Interaction of Ultrasound with Vortices in Type-II Superconductors

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(Received 14 September 1995)

The theory of ultrasound in the mixed state of type-II superconductors is suggested which takes into account the Magnus force on vortices, the anti-Magnus force on ions, and diamagnetism of the mixed state. The acoustic Faraday effect (rotation of polarization of the transverse ultrasonic wave propagating along vortices) is linear in the Magnus force in any regime of the flux flow for wavelengths now used in the ultrasound experiments. Therefore, in contrast to previous predictions, the Faraday effect should be looked for only in clean superconductors with a strong Magnus force.

PACS numbers: 74.60.Ge

Investigation of ultrasound propagation in the mixed state of a type-II superconductor has proved to be an effective method of studying low-T_c and high-T_c superconductors [1,2]. Recently Dominguez *et al.* [3] have attracted attention to an interesting manifestation of interaction between the ultrasound and vortices: the *acoustic Faraday effect*. The effect results from a force on ions transverse to the ion velocity: the velocity of the transverse sound propagation depends on the sign of circular polarization and the plane of linear polarization should rotate when the sound wave propagates along vortices.

However, the equations of Dominguez *et al.* [3] (see also [4]) are not Galilean invariant and do not satisfy the momentum-conservation law for the whole system "ions + electrons" since they failed to include in the ion equation of motion the so-called anti-Magnus force which is especially important for dirty superconductors. In the present work I use dynamic equations which are free from these deficiencies. The correct theory predicts other functional dependences on the physical parameters and different conditions for observation of the Faraday effect. In particular, our analysis does not confirm the conclusion of Dominguez *et al.* on the strong Faraday effect without the Magnus force in dirty superconductors at high temperatures and predicts a many orders of magnitude weaker effect at low temperatures. Finally, for the ultrasound wavelengths used in the experiments the best conditions for observation of the acoustic Faraday effect are at high temperatures in clean superconductors. The present work includes the theory of Refs. [1,2] as a particular case, but generalizes the theory on a larger domain of physical parameters: the nonzero Magnus force (clean superconductors), low magnetic fields, where the diamagnetism of the mixed state is essential, wavelengths short compared to the London penetration depth.

We adopt the following picture of electron and ion motion induced by the ultrasound. The electron liquid consists of two parts: normal and superfluid. The normal component is effectively clamped to the crystal ions by viscous forces responsible for the normal resistance: they move together with the ion velocity $\vec{v}_i = d\vec{u}_i/dt$ ($\vec{u}_i$ is

the ion displacement). In contrast, the superfluid electrons move with the superfluid velocity $\vec{v}_s$ which is different from the ion velocity in general. Such a picture holds until the magnetic field is weak compared to the upper critical magnetic field H_{c2} and the flux-flow resistance is much less than the normal resistance. Then the normal current proportional to the velocity of the normal electrons with respect to ions may be neglected.

Thus our three-component system (ions, normal electrons, and superfluid electrons) becomes effectively two component as in the two-fluid model for superfluids: there is a superfluid with the velocity $\vec{v}_s$ and the mass density $m_e n_s$ and a heavy normal fluid with velocity $\vec{v}_i$ and mass density $m_i n + m_e(n - n_s) \approx m_i n$. The charge densities of these two components are $e n_s$ and $-e n_s$. Here e is the electron charge, m_e and m_i are the electron and ion masses, and n_s and n are the superfluid electron and total charge number densities, respectively. We can write the equations of motion for the superfluid and the normal fluid immediately using a close analogy with the two-fluid hydrodynamics for rotating superfluids [5]:

$$\frac{\partial \vec{v}_s}{\partial t} = \frac{e}{m_e}\left[\vec{E} + \frac{1}{c}(\vec{v}_L \times \vec{B})\right]. \tag{1}$$

$$\frac{\partial^2 \vec{u}_i}{\partial t^2} - c_t^2 \vec{\nabla}^2 \vec{u}_i = -\frac{e}{m_i}\frac{n_s}{n}\left\{\vec{E} + \frac{1}{c}[\vec{v}_i \times \vec{B}] + \frac{1}{c}[(\vec{v}_L - \vec{v}_{sl}) \times \vec{B}]\right\}, \tag{2}$$

