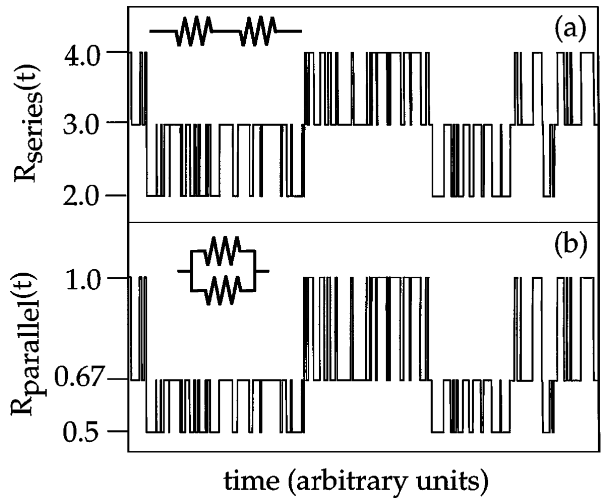


FIG. 1. A Monte Carlo simulation of the noise signal from series (a) and parallel (b) arrangements of two resistors that are statistically independent TLS's fluctuating between 1 and 2 a.u. with a ratio of characteristic frequencies of 30.

[4]. Parman *et al.* [5] report power-law behavior in $\mathbf{s}^{(2)}$ at low frequencies and infer long-range cooperative rearrangements of hydrogen-bonding configurations. The result may be more reasonably attributed to DCR with only shorter-range structural rearrangements of the material.

Second, farther from the percolation threshold, a large-dimensioned (0.2 mm long $\times$ 1 mm wide $\times$ 1 $\mu$m thick) granular carbon-composite resistor was recently found to have a large nonwhite $\mathbf{s}^{(2)}$ [19]. Fluctuator interactions spanning the dimensions of the sample would be unphysical, but large DCR is expected for such a system. Granular materials should be a valuable testing ground for the importance of DCR in resistance noise.

Third, the small fluctuation limit is inappropriate for doped crystalline semiconductors; fluctuations in the state of the charge trapping centers can have large effects on both the local carrier concentration and mobility [14]. An extreme case where DCR is important is for planar heterostructures with interfacial roughness. For example, inhomogeneous current distributions resulting from interface roughness in GaAs/AlGaAs tunnel structures are well documented [26,27]. The effective network for such a device consists of 2D sheets of low-resistance elements (representing the GaAs) connected by a low density of higher-resistance elements (representing the thinnest regions of the AlGaAs). As transport through the thin points in the barriers is sensitive to nearby charge trapping centers [26], large fluctuations of the local tunnel current are present and DCR is expected.

An experimental study of $\mathbf{s}^{(2)}$ for GaAs/$Al_{0.3}Ga_{0.7}As$ superlattices (SL's) is reported below. The samples studied were 50-period GaAs/$Al_{0.3}Ga_{0.7}As$ SL's with quantum well and barrier thicknesses of 80 and 20 Å, respectively. The barriers were $\delta$ doped with Si to achieve a final carrier concentration of $10^{17}$ $cm^{-3}$ in the quantum wells. The samples were mesas with cross section $\sim$ 100 $\mu$m $\times$ 100 $\mu$m. All measurements were performed with the applied bias in the growth direction, i.e., from the base of the SL mesa to its top. Previous TEM and transport measurements of devices from the same wafer demonstrated large interfacial roughness, resulting in serpentinelike current paths through the device [28,29]. The data reported here are in the low-bias Ohmic limit, far below the electric-field-induced localization regime [28,30]. The data presented below were obtained with standard constant-current dc four-probe noise measurements [12] at room temperature. The experimental arrangement and the computations involved in the calculation of $\mathbf{s}^{(2)}$ and of the expectation of $\mathbf{s}^{(2)}$ for Gaussian noise have recently been presented in detail [19].

