Large Time Out-of-Equilibrium Dynamics of a Manifold in a Random Potential

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We study the out-of-equilibrium dynamics of an elastic manifold in a random potential using mean-field theory. We find two asymptotic time regimes: (i) stationary dynamics; (ii) slow aging dynamics with violation of equilibrium theorems. We obtain an analytical solution valid for all large times with universal scalings of two-time quantities with space. A nonanalytic scaling function crosses over to ultrametricity when the correlations become long range. We propose procedures to test numerically or experimentally the extent to which this scenario holds for a given system.

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The dynamics of an elastic manifold in a quenched random potential is relevant for a large number of experimental systems. Examples are flux lattices in high-T_c superconductors [1], interfaces in random fields [2], charge density waves, and surface growth on disordered substrates [3]. The competition between elasticity and disorder produces a "glass" state with pinning, slow dynamics, and nonlinear macroscopic response (e.g., leading to zero linear resistivity in superconductors [1]). While there is a phenomenological picture [1] based on scaling arguments (droplets) no satisfactory analytical approach is available at present for the low-temperature dynamics.

The *statics* of a d-dimensional elastic manifold embedded in a N-dimensional space in the presence of a random potential was studied by Mézard and Parisi (MP) who applied a replica variational Gaussian approximation (Hartree) for N finite which becomes exact at $N = \infty$ [4]. The replica symmetry breaking (RSB) solution captures some of the essential physics in finite N dimension, such as sample-to-sample susceptibility fluctuations [4,5], and predicts the nontrivial (Flory-like) roughness exponent ζ; it allows for a theory [6] of the statics of the vortex glass state in superconductors relevant to experiments. Other analytical approaches are based on renormalization group (RG) methods [6–8] and it is as yet unclear whether they capture all the physics [9]. Despite the obvious interest of the static approach, it applies by construction to equilibrium (Gibbs measure) properties, which may not hold for experimental times in a glassy system [10].

In this Letter we study, also within the Hartree approximation, the dynamics of this problem starting from a random configuration as in a temperature quench. We find that at low enough temperature there is an aging regime and that the system never reaches equilibrium. Correlation functions depend not only on time differences but also on the waiting time after the quench. We obtain two-time scaling with explicit space dependence, a new feature with respect to mean-field analytical results for glassy systems obtained so far [11–14]. Details will be presented elsewhere [15].

This treatment is exact for $N = \infty$. At finite N it holds *a priori* only within the Gaussian variational ansatz. Whether the properties hold qualitatively for true finite N-dimensional models cannot be answered analytically at present. Good qualitative agreement [11] of the mean field dynamical analytical solution with experiments on spin glasses suggests that our results for $N = \infty$ may be relevant for some systems related to the present model.

Our main purpose is to suggest, on the basis of the exact solution for $N = \infty$, *definite* predictions for the nonequilibrium dynamics which can be checked numerically and experimentally. Our results provide a basis for a finite-N analysis. Remarkably, despite the system being out of equilibrium, some of the results of the MP replica calculation, e.g., ζ, are shown to carry through to the dynamics, albeit with a different interpretation in terms of directly observable time-dependent physical quantities.

The model of a manifold of internal dimension d embedded in a random medium of dimension N is described, in terms of an N-component displacement field $\boldsymbol{\phi}$, $\boldsymbol{\phi} = (\phi_1, \phi_2, \ldots, \phi_N)$, by the Hamiltonian [4]

$$H = \int d^d x \left[\frac{1}{2} (\nabla \boldsymbol{\phi})^2 + V(\boldsymbol{\phi}(\boldsymbol{x}), \boldsymbol{x}) + \frac{\mu}{2} \boldsymbol{\phi}^2 \right].$$

μ is a mass, which effectively constrains the manifold to fluctuate in a restricted volume of the embedding space. V is a Gaussian random potential with correlations

$$\overline{V(\boldsymbol{\phi}, \boldsymbol{x}) V(\boldsymbol{\phi}', \boldsymbol{x}')} = -N \delta^d(\boldsymbol{x} - \boldsymbol{x}') \mathcal{V}((\boldsymbol{\phi} - \boldsymbol{\phi}')^2/N).$$