Here c_t is the "bare" sound velocity ignoring interaction with superfluid electrons, $\vec{B}$ is the magnetic induction, Φ_0 is the magnetic flux quantum, $\vec{v}_L = d\vec{u}_L/dt$ is the vortex velocity, $\vec{u}_L$ is the vortex displacement, and $\vec{v}_{sl}$ is the *local* superfluid velocity at the points of the vortex line which differs from the *average* superfluid velocity $\vec{v}_s$ due to deformations of the vortex lattice. If the transverse sound wave propagates along vortices (the axis $\hat{z}$), then

$$\vec{v}_{sl} = \vec{v}_s + \frac{c}{e n_s B}C_{44}^*\left(\hat{z} \times \frac{\partial^2 \vec{u}_L}{\partial z^2}\right). \tag{3}$$

The renormalized tilt modulus C_{44}^* relates only to the vortex line tension, without including the elastic energy of the average magnetic field [6]. For an isotropic superconductor not very close to H_{c1} $C_{44}^* = (\Phi_0 B/4\pi\lambda^2)\ln(a/r_c)$ (for an anisotropic case, see, e.g., [7] and references therein). Here $a \sim \sqrt{\Phi_0/B}$ and r_c are the intervortex distance and the vortex core radius, respectively.

The electrical field $\vec{E}$ and the magnetic field $\vec{h}$ generated by the ultrasound satisfy the Maxwell equations:

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial\vec{h}}{\partial t}, \qquad \frac{4\pi}{c}\vec{j} = \vec{\nabla} \times \vec{h}, \qquad (4)$$

where $\vec{j} = en_s(\vec{v}_s - \vec{v}_i)$ is the average electric current. The supercurrent $\vec{j}_s = en_s\vec{v}_s$ satisfies the London equation:

$$\frac{4\pi\lambda^2}{c}\vec{\nabla} \times \vec{j}_s = \vec{b}_v - \vec{h}. \qquad (5)$$

Here $\lambda = \sqrt{m_e c^2/4\pi e^2 n_s}$ is the London penetration depth and $\vec{b}_v$ is the ac component of the vortex field [8] (the vortex induction in [9]) which coincides with $\vec{h}$ only in a uniform vortex lattice. For the transverse sound propagating along vortices $\vec{b}_v = B\partial\vec{u}_L/\partial z$.

We need also the equation of vortex motion which connects three velocities $\vec{v}_i$, $\vec{v}_{sl}$, and $\vec{v}_L$:

$$-\eta(\vec{v}_L - \vec{v}_i) + \eta'[\hat{z} \times (\vec{v}_L - \vec{v}_i)]$$
$$= \pi\hbar n_s[\hat{z} \times (\vec{v}_{sl} - \vec{v}_i)]. \qquad (6)$$

Equation (6) generalizes the equation of the vortex motion known for a crystal at rest: the velocities $\vec{v}_L$ and $\vec{v}_{sl}$ are replaced by the relative velocities $\vec{v}_L - \vec{v}_i$ and $\vec{v}_{sl} - \vec{v}_i$ in accordance with Galilean invariance. The Lorentz force on the right-hand side is balanced by the viscous force $\propto \eta$ and the Magnus force $\propto \eta'$. The parameter η' was known to vary from $\pi\hbar n_s$ for superclean superconductors [10] to 0 for dirty superconductors [11]. However, η' may even exceed $\pi\hbar n_s$, being equal $\pi\hbar n$, when the Iordanskii force is essential (see Sec. X of Ref. [5]).

Equations (1)–(6) are a closed system of equations averaged over the vortex-array cell. Equation (1) for electrons does not differ from the case of the ion lattice at rest. As for Eq. (2) for ions, I shall show now that the force from vortices in this equation (the last term on the right-hand side) is required by the momentum conservation and the third Newton law.