The power spectrum for one of the SL mesas studied is shown in the inset of Fig. 3, for $I_{dc} = 0.5$ mA and $\langle V(t)\rangle = 151$ mV. The deviations from simple power law behavior are typical of doped semiconductors in which the noise is dominated by charge-trapping centers with well-defined characteristic activation energies

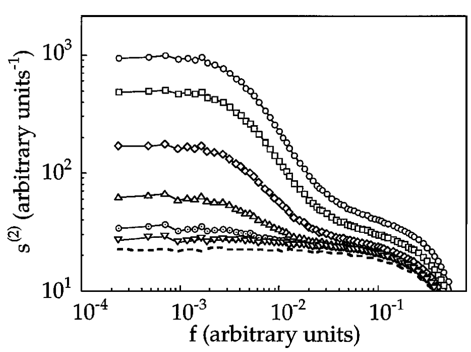


FIG. 2. Simulated $\mathbf{s}^{(2)}$ for the circuits of TLS resistors from Fig. 1. The chosen frequency band is an octave wide below the characteristic frequency of the faster TLS. The dashed curve is $\mathbf{s}^{(2)}$ for the series circuit. The remaining six curves, from top to bottom in the figure, are $\mathbf{s}^{(2)}$ for the parallel circuit with $\Delta r/\langle r\rangle = 0.8$, 0.4, 0.2, 0.1, 0.05, and 0.025, respectively.

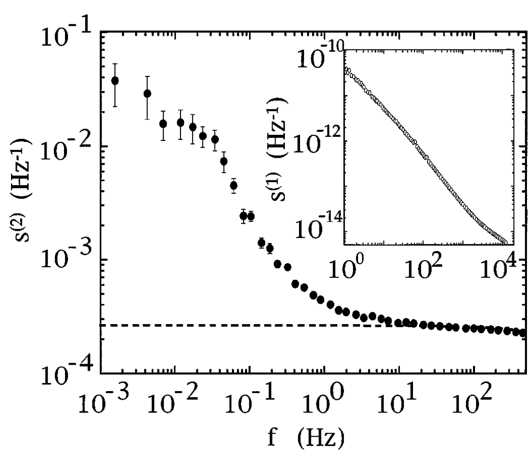


FIG. 3. Inset: The normalized power spectrum $\mathbf{s}^{(1)} = \mathbf{S}^{(1)}/\langle V\rangle^2$ for a GaAs/$Al_{0.3}Ga_{0.7}As$ SL, as described in the text. Main panel: $\mathbf{s}^{(2)}$ for the same device. The selected frequency band is (2.4 kHz, 12.0 kHz).

[12]. In the main part of Fig. 3, $\mathbf{s}^{(2)}$ is presented for a band $(f_L, f_H) = (2.4\ \text{kHz}, 12.0\ \text{kHz})$. The deviation from Gaussian behavior varies as $1/f_2^{1.5}$ down to $f_2 \sim 0.05$ Hz. Also, we note that the integrated power in nonoverlapping bands at high frequencies ($f > 1$ kHz) exhibits large covariances. This feature and its relation to DCR will be discussed elsewhere [31].

Measurements of a second mesa yielded qualitatively similar $\mathbf{s}^{(1)}$ and $\mathbf{s}^{(2)}$. It is prohibitively unlikely for the fluctuators contributing to the noise power in the chosen band to be located near enough to each other so as to have strong interactions with the same lower-frequency fluctuators. Again, the non-Gaussianity is explained by DCR for noninteracting fluctuators.

In conclusion, a mechanism by which a resistor composed of statistically independent microscopic fluctuators may exhibit strongly NG noise was demonstrated. When the local resistivity fluctuations are large or the transport is strongly inhomogeneous, the dynamical redistribution of the transport current gives rise to long-range correlations in the contribution to the total noise power by statistically independent fluctuators. Hence, higher-order statistics of $1/f$ resistance noise [2] which have previously been used to characterize fluctuator interactions in clean systems [7,8,12] may also be used to study conduction connectivity near the percolation threshold or the metal-insulator transition.