We consider the Langevin dynamics $\partial_t \boldsymbol{\phi} = -\delta_\phi H + \boldsymbol{\eta}$ with $\langle \eta_\alpha(\boldsymbol{x}, t) \eta_\beta(\boldsymbol{x}', t') \rangle = 2T \delta_{\alpha\beta} \delta^d(\boldsymbol{x} - \boldsymbol{x}') \delta(t - t')$. We let the system evolve from a spatially translationally invariant (STI) configuration at $t = 0$. It remains STI at subsequent times. We use the dynamical Hartree approximation, exact for $N \to \infty$, and which, for N finite, amounts to substituting $\mathcal{V}$ by an effective $\hat{\mathcal{V}}$ [4,14]. The "equilibrium" dynamics (*à la* Sompolinsky [16]) was studied for general d in [17]. $d = 0$ was studied in [12] and an analytical solution at large times was given in [14].

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The quantities of interest in the large-time off-equilibrium dynamics are the two-time correlation $C_{xx'}(t,t') = 1/N\overline{\langle \boldsymbol{\phi}(x,t)\cdot\boldsymbol{\phi}(x',t')\rangle}$ and the response $R_{xx'}(t,t') = 1/N\overline{\delta\langle\boldsymbol{\phi}(x,t)\rangle/\delta\boldsymbol{f}(x',t')}|_{f=0}$ where $\boldsymbol{f}(x',t')$ is a small perturbation applied at the space point x' at time t'. We also define the mean squared displacement $D_{xx'}(t,t') = 1/N\overline{\langle[\boldsymbol{\phi}(x,t) - \boldsymbol{\phi}(x',t')]\rangle}$ and the correlation $B_{xx'}(t,t') = 1/N \times \overline{\langle[\boldsymbol{\phi}(x,t) - \boldsymbol{\phi}(x,t')][\boldsymbol{\phi}(x',t) - \boldsymbol{\phi}(x',t')]\rangle}$. The brackets and overline represent the average over the thermal noise and the quenched disorder, respectively. The Fourier transforms B_k and R_k in the space difference $x - x'$ are used below, and we denote with tilde equal space ($x = 0$) two-time functions, $\tilde{B} = \int_k B_k$.

A common choice is $\mathcal{V}(z) = (\theta + z)^{1-\gamma}/2(1-\gamma)$: "short-range" (SR) correlations correspond to $\gamma(1 - d/2) > 1$ and "long range" (LR) to $\gamma(1 - d/2) < 1$. The static solution [4] is characterized by two exponents: in the LR case, an ultrametric ansatz gives $D_x^{\rm st} \sim x^{2\zeta}$ with a roughness exponent $\zeta_{\rm LR} = (2 - d/2)/(1 + \gamma)$ and a free-energy fluctuation exponent θ. In the SR case a one step RSB ansatz gives $\zeta_{\rm SR} = (2 - d)/2$. Statistical rotational symmetry imposes $\theta = 2\zeta + d - 2$. The $d = 2$ sine-Gordon model is marginal and solved [6] by a one step RSB.

Aging.—Let us describe the picture that emerges for large times and $\mu > 0$. Equal-time quantities reach their asymptotic values which do not necessarily coincide with the equilibrium ones [11,15]. For two-time quantities we consider two different regimes of the times.

(i) After a large waiting time t_w, $B_k(\tau + t_w, t_w)$ first grows with τ in a manner independent of t_w, from 0 up to the Edwards-Anderson parameter for the mode k defined as $b_k^1 \equiv \lim_{\tau\to\infty}\lim_{t_w\to\infty} B_k(\tau + t_w, t_w)$. In this time regime the displacement is time-translation invariant (TTI); we denote it $b_k^F(\tau) = \lim_{t_w\to\infty} B_k(\tau + t_w, t_w)$. The response $r_k^F(\tau) = \lim_{t_w\to\infty} R_k(\tau + t_w, t_w)$ satisfies fluctuation dissipation theory (FDT) $r_k^F(\tau) = (1/2T)\partial_\tau b_k^F(\tau)\theta(\tau)$. The "FDT regime" is very much like an equilibration in a state; the manifold looks pinned with an effective mass $\overline{M}$, since $b_k^1 = 2T/(k^2 + \overline{M})$ (for $\mu \to 0$).