If there is no external forces on the electron superfluid, the Helmholtz theorem tells that $\vec{v}_L = \vec{v}_{sl}$ [5]. This means that $\eta = 0$ and $\eta' = \pi\hbar n_s$. Bearing this in mind, one can present $\vec{v}_L$ as $\vec{v}_L = \vec{v}_{sl} + (\vec{v}_L - \vec{v}_{sl})$ on the right-hand side of Eq. (1). Then the contribution $\propto [(\vec{v}_L - \vec{v}_{sl}) \times \vec{B}]$ is an external force due to interaction with the normal fluid. Correspondingly, the same force must appear in the ion (normal fluid) equation (2), but with the opposite sign. In superfluid hydrodynamics this force is called *mutual friction force* [5]. But for superconductors it is better to call it *coupling force,* since the force may incorporate not only friction, but also elastic pinning (see below). The coupling force, being external for the superfluid component, is internal for the system ions + electrons as a whole. Therefore this force does not contribute to the time variation of the total momentum

$$\frac{\partial}{\partial t}\left[m_i n\vec{v}_i + m_e(n - n_s)\vec{v}_i + m_e n_s\vec{v}_s\right] \approx m_i n\frac{\partial\vec{v}_i}{\partial t} + m_e n_s\frac{\partial\vec{v}_s}{\partial t}$$
$$= m_i n c_t^2\vec{\nabla}^2\vec{u}_i + \frac{1}{c}\left[\vec{j}_l \times \vec{B}\right] = \frac{\partial}{\partial z}\left(m_i n c_t^2\frac{\partial\vec{u}_i}{\partial z} + \frac{B\vec{h}}{4\pi} + C_{44}^*\frac{\partial\vec{u}_L}{\partial z}\right). \qquad (7)$$

Here $\vec{j}_l = en_s(\vec{v}_{sl} - \vec{v}_i)$ is the *local* current on the vortex line, in contrast to the *average* current $\vec{j} = en_s(\vec{v}_s - \vec{v}_i)$ in the Maxwell equations (4). Equation (7) demonstrates the momentum conservation law for the case when the wave propagates along vortices (the axis z): the time variation of the momentum is determined by the divergence of the momentum flux which consists of (i) the stress tensor of the ion lattice, (ii) the stress tensor of the magnetic field linearized with respect to the ac field $\vec{h}$, and (iii) the vortex-lattice stress tensor given by the tilt modulus C_{44}^*. In the long-wavelength limit $\lambda k \to 0$ (see below) the term $\propto \partial\vec{v}_s/\partial t$ in the momentum variation may be neglected, and Eq. (7) has a simpler form,

$$m_i n(\partial^2\vec{u}_i/\partial t - c_t^2\vec{\nabla}^2\vec{u}_i) = \frac{1}{c}\left[\vec{j}_l \times \vec{B}\right], \qquad (8)$$

used by Shapira and Neuringer [1]. But they neglected a difference between the local and average electric current responsible for the diamagnetism of the mixed state.

Replacing the coupling force in the equation of ion motion Eq. (2) with the help of Eq. (6) one obtains

$$\frac{\partial^2\vec{u}_i}{\partial t^2} - c_t^2\vec{\nabla}^2\vec{u}_i = -\frac{e}{m_i}\frac{n_s}{n}$$
$$\times \left\{\vec{E} + \frac{1}{c}[\vec{v}_i \times \vec{B}] - \frac{B}{c}\tilde{\eta}(\vec{v}_L - \vec{v}_i)\right.$$
$$\left. - \frac{\alpha_M - 1}{c}[(\vec{v}_L - \vec{v}_i) \times B]\right\}, \qquad (9)$$

where $\tilde{\eta} = \eta/\pi\hbar n_s$ and $\alpha_M = \eta'/\pi\hbar n_s$ are the dimensionless amplitudes of the viscous and the Magnus force, respectively. The coupling force has a component $\propto (\alpha_M - 1)$ transverse to the ion velocity which is maximal in a dirty superconductor when the Magnus force vanishes ($\alpha_M = 0$). Indeed, suppression of the Magnus force in the dirty superconductor means that impurities produce a force which cancels the Magnus force on vortices, i.e.,

on superfluid electrons. Then in accordance with the third Newton law one must expect that the force of opposite sign acts from vortices on impurities, i.e., on crystal ions. This explains the force $\propto (\alpha_M - 1)$ which may be called the anti-Magnus force.

