Experimental Test of the Quantum Shot Noise Reduction Theory

A. Kumar, L. Saminadayar, and D. C. Glattli
Service de Physique de l'Etat Condensé, Centre d'Etudes de Saclay, F-91191 Gif-sur-Yvette, France

Y. Jin and B. Etienne
Laboratoire de Microstructures et Microélectronique, Centre National de la Recherche Scientifique, B.P. 107, F-92225 Bagneux, France

(Received 14 December 1995)

The quantum suppression of shot noise predicted for mesoscopic conductors is observed using absolute measurements of the equilibrium and nonequilibrium electrical fluctuations of a quantum point contact. The small energy (30−600 mK) and low frequency (1−10 kHz) used for measurements allow for a reliable quantitative experimental confirmation of the quantum noise theory. The noise reduction factor is found to be in excellent agreement with theoretical expectations, evolving from nearly unity at low electronic wave transmission to nearly zero on a conductance plateau.

PACS numbers: 72.70.+m, 05.30.Fk, 73.23.Ps, 73.40.Lq

Shot noise refers to the time-dependent fluctuations in the electrical current as a consequence of the *particle* nature of the electrons. In a quantum conductor, it is expected to provide information complementary to the current which probes the transmission of electron *waves*. In particular, shot noise is sensitive to the carrier statistics. A recent striking prediction is that the Pauli exclusion principle *correlates* the flow of electrons participating in the current I [1–6]. As a result, the spectral density of current fluctuations is reduced below [1] the Poissonian value $S_I = 2eI$, which characterizes noise in classical conductors like vacuum diodes [7]. The aim of this work is to show the quantum suppression of Poissonian noise and to provide an accurate experimental test of the quantum noise reduction theory in mesoscopic systems.

A quantum conductor, defined over the size of the carrier coherence length l_ϕ, elastically scatters the waves of the incoming carriers emitted by the left and right leads, thus producing electrical resistance. The multichannel Landauer formula [8] relates the conductance $G = 2e^2/h \sum_n \mathcal{T}_n$ to the transmission $\mathcal{T}_n$ of the nth propagating mode at the Fermi energy. Associated with scattering, a partition noise is generated because a carrier emitted by a reservoir in an initial incoming state is statistically scattered into one of the final outgoing states. The complete shot noise is in general the combination of the thermal emission noise of the reservoirs and of the partition noise regulated by Fermi statistics. At zero temperature, however, the flow of carriers emitted by the reservoirs becomes noiseless and shot noise reaches the *partition noise limit.* For a single conduction channel, a noise reduction factor $1 - \mathcal{T}_1$ from the Poissonian value $S_I = 2eI$ is expected with vanishing noise for unit transmission. For many channels the reduction factor is $\sum_n \mathcal{T}_n(1 - \mathcal{T}_n)/(\sum_n \mathcal{T}_n)$ [1,2]. For a diffusive conductor it is $1/3$ on average [3]. For a macroscopic conductor of size L a further (classical) reduction l_ϕ/L occurs as each region of length equal to l_ϕ produces uncorrelated noise. At finite temperature T, the complete quantum shot noise, written as equivalent Johnson-Nyquist noise temperature $T^* = S_I/4Gk_B$, is [4,5]

$$T^* = T\left[1 + \frac{\sum_n \mathcal{T}_n(1 - \mathcal{T}_n)}{\sum_n \mathcal{T}_n}\left(\frac{eV_{DS}/2k_BT}{\tanh(\frac{eV_{DS}}{2k_BT})} - 1\right)\right], \tag{1}$$

where eV_{DS} is the difference in chemical potential between reservoirs. Note that Eq. (1) reduces to Johnson-Nyquist noise $S_I = 4Gk_BT$ for $T \gg eV_{DS}/2k_B$. For $eV_{DS}/2k_B \gg T$, the noise increases linearly with voltage (or current) and the reduction factor is given by the asymptotic slope of T^* versus $eV_{DS}/2k_B$.