(ii) However, for all t_w and sufficiently large τ, $B_k(\tau + t_w, t_w)$ continues to grow beyond b_k^1 up to $b_k^0 \equiv \lim_{\tau\to\infty} B_k(\tau + t_w, t_w)$. The growth of B_k now depends on t_w: the larger t_w the slower the motion of the system, it *ages*. Thus pinning is gradual: the older the system the longer it is pinned but it is not pinned forever. The aging time regime is defined (for $t_w \to \infty$) as the times τ such that $B_k(\tau + t_w, t_w) > b_k^1$. We denote by $b_k(\tau + t_w, t_w)$ and $r_k(\tau + t_w, t_w)$ the displacement and response in the aging regime, where both TTI and FDT are violated.

As regards the measurements of noise and susceptibility (i) corresponds to high frequencies while (ii) corresponds to low frequencies (scaling with the waiting time) such that noise and susceptibility depend on t_w. In a domain-growth process (i) corresponds to the fast thermal fluctuations of the spins around their mean magnetization while (ii) corresponds to the actual growth of the domains. An important measurable quantity is the susceptibility $\chi_k(t,t') = \int_{t'}^t ds\, R_k(t,s)$ of the mode k, i.e., the total linear response to an external force of spatial modulation k applied during the interval $[t', t]$. The results below imply $\chi_k(t_w + \tau, t_w) = k^{-2}F[k^2\chi_0(t_w + \tau, t_w)]$, where $\chi_0(t_w + \tau, t_w)^{-1}$ is a "running" effective mass which exclusively depends on times through the local displacement, $\chi_0 = \chi_0[\tilde{b}]$, and goes to zero at large time separation τ. The typical internal distance $\sim\sqrt{\chi_0} \sim \tilde{b}^{1/\zeta}$ grows slower with τ as the age t_w increases.

We study $\mu \to 0$ *after* the large-time limit. If one takes $\mu = 0$ from the start, one must take into account diffusion: $B_{xx}(t, 0) \to \infty$ at large t since $\int dk\, b_k^0 = \infty$. In addition to the *aging* regime defined as t, $t' \to \infty$ with $B_{xx}(t,t')$ fixed, there is then a *diffusion* regime where $B_{xx}(t,t')/B_{xx}(t,0)$ is fixed [15]. Our results for B_k and R_k are expected to hold also at $\mu = 0$ in the aging regime.

Generalization of equilibrium theorems, two-time scalings.—Let us describe in more detail the aging regime as derived below. For large times, the precise manner in which TTI is violated is described by "triangle" relations [11] involving any three times

$$B_k(t_{\min}, t_{\max}) = f_k(B_k(t_{\min}, t_{\rm int}), B_k(t_{\rm int}, t_{\max}))\,, \qquad (1)$$

while the violation of FDT is given by

$$R_k(t,t') = X_k[B_k(t,t')]\partial_{t'}B_k(t,t')\,, \qquad (2)$$

where (2) means that X_k depends on the times only through $B_k(t,t')$. In order to complete this ansatz, we have to specify how f_k and X_k for different k are related to one another. This is done below in an algebraic manner, implying no new hypotheses with respect to the $d = 0$ case [11,14]. We find that

$$B_k(t,t') = B_k[k, B_0(t,t')]\,, \qquad (3)$$

i.e., all k modes depend on times *only through* the dependence of one of them. Thus one can use any two-time function, e.g., $B_0(t,t')$ or $\tilde{B}(t,t')$, to parametrize the two-time dependence. We also show

$$X[B_k] = X_k[B_k[k, B_0]] = X_0[B_0] = \tilde{X}[\tilde{B}]\,, \qquad (4)$$

i.e., the values of all X_k are the same for times such that B_0 takes the same value. We have also defined $\tilde{X}[\tilde{b}(t,t')] = \tilde{r}(t,t')/\partial_{t'}\tilde{b}(t,t')$. Equation (4) implies that if X is time independent in the aging regime, then it takes the same value for all k. Hence, for the k-mode susceptibility we have $\chi_k(t,t') = \chi_k[k, \chi_0(t,t')]$. These functional dependences are testable using two-time parametric plots. Their explicit forms are determined below.

We find the following: (i) in the FDT regime $B_k < b_k^1$, $X_k[B_k] = -1/2T$ and (ii) in the aging regime $b_k^0 > b_k > b_k^1$ there are two distinct cases. For SR correlations $X_k(b_k) = X$ and f_k has the form $f_k(u,v) = \jmath_k^{-1}(\jmath_k(u)\jmath_k(v))$. This implies $B_k(t,t') = \jmath_k^{-1}(h(t')/h(t))$ where $h(t)$ is increasing and independent of k. For LR correlations $X_k(b_k)$ is a nonconstant function of b_k. The function f_k is $f_k(u,v) = \max(u,v)$.