Later on we assume that the transverse ultrasound plane wave $\propto \exp(ikz - i\omega t)$ propagates along the vortices. From Eqs. (3)–(5) one can derive relations connecting the electric field and the local superfluid velocity with the ion and the vortex velocities and displacements:

$$\vec{E} = -\frac{m_e}{e} \omega^2 \vec{u}_i - \frac{1}{1 + \lambda^2 k^2} \frac{1}{c} [\vec{v}_L \times \vec{B}], \qquad (10)$$

$$\vec{v}_{sl} = \frac{\vec{v}_i}{1 + \lambda^2 k^2} - \frac{c}{e n_s B} C_{44}(k) k^2 [\hat{z} \times \vec{u}_L]. \qquad (11)$$

where now

$$C_{44}(k) = \frac{B^2}{4\pi} \frac{1}{1 + \lambda^2 k^2} + C_{44}^* \qquad (12)$$

is the k-dependent tilt modulus related to the total energy of deformation [12]. The contribution $\propto \omega^2$ to the electric field $\vec{E}$ will be neglected later on, since it is not connected with vortices. It yields a small correction to the sound velocity of the relative order $m_e n_s / m_i n$ which is present even in the Meissner state.

Finally, with the help of Eqs. (6), (10), and (11), one can rewrite the equation of the ion motion, Eq. (2), in terms of the ion velocity and displacement only:

$$-(\omega^2 - c_t^2 k^2) m_i n \vec{u}_i = \frac{e n_s B}{c} [f_\parallel \vec{v}_i + f_\perp (\hat{z} \times \vec{v}_i)]. \qquad (13)$$

The longitudinal and the transverse forces on ions from vortices are given by the dimensionless force parameters:

$$f_\parallel = \frac{Dk^2}{i\omega} - \frac{1}{(\tilde{\eta} - Dk^2/i\omega)^2 + \alpha_M^2}$$
$$\times \left\{ \left(\tilde{\eta} - \frac{Dk^2}{i\omega} \right) \times \left[\left(\frac{\lambda^2 k^2}{1 + \lambda^2 k^2} \right)^2 - \left(\frac{Dk^2}{i\omega} \right)^2 \right] \right.$$
$$\left. + 2\alpha_M \frac{\lambda^2 k^2}{1 + \lambda^2 k^2} \right\}, \qquad (14)$$

$$f_\perp = \frac{2\lambda^2 k^2}{1 + \lambda^2 k^2} + \frac{1}{(\tilde{\eta} - Dk^2/i\omega)^2 + \alpha_M^2}$$
$$\times \left\{ 2 \left(\tilde{\eta} - \frac{Dk^2}{i\omega} \right) \frac{\lambda^2 k^2}{1 + \lambda^2 k^2} \frac{Dk^2}{i\omega} \right.$$
$$\left. + \alpha_M \left[\left(\frac{Dk^2}{i\omega} \right)^2 - \left(\frac{\lambda^2 k^2}{1 + \lambda^2 k^2} \right)^2 \right] \right\}. \qquad (15)$$

Here $D = cC_{44}/e n_s B$.