While theory is now well advanced, few experiments are available. Low frequency measurements [9] have found indications of noise reduction using a quantum point contact (QPC), a ballistic quantum conductor which offers good control of the conduction channel transmission $\mathcal{T}_n$. However, they failed to observe the characteristic linear variation of S_I with current because of $1/f$ or telegraphic conductance noise. Shot noise has been found in a diffusive conductor with a reduction factor close to $1/3$, but in the incoherent transport regime [10]. Reduced shot noise has been recently observed in a QPC at high frequency using 10 GHz bandwidth measurements [11]. The oscillatory variation of the noise observed for moderate transmission qualitatively follows predictions but, at low transmission, the recovery of full shot noise was lacking probably due to the high voltage bias used for measurements. Up to now a quantitative test of the quantum reduction of shot noise was missing.

The present experiment fills this gap and provides an experimental validation of the theory. A noise correlation technique allows us to perform absolute Johnson-Nyquist and shot noise measurements with noise temperature resolution of 10 mK on a QPC realized in a 2D electron gas (2DEG). The measurement energy range, 50 times smaller than used in previous experiments, and the kHz frequency meet the requirement of the linear transmission regime and dc limit for reliable comparison with theory. The shot noise, white over at least one frequency decade, varies linearly with current for voltage $V_{DS} > 4k_BT/e$. It shows a crossover to Johnson-Nyquist noise for $k_BT > eV_{DS}/2$ in excellent agreement with the predictions. The shot noise, Poissonian at low transmission $\mathcal{T}_1$, shows the expected $1 - \mathcal{T}_1$ reduction at larger transmission. For $\sum_n \mathcal{T}_n > 1$, electron heating becomes important but can be quantitatively included. Good agreement with predictions for an ideal QPC is recovered with a small magnetic field improving the conductance quantization and the 2D thermal conduction.

The QPC is realized by electron beam lithography using a split gate scheme [12] in a high mobility Si δ-doped GaAs/Ga(Al)As heterojunction. The 100 nm deep 2DEG of density 1.1×10^{15} m^{-2} and mobility 200 m^2 V^{-1} s^{-1} has a Fermi energy of 45 K and elastic mean free path $l_e \simeq 12$ μm. The measurements detect the spectral density of the voltage fluctuations across the QPC at fixed transmission for a series of fixed currents [13]. They are realized in the spirit of noise thermometry. First, at zero current, the equilibrium noise is measured while sweeping the temperature from 30 to 600 mK. The accurate knowledge of the voltage gain provides a check that the Johnson-Nyquist relation $S_V = 4k_BT/G$ is obeyed for the mesoscopic conductor within a few percent accuracy. Then, at fixed temperature, the current is swept and S_V is converted into temperature using the previous calibration. The absolute noise measurements are limited only by the thermometer accuracy and statistical errors.

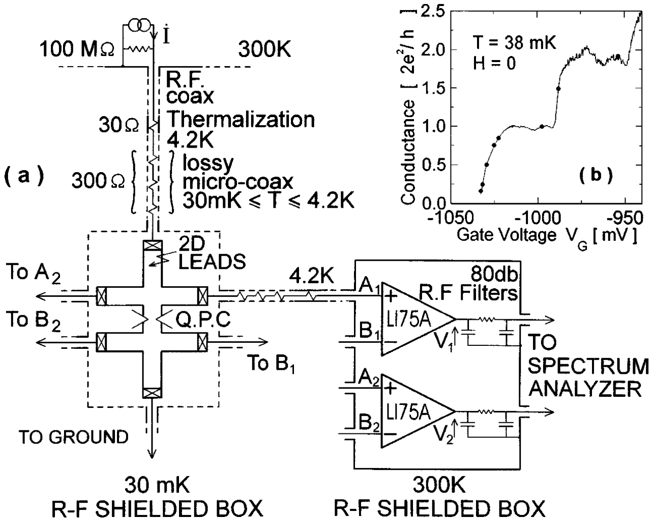

FIG. 1. (a) Schematic measurement circuit. (b) QPC conductance vs gate voltage for $H = 0$. The black points show the values of G and gate voltage where noise is studied.