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[1] J. S. Bendat and A. G. Piersol, *Random Data* (John Wiley & Sons, New York, 1986), 2nd ed.
[2] P. J. Restle *et al.*, Phys. Rev. B **31**, 2254 (1985).
[3] I. Pocsik *et al.*, Solid State Commun. **76**, 103 (1990).
[4] C. E. Parman *et al.*, Phys. Rev. B **44**, 8391 (1991).
[5] C. E. Parman *et al.*, Phys. Rev. Lett. **69**, 1097 (1992).
[6] P. J. Restle *et al.*, Phys. Rev. B **34**, 4419 (1986).
[7] G. A. Garfunkel *et al.*, Phys. Rev. B **41**, 4901 (1990).
[8] N. E. Israeloff *et al.*, Phys. Rev. B **44**, 12 613 (1991); M. B. Weissman *et al.*, J. Magn. Magn. Mater. **114**, 87 (1993); M. B. Weissman, Rev. Mod. Phys. **65**, 829 (1993).
[9] A. C. Marley *et al.*, Phys. Rev. Lett. **74**, 3032 (1995).
[10] R. Rammal *et al.*, Phys. Rev. A **31**, 2662 (1985); R. Rammal *et al.*, Phys. Rev. Lett. **54**, 1718 (1985); A.-M. S. Tremblay *et al.*, Phys. Rev. B **33**, 2077 (1986); D. C. Wright *et al.*, Phys. Rev. B **33**, 396 (1986); P. M. Hui and D. Stroud, Phys. Rev. B **34**, 8101 (1986); R. Rammal and A.-M. S. Tremblay, Phys. Rev. Lett. **58**, 415 (1987); I. Balberg *et al.*, Phys. Rev. Lett. **60**, 1887 (1988); D. Lubin *et al.*, Phys. Rev. B **38**, 5899 (1988); A.-M. S. Tremblay *et al.*, Physica (Amsterdam) **157A**, 89 (1989); A. Kolek and A. Kusy, Phys. Rev. B **43**, 11 274 (1991).
[11] R. M. Cohn, Proc. Am. Math. Soc. **1**, 316 (1950).
[12] M. B. Weissman, Rev. Mod. Phys. **60**, 537 (1988).
[13] A. Van der Ziel Physica (Utrecht) **16**, 359 (1950); F. K. Du Pre, Phys. Rev. **78**, 615 (1950).
[14] P. Dutta *et al.*, Phys. Rev. Lett. **43**, 646 (1979).
[15] P. Dutta and P. M. Horn, Rev. Mod. Phys. **53**, 497 (1981).
[16] V. M. Vinokur and S. P. Obukhov, Sov. Phys. JETP **68**, 126 (1989).
[17] R. T. Wakai and D. J. Van Harlingen, Phys. Rev. Lett. **58**, 1687 (1987); K. S. Ralls and R. A. Buhrman, Phys. Rev. Lett. **60**, 2434 (1988).
[18] P. J. Restle, Ph.D. dissertation, University of Illinois, 1985.
[19] G. T. Seidler and S. A. Solin, Phys. Rev. B (to be published).
[20] B. D. H. Tellegen, Philips Res. Rep. **7**, 259 (1952).
[21] G. T. Seidler (unpublished).
[22] J. W. Essam, Rep. Prog. Phys. **43**, 833 (1980).
[23] L. M. Lust and J. Kakalios, Phys. Rev. Lett. **75**, 2192 (1995).
[24] J. L. Williams and R. K. Burdett, J. Phys. C **2**, 298 (1969); C. C. Chen and Y. C. Chou, Phys. Rev. Lett. **55**, 296 (1985); G. A. Garfunkel and M. B. Weissman, Phys. Rev. Lett. **55**, 296 (1985); J. V. Mantese and W. W. Webb, Phys. Rev. Lett. **55**, 2212 (1985); D. A. Rudman *et al.*, Phys. Rev. B **33**, 1456 (1986); M. Octavio *et al.*, Phys. Rev. B **36**, 2461 (1987); R. H. Koch *et al.*, Phys. Rev. B **32**, 6932 (1985).
[25] The boundary conditions at the vertices of a fluctuating element can be satisfied by a current source and sink on the fluctuating element [N. Wingreen (private communication)].
[26] P. M. Campbell *et al.*, Phys. Rev. Lett. **67**, 1330 (1991).
[27] T. Judd *et al.*, Appl. Phys. Lett. **49**, 1652 (1986); R. E. Cavicchi and M. B. Panish, J. Appl. Phys. **67**, 873 (1990); E. S. Snow *et al.*, Appl. Phys. Lett. **56**, 117 (1990).
[28] M. Lee *et al.*, Phys. Rev. B **48**, 11 921 (1993).
[29] M. Lee *et al.*, Solid State Commun. **89**, 687 (1994).
[30] M. Lee and S. A. Solin, Solid State Commun. **83**, 673 (1992).
[31] G. T. Seidler *et al.* (unpublished).