The previous experimental and theoretical investigations [1–3] dealt with the long-wavelength case $\lambda k \to 0$. One may call it the electrodynamic limit since in this case all forces from vortices on ions can be expressed in terms of the electrodynamic parameters: the Ohmic and the Hall conductivities, $\sigma_O = \eta c^2/\Phi_0 B$, $\sigma_H = \eta' c^2/\Phi_0 B$,

related to the viscous and the Magnus force, respectively, and the magnetic permeability

$$\mu = \frac{B^2/4\pi}{B^2/4\pi + C_{44}^*} = \frac{B^2}{4\pi C_{44}(0)}, \qquad (16)$$

which describes the diamagnetism due to circular currents over the vortex-lattice cell [6]. Here $C_{44}(0)$ is the tilt modulus in the limit $k \to 0$ (the Labusch modulus). Equation (16) yields the differential magnetic permeability $\mu = B/H$ for the case when the variation of the magnetic field is normal to the vortices. Here $H = (1/4\pi)\partial F(B)/\partial B$ is the thermodynamic magnetic field along the equilibrium magnetization curve. Equation (6) of vortex motion is simply Ohm's law

$$\vec{j}_l = \sigma_O \vec{E}_i - \sigma_H(\hat{z} \times \vec{E}_i), \qquad (17)$$

which connects the local current $\vec{j}_l = \vec{j}/\mu$ and the electric field $\vec{E}_i = \vec{E} + (1/c)[\vec{v}_i \times \vec{B}]$ in the coordinate frame moving with the ion velocity. Then Eq. (13) may be derived from Eq. (8) together with Ohm's law [Eq. (17)] and the relation $\vec{E} = (4\pi i\omega/c^2 k^2)\vec{j} = (4\pi i\omega\mu/c^2 k^2)\vec{j}_l$ which follows from the Maxwell equations (4). This yields the following values of the force parameters:

$$f_\parallel = \frac{B}{e n_s c} \left(\frac{c^2 k^2}{4\pi i\omega\mu} \right)^2$$
$$\times \left(\frac{4\pi i\omega\mu}{c^2 k^2} + \frac{\sigma_O - c^2 k^2/4\pi i\omega\mu}{(\sigma_O - c^2 k^2/4\pi i\omega\mu)^2 + \sigma_H^2} \right), \qquad (18)$$

$$f_\perp = \frac{B}{e n_s c} \left(\frac{c^2 k^2}{4\pi i\omega\mu} \right)^2 \frac{\sigma_H}{(\sigma_O - c^2 k^2/4\pi i\omega\mu)^2 + \sigma_H^2}. \qquad (19)$$

In the electrodynamic limit $\lambda k \ll 1$ our theory is valid in a wider interval of the magnetic fields than in the case of arbitrary λk. One may use the equations averaged over the vortex-array cell until the wavelength $2\pi/k$ exceeds the intervortex distance $a \sim \sqrt{\Phi_0/B}$. But because of $\lambda \ll 1/k$, this condition may be satisfied even if $a \gg \lambda$, i.e., rather close to the lower critical field H_{c1} where the diamagnetism is important; i.e., μ is small. On the other hand, in the electrodynamic approach our assumption in the beginning of this Letter that the current of normal electrons is negligible is not necessary: one can use the conductivity tensor taking into account this current. Then the theory is valid even close to H_{c2}.

For the sake of simplicity we did not include the elastic pinning force into our analysis explicitly, but it is easy to do simply by replacing the viscous coefficient η by $\eta - \alpha_P/i\omega$ in all equations. Here α_P is the bulk pinning coefficient which may, in principle, depend on the frequency as assumed by Dominguez et al. [3]. Thus our analysis holds for any regime of vortex motion, either the nonactivated flux flow or the thermally assisted flux flow (TAFF) with flux jumps over pinning barriers. But

different regimes of the flux flow correspond to different expressions for the conductivities.

One can obtain the results of Refs. [1,2] from Eqs. (18) and (19) neglecting the Magnus force and the finite diamagnetism of the mixed state ($\sigma_H \sim \alpha_M = 0$, $\mu = 1$). Note that the magnetic permeability μ of Shapira and Neuringer [1] relates to the atomic magnetism, whereas the latter was ignored in the present work and our μ is due to the circular currents in the vortex-lattice cell and the difference between the local and average currents as a result of them. However, finally the role of μ in the equations is similar in both cases, as one might expect for an electrodynamic theory.