To ensure accuracy and reliability, the voltage fluctuations of two symmetric pairs of contacts located 20 μm from the QPC are independently measured by two ultralow noise amplifiers and a spectrum analyzer calculates the cross-correlation spectrum of the outputs as shown in Fig. 1(a). This technique removes from the detected signal the uncorrelated voltage noise of the amplifiers and the thermal noise of the leads. Using the calibration above, the noise temperature T^* can be reliably extracted from the total detected signal $4k_BT^*/G + S_I^c/G^2$. Here S_I^c is the small white current noise of the external circuit not removed by correlation. Typically, $S_I^c/4Gk_B \simeq 450$ mK for $G = 2e^2/h$. Drifts in G and S_I^c, less than 1%, lead to errors below the statistical errors $\simeq T_N(\Delta f \tau)^{-1/2}$ $\simeq$6 mK for a bandwidth Δf acquisition time τ product of 2×10^5 and detector noise temperature T_N ($T_N = 2.3$ K for $G = 2e^2/h$). The measurements, in the kHz frequency range, show negligible $1/f$ noise because of the small currents and temperatures used. Indeed, the conductance noise spectral density $S_G \sim k_BT/f$ [14] gives a $1/f$ to shot noise ratio scaling as Ik_BT/f: By using currents and temperatures 50 times less than in previous experiments, the 100 kHz $1/f$ corner frequency typically observed [9] is lowered below 100 Hz. Finally, an important point contributing to the success of the experiment is the efficient low temperature photon filtering, from radiofrequency (100 MHz) to far infrared (THz) range. Lossy microcoaxes for all leads and a shielded low temperature experimental box result in a photon temperature close to the sample temperature [15]. This filtering ensures thermalization of electron traps contributing to $1/f$ noise and reduces the broadband voltage fluctuations from the hot parts of the external circuit which on microscopic time scales may affect electron transport [16,17].

Figure 1(b) shows the two first plateaus of the QPC conductance versus gate voltage measured at 38 mK and zero magnetic field. The black points correspond to the fixed values of G where the noise is studied. Here, the QPC conductance is calculated from an ac lock-in measurement of the resistance. The differential resistance changes by less than 5% as the dc current is varied maintaining the bias voltage V_{DS} less than 100 μV. For the following noise measurements where $eV_{DS}/2k_B <$ 600 mK this ensures that the transmission is nearly energy independent. But, for better accuracy, the differential conductance is monitored during noise measurements and kept constant using gate voltage feedback.

Figure 2 shows the results of noise measurements for a conductance e^2/h corresponding to a transmission $\mathcal{T}_1 = 1/2$ of the first mode. The voltage fluctuation spectral density S_V, averaged over a 500 Hz bandwidth around 6 kHz, is plotted versus the dc current. The three series of experimental points correspond to sample temperatures of 38, 80, and 180 mK. At low temperature, the noise varies linearly with current within the statistical accuracy. The slope of $T^* = S_VG/4k_B$ plotted against $eV_{DS}/2k_B$ is found to be 0.49. It is smaller than the unit slope expected for Poissonian shot noise and very close to the