Formal solution for the spatial scaling.—In order to extract the scaling properties and justify the above ansatz, we encode the correlation and response functions in the superspace order parameter [18]. At the saddle point, using causality $\boldsymbol{Q}_{xx'}(1,2) = C_{xx'}(t_1,t_2) + (\overline{\theta}_2 - \overline{\theta}_1)[\theta_2 R_{xx'}(t_1,t_2) + \theta_1 R_{x'x}(t_2,t_1)]$. The θ's are Grassmann variables and we denote $1 \equiv (t_1,\theta_1,\overline{\theta}_1)$, $d1 \equiv dt_1 d\theta_1 d\overline{\theta}_1$, $\boldsymbol{\delta}(1-2) \equiv (\theta_2-\theta_1)(\overline{\theta}_2-\overline{\theta}_1)\delta(t_2-t_1)$, and $D^{(2)}(1) \equiv \partial_{\theta_1}(\partial_{\overline{\theta}_1} - \theta_1\partial_{t_1})$. We use two types of functions of the superorder parameter: "operator" as in $\boldsymbol{Q}^2_{xx'}(1,2) = \int d3\boldsymbol{Q}_{xx'}(1,3)\boldsymbol{Q}_{xx'}(3,2)$ and "pointwise" as in $\boldsymbol{Q}^{\bullet 2}_{xx'}(1,2) = [\boldsymbol{Q}_{xx'}(1,2)]^2$.

The equation of motion, exact for $N = \infty$, is

$$\left(D^{(2)} - \nabla^2 + \mu + \int d3\, \mathcal{V}'^{\bullet}(B_{xx}(1,3))\right)\boldsymbol{Q}_{xx'}(1,2) - \delta^d(x-x')\boldsymbol{\delta}(1-2) - 2[\mathcal{V}'^{\bullet}(\boldsymbol{B}_{xx})\boldsymbol{Q}_{xx'}](1,2) = 0, \quad (5)$$

where $\boldsymbol{B}_{xx'}(1,2) = \boldsymbol{Q}_{xx'}(1,1) + \boldsymbol{Q}_{xx'}(2,2) - 2\boldsymbol{Q}_{xx'}(1,2)$.

Using space translational invariance one finds that all Fourier modes $\boldsymbol{Q}_k$ can be expressed in terms of the zero mode $\boldsymbol{Q}_{k=0} \equiv \boldsymbol{Q}_0$ through the operator relation

$$\boldsymbol{Q}_k(1,2) = [k^2\boldsymbol{\delta} + \boldsymbol{Q}_0^{-1}]^{-1}(1,2). \quad (6)$$

This scaling relation encodes the correlation and response functions with two times and space. Substituting Eq. (6) into Eq. (5) one can write a separate equation for each mode and map the problem of each $B_k(t,t')$, $R_k(t,t')$ into an effective $d = 0$ problem with a complicated memory kernel [19]. Up to now we have made no approximations. Note that one can now solve numerically the equation for one of the modes and recover the space dependence algebraicly from Eq. (6). This implies that an ansatz as in problems without space dependence [11,14] applies to each mode and justifies Eqs. (1) and (2).

Now, in the large time limit any operator function $F[\boldsymbol{Q}_A]$ of an order parameter $\boldsymbol{Q}_A$, which can be parametrized by f_A, X_A, yields a new $\boldsymbol{Q}_B = F[\boldsymbol{Q}_A]$ which can also, at long times, be parametrized with f_B, X_B. The explicit computation of f_B, X_B in terms of f_A, X_A has been done [13] and when applied to Eq. (6) yields explicit functional relations between $\boldsymbol{Q}_k$, $\boldsymbol{B}_k$, and $\boldsymbol{Q}_0$ and, in particular, Eqs. (3) and (4) for the components B_k, R_k.