Our analysis seriously differs from that of Dominguez *et al.* [3]: (i) They missed the anti-Magnus force in the equation of ion motion. This caused violation of the momentum conservation law and the wrong prediction for the Faraday effect in the dirty superconductors in which the anti-Magnus force is especially important. (ii) In Ref. [3] the equation of vortex motion was not Galilean invariant, since it contained the vortex velocity in the laboratory frame as if the latter were a preferential frame. But for our problem the only preferential coordinate system is one that moves with the ion velocity.

Experiment and theory [2] have shown that in a Bi superconductor there is a crossover between two temperature regions: the low-temperature region of high Ohmic conductivity due to high activation barriers in the TAFF model where $\omega \tilde{\eta}/Dk^2 = c_t \tilde{\eta}/Dk = 4\pi \omega \mu \sigma_O/c^2 k^2 > 1$, and high-temperature regions of low conductivity where $4\pi \omega \mu \sigma_O/c^2 k^2 < 1$. The predictions of Ref. [3] must be revised for both regions. The Faraday rotation (the angle of polarization rotation per unit length) is given by

$$d\theta/dz = \frac{en_s B}{2m_i nc_t c}\,\mathrm{Re} f_\perp . \qquad (20)$$

At low temperatures Dominguez *et al.* [3] obtained that $d\theta/dz = \sigma_H B^2/2m_i nc_t c^2$, whereas Eqs. (19) and (20) yield the Faraday rotation by the factor $(c^2 k^2/4\pi \omega \mu \sigma_O)^2$ smaller. This factor is of order unity at the crossover temperature $T \sim 60$ K, but exponentially small at low temperatures since $\sigma_O \propto \exp(U/T)$ where $U \sim 500$ K for typical magnetic fields [2]. At higher temperatures Dominguez *et al.* obtained the Faraday rotation $d\theta/dz = eB/2m_i c_t c$ which did not depend on the Hall conductivity. This meant that the Faraday effect was possible in a dirty superconductor without the Magnus force even in the limit $\lambda k \to 0$ in disagreement with Refs. [1,2]. Dominguez *et al.* explained it by the effect of the electromagnetic force $\propto [\vec{E} + \frac{1}{c}(\vec{v}_i \times \vec{B})]$ in Eq. (9), which was neglected in Ref. [2]. However, as explained after Eq. (9), this equation should include also the anti-Magnus force which cancels the electrodynamic force in the limit $\lambda k \to 0$ restoring the result of Ref. [2]. According to Eqs. (15) and (20) the Faraday rotation without the Magnus force ($\alpha_M \sim \sigma_H = 0$) is

$$\frac{d\theta}{dz} = \frac{en_s B}{m_i nc_t c}\,\frac{\lambda^2 k^2}{1 + \lambda^2 k^2}\,\frac{1}{1 + (c^2 k^2/4\pi \omega \mu \sigma_O)^2} . \qquad (21)$$

For the frequency 3 MHz from Ref. [2] this expression yields at $T > 60$ K the Faraday rotation at least by a factor of 10^{-5} smaller than predicted by Dominguez *et al.* [3]. So the Faraday effect in dirty superconductors is possible only at $\lambda k \gtrsim 1$ when the frequency of ultrasound is very high (about a few GHz). For the frequencies used in the experiments now, the best conditions for observation of the Faraday effect are clean superconductors with large Hall angle at high temperatures. Then the Faraday rotation can be strongly amplified close to the resonances with vortex modes [zeros of the denominator in Eqs. (18) and (19)], which exist in superclean high-T_c superconductors [7,13]. Thus the ultrasound experiment could reveal these modes [14].

In summary, the theory of interaction between the ultrasound and the vortices in the type-II superconductors has been suggested. For the first time the theory takes into account the Magnus force on vortices, the anti-Magnus force on ions, and the diamagnetism of the mixed state self-consistently. This results in a serious revision of previous predictions concerning the acoustic Faraday effect. Despite this revision, our analysis confirms that possible observation of the acoustic Faraday effect is expected to provide a valuable information on vortex dynamics in type-II superconductors.

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