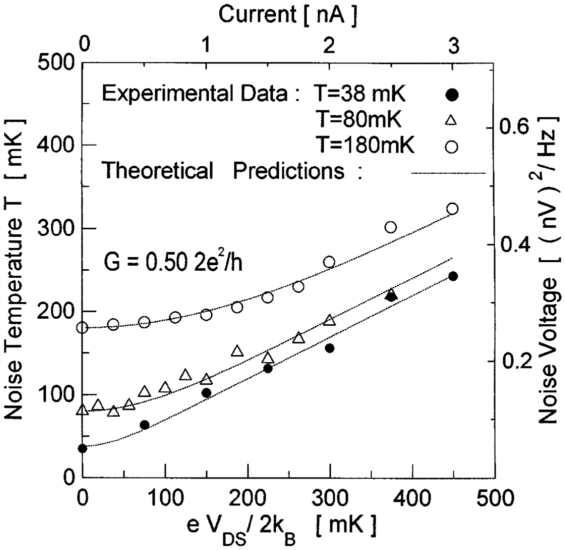

FIG. 2. Spectral density of the QPC voltage fluctuations, also expressed as noise temperature, for transmission $\mathcal{T}_1 = 0.5$ and for $T = 38$, 80, and 180 mK, as a function of the current or of the average voltage expressed in relevant temperature units. The dotted lines, not fits, are predictions of Eq. (1).

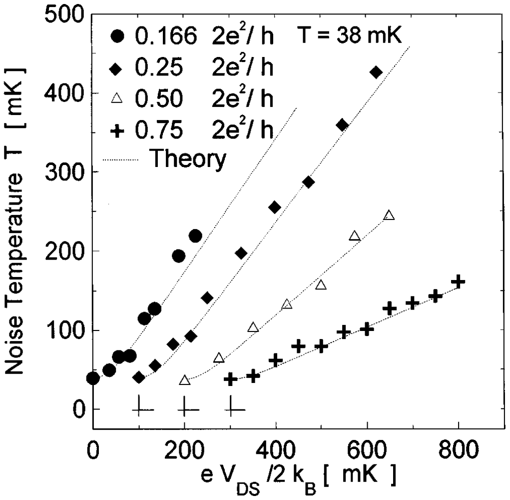

FIG. 3. QPC noise temperature vs bias in temperature units for $G/(2e^2/h) = 1/6$, $1/4$, $1/2$, and $3/4$ at fixed $T = 38$ mK. For clarity, data for different G are offset by 100 mK. The dotted lines are not fits but predictions of Eq. (1).

predicted value of $1 - \mathcal{T}_1 = 0.5$. At 4 and 9 kHz a similar study gives values of 0.50 and 0.52, respectively. This consistency accurately confirms that the noise is white over this frequency range. When increasing the temperature the zero bias noise increases according to the Johnson-Nyquist formula. For finite bias, the variation with I shows the crossover from a quadratic to nearly linear law when $V_{DS} \simeq 2k_BT/e$. This demonstrates the predicted transition from Johnson-Nyquist to shot noise. We emphasize that the three dotted curves are not fits but a comparison of the data with Eq. (1) using $\mathcal{T}_1 = 1/2$ (there is no adjustable parameter). *The agreement with theoretical expectations, within the calculable statistical deviations, is nearly perfect.*

The transition from nearly suppressed shot noise to nearly classical shot noise is well demonstrated by Fig. 3, which shows the data in temperature units for transmissions $\mathcal{T}_1 = 3/4$, $1/2$, $1/4$, and $1/6$ at low temperature. When decreasing $\mathcal{T}_1$, the slope of the linear variation with current increases as predicted, in contrast with the high bias result of Ref. [11]. From the best slopes using only the data for $eV_{DS} > 3k_BT$, one finds noise reduction factors of 0.25, 0.49, 0.74, and 0.86, respectively. The excellent agreement with the $1 - \mathcal{T}_1$ prediction is also demonstrated by the comparison with the dotted curves calculated using Eq. (1). This is also shown by Fig. 4 where the reduction factor (filled triangle) is plotted as a function of the QPC conductance normalized to $2e^2/h$, i.e., $\sum_n \mathcal{T}_n$. The dotted curve is the calculation for an ideal QPC where, when increasing transmission from zero, each mode is successively transmitted. The first four points agree with this ideal picture. When increasing the transmission, an additional source of noise becomes important. As shown later, the deviation from the ideal picture is, however, strongly reduced upon applying a small perpendicular magnetic field H.