Explicit calculations.—We can now establish the large time equations in both regimes. In the FDT regime $r_k^F = -x_F\partial_\tau b_k^F(\tau)$ with $x_F = -1/2T$ and one finds

$$\frac{db_k^F(\tau)}{d\tau} = 2T - [k^2 + \overline{M} + 4x_F\mathcal{V}'(\tilde{b}^1)]b_k^F(\tau) + 4x_F\frac{d}{d\tau}\int_0^\tau d\tau'\, \mathcal{V}'(\tilde{b}^F(\tau-\tau'))b_k^F(\tau'). \quad (7)$$

Neglecting the time derivatives on the left hand side of the full dynamical equations and integrating over the FDT regime (see [11,14]) one finds the equations for the aging regime

$$0 = r_k(t,t')(k^2 + \overline{M}) + \frac{2b_k^1}{T}\mathcal{V}''(\tilde{b}(t,t'))\tilde{r}(t,t') + 4\int_{t'}^t ds\, \mathcal{V}''(\tilde{b}(t,s))\tilde{r}(t,s)r_k(s,t'), \quad (8)$$

$$\begin{aligned}0 = &- b_k(t,t')(k^2 + \overline{M}) + \frac{2b_k^1}{T}[\mathcal{V}'(\tilde{b}^1) - \mathcal{V}'(\tilde{b}(t,t'))] + 4\int_0^t ds[\mathcal{V}'(\tilde{b}(t,s))r_k(t,s) + \mathcal{V}''(\tilde{b}(t,s))\tilde{r}(t,s)b_k(t,s)] \\ &+ 2T - 4\int_0^{t'} ds[\mathcal{V}'(\tilde{b}(t,s))r_k(t',s) + \mathcal{V}''(\tilde{b}(t,s))\tilde{r}(t,s)b_k(t',s)] - 4\int_{t'}^t ds\, \mathcal{V}''(\tilde{b}(t,s))\tilde{r}(t,s)b_k(s,t'). \quad (9)\end{aligned}$$

$\overline{M} \equiv -4\lim_{t\to\infty}\int_0^t ds\, \mathcal{V}''(\tilde{b}(t,s))\tilde{r}(t,s)$ is the "anomaly." These equations have time-reparametrization invariance which prevent determining $h(t)$. "Quasistatic" values $\tilde{b}^1$, b_k^1 follow from letting $t' \to t_-$ in Eq. (9):

$$b_k^1 = 2T/(k^2 + \overline{M}). \quad (10)$$

Similarly, Eq. (8) integrated over k yields either the high-T solution $\tilde{r}(t,t_-) = 0$ or the low-T condition

$$1 = -4\mathcal{V}''(\tilde{b}^1)\int_k (k^2 + \overline{M})^{-2}. \quad (11)$$

This implies $\overline{M} = [-4c_d\mathcal{V}''(\tilde{b}^1)]^{2/(4-d)}$ and $\tilde{b}^1 = -4Tc_d/(d-2)[-4c_d\mathcal{V}''(\tilde{b}^1)]^{(d-2)/(4-d)}$ for $d < 4$, where $c_d = \int_k (k^2+1)^{-2}$. Thus it is not necessary to know the details of the aging solution to determine b_k^1, $\tilde{b}^1$, and $\overline{M}$.

FDT regime.—Defining $\phi(\tau) = 4[\mathcal{V}'(\tilde{b}^F(\tau)) - \mathcal{V}'((\tilde{b}^1))]$, the Laplace transform in τ of Eq. (7) yields

$$b_k^F(s) = (x_F s)^{-1}[k^2 + \overline{M} + s - x_F s\phi(s)]^{-1}. \quad (12)$$

At small τ, $b_k^F(\tau) \sim [1 - \exp(-A_k\tau)]/x_F A_k$ with $A_k = k^2 + \overline{M} - x_F\phi(0)$. When $b_k^{\rm int} \sim 1/xA_k$ there is a crossover to a slower regime where one can neglect the term $s\ (d/d\tau)$ and find a power law behavior: $b_k^F(\tau) = b_k^1 - c(b_k^1)^2\tau^{-\beta}$ with β determined by [17] $\Gamma[1-2\beta]\Gamma[1-\beta]^{-2} = x_F Y(\tilde{b}^1)$ and $Y(\tilde{b}^1) = 4\mathcal{V}''(\tilde{b}^1)^2\mathcal{V}'''(\tilde{b}^1)^{-1} \times \partial_{\overline{M}}\ln\int_k (k^2 + \overline{M})^{-2}$. An explicit calculation gives $Y(\tilde{b}^1) = 4/\tilde{X}(\tilde{b}^1)$ in terms of the function $\tilde{X}(\tilde{b})$ defined by Eq. (17).