The extra noise found for $H = 0$ at large transmission arises from electron heating and also probably from mode mixing. Heating, in zero field, is easy to predict and can be included quantitatively. The corresponding increase of thermal noise varies linearly with current in the regime of electronic thermal conduction and can be mistaken for shot noise [6,18]. Indeed, below 200 mK the thermalization length of electrons with phonons is larger than the sample size [19]. Therefore a heat flux $I^2/2G$ flows on each side of the QPC through the 2D leads up to the Ohmic contacts by electronic thermal conduction. According to the Wiedemann-Franz law, the electron temperature T_e defined within a few electron-electron collision lengths l_{in} is given by $T_e^2 = T^2 + (24/\pi^2)(G/G_m)(1 + 2G/G_m)(eV_{DS}/2k_B)^2$, where G_m is the conductance of all 2D leads in parallel. Here, Joule heating in the leads has been included and the metallic Ohmic contacts are assumed to be at the lattice temperature T. One easily sees that for a conductance plateau ($\mathcal{T}_n = 0$ or 1), the Johnson-Nyquist noise is no longer constant with current but asymptotically increases linearly with bias, like shot noise, with a noise temperature $T^* = T_e \simeq (24G/\pi^2G_m)^{1/2}(1 + G/G_m)^{1/2}(eV_{DS}/2k_B)$. More generally, replacing T by $T_e(V_{DS}, T)$ in Eq. (1) gives for any transmission an effective increase of the noise reduction factor for large conductance. The dashed curve in Fig. 4 gives the new reduction factor for an ideal QPC calculated using the experimental value of $G_m = 0.004\ \Omega^{-1}$

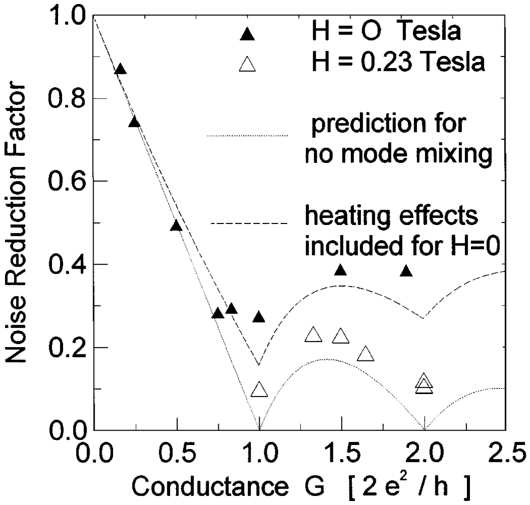

FIG. 4. Noise reduction factor vs the conductance for $H = 0$ (filled triangles) and $H = 0.23$ T (open triangles). Predictions for no mode mixing without (dotted curve) and with (dashed curve) the calculable heating effects for $H = 0$.

known with 20% accuracy. The quantitative account for heating effects considerably improves the agreement with the data. The remaining deviations can be attributed to mode mixing. Indeed, the $\simeq 0.1(2e^2/h)$ modulation of the conductance plateaus observed below 200 mK are characteristic of a nonideal QPC. Mode mixing gives more freedom to an electron to be partitioned and thus increases noise. For example, for a conductance $G = 2e^2/h$ obtained with $\mathcal{T}_1 = 0.9$ and $\mathcal{T}_2 = 0.1$ the noise suppression is only 82%.

The suppression of the additional noise upon applying a magnetic field, even weak, is the result of two effects. First, mode mixing is reduced because of the rapid increase of the 1D subband energy separation. Indeed for a field of 0.23 T the first and second plateaus already show less than 2% modulation. Second, the field corresponds to a Shubnikov–de Haas resistance minima of the leads $\simeq 1/10$ the zero field resistance (filling factor 20). The estimation of the resulting increase of the 2D thermal conduction tells us that heating effects are strongly reduced. However, the fact that the resistance of each lead is not known in detail precludes us from a quantitative description of heating in this regime. The agreement between the data in magnetic field and the prediction for an ideal QPC is, however, satisfying.

In summary, absolute noise measurements at very low energies and in the dc limit have been performed using the simplest mesoscopic system, a QPC. Two key predictions of mesoscopic noise theory—the suppression of Poissonian shot noise and the evolution with temperature from shot noise to Johnson-Nyquist noise—have been convincingly validated. While the scattering picture of quantum transport has been well tested through conductance measurements, the present work shows that, within reasonable experimental accuracy, this picture is also the correct one to accurately describe noise, a fundamentally different physical quantity. The method of noise measurements based on correlations has proven to be very efficient and can be easily reproduced provided care is taken about rf noise filtering and heating effects. We thus expect that in the future noise study will become more common to provide complementary information on mesoscopic conduction not given by conductance.

We thank P. Jacques for valuable technical help, P. Pari and P. Forget for cryogenic support, P. Grousset for help in measurements, and M. H. Devoret, R. Landauer, G. Lesovik, and Th. Martin for stimulating discussions.

---

[1] G. B. Lesovik, Pis'ma Zh. Eksp. Teor. Fiz. **49**, 513 (1989) [Sov. Phys. JETP Lett. **49**, 592 (1989)].
[2] M. Büttiker, Phys. Rev. Lett. **65**, 2901 (1990).
[3] C. W. J. Beenakker and M. Büttiker, Phys. Rev. B **46**, 1889 (1992).
[4] Th. Martin and R. Landauer, Phys. Rev. B **45**, 1742 (1992); Physica (Amsterdam) **175B**, 167 (1991).
[5] M. Büttiker, Phys. Rev B **46**, 12 485, (1992).
[6] M. J. M. de Jong, thesis, University of Leiden, 1995.
[7] W. Schottky, Ann. Phys. (Leipzig) **57**, 541 (1918).
[8] R. Landauer, IBM J. Res. Dev. **1**, 223 (1957); **32**, 306 (1988).
[9] Y. P. Li *et al.,* Appl. Phys. Lett. **57**, 774 (1990); S. Washburn *et al.,* Phys. Rev. B **44**, 3875 (1991).
[10] F. Liefrink *et al.,* Phys. Rev. B **49**, 14 066 (1994).
[11] M. Reznikov *et al.,* Phys. Rev. Lett. **75**, 3340 (1995).
[12] B. J. van Wees *et al.,* Phys. Rev. Lett. **60**, 848 (1988); D. A. Wharam *et al.,* J. Phys. C **21**, L209 (1988).
[13] The high impedance seen by the QPC, 33 MΩ below 10 Hz, decreases with frequency to reach $\simeq 50\ \Omega$ above 50 MHz, and to provide a voltage source for electronic frequencies.
[14] F. N. H. Robinson, *Noise and Fluctuations in Electronic Devices and Circuits* (Clarendon Press, Oxford, 1974), p. 76.
[15] D. C. Glattli *et al.* (to be published).
[16] The sum of the thermal voltage fluctuations of the external circuit (impedance $Z_e \simeq 50\ \Omega$ and temperature Θ) is $\pi(Z_e 2e^2/3h)^{1/2} k_B \Theta/e = 0.112 k_B \Theta/e$ and must be $\ll k_B T/e$. Here, $\Theta \ll 0.1$ K meets this requirement.
[17] G. B. Lesovik, JETP Lett. **60**, 815 (1994).
[18] A. Steinbach, J. Martinis, and M. H. Devoret, Bull. Am. Phys. Soc. **40**, 400 (1995). They observe a similar heating effect in large metallic resistors. They have recently found *pure shot noise* using samples smaller than l_ϕ (to be published).
[19] For thermal coupling with phonons, see, for example, Y. Ma *et al.,* Phys. Rev. B **43**, 9033 (1991).