Aging regime in short-range models.—Power law models are short range for $\gamma > \gamma_c = 2/(2-d)$ and $d \le 2$. With the ansatz $X_k[b_k(t,t')] = X$ Eqs. (8) and (9) reduce to a single equation for $b_k(t,t')$. One must have $X = -\overline{M}/4\mathcal{V}'(\tilde{b}^1)$. As discussed above $b_k(t,t') = j_k^{-1}(h(t')/h(t))$. Defining $u = \ln h(t)$ one has $b_k(t,t') = \mathcal{B}_k(u - u')$ where $0 < u < \infty$ and $\mathcal{B}_k(0) = b_k^1$ and $\mathcal{B}_k(\infty) = b_k^0$.

Remarkably, we obtain

$$0 = 2T - [k^2 + \overline{M} + 4X\mathcal{V}'(\tilde{b}^1)]\mathcal{B}_k(u) + 4X\frac{d}{du}\int_0^u du'\,\mathcal{V}'(\tilde{\mathcal{B}}(u-u'))\mathcal{B}_k(u') + 4(X - x_F)[\mathcal{V}'(\tilde{b}^1) - \mathcal{V}'(\tilde{\mathcal{B}}(u))],$$

which is formally similar to Eq. (7) though in a completely different variable. Laplace transforming in u one gets (with $\phi(u) = 4[\mathcal{V}'(\tilde{\mathcal{B}}(u)) - \mathcal{V}'((\tilde{b}^1))]$)

$$\mathcal{B}_k(s) = b_k^1 + (Xs)^{-1} \times \{(k^2 + \overline{M})^{-1} - [k^2 + \overline{M} - Xs\phi(s)]^{-1}\}.$$

At the beginning of the aging regime $u \ll 1$, we obtain $\mathcal{B}_k(u) = b_k^1 - 4\mathcal{V}''(\tilde{b}^1)(k^2 + \overline{M})^{-2}u^\alpha$, and thus for $t' \sim t$

$$\tilde{b}(t,t') - \tilde{b}^1 \sim \ln^\alpha[h(t)/h(t')] \sim c(t_w)(\tau/t_w)^\alpha, \quad (13)$$

where $c(t_w) = [d\ln h(t_w)/d\ln t_w]^\alpha$. If $h(t) = t^\delta$, we recover the trap-model scaling where $c(t_w)$ is a constant [20]. The exponent α is determined by

$$\Gamma[1 + 2\alpha]\Gamma[1 + \alpha]^{-2} = XY(\tilde{b}^1). \quad (14)$$

For the power law model, $xY(\tilde{b}^1) = (4 - d)\gamma/2(1 + \gamma)$ which $\alpha \to 0$ when $\gamma \to \gamma_{\rm cr}^+ = 2/(2 - d)$ and shows how ultrametricity in $\gamma < \gamma_{\rm cr}$ is approached.

At widely separated times $u \to \infty$, the approach to b_k^0 is described by a scaling form of k^2u:

$$b_k(t,t') = b_k^1\left(1 - \frac{x_F}{X}\right) - \frac{1}{Xk^2}\left[1 - \left(\frac{h(t')}{h(t)}\right)^{k^2/XZ}\right],$$

where $Z = \int du\,\phi(u)$ is a constant, finite for $\gamma > \gamma_{\rm cr}$. Integrating over k we also obtain the large time separation behavior $\tilde{b}(t,t') \propto \ln^{1-d/2}[h(t)/h(t')]$.

Aging regime for long-range models.—These models are solved by the ultrametric ansatz $b_k(t,t') = \max[b_k(t,s), b_k(s,t')]$, which inserted into (8) and (9) leads [15] to a single equation parametrized by $\tilde{b}$,

$$\chi_k(t,t') = \chi_k[\tilde{b}] = [k^2 + \overline{M} - \overline{M}(\tilde{b})]^{-1} \quad (15)$$

after simple manipulations as in [14]. We define $M[\tilde{b}] = -4\int_{\tilde{b}}^{\tilde{b}^1}\mathcal{V}''(\tilde{b}')\tilde{X}[\tilde{b}']d\tilde{b}'$ and use (2)–(4). Taking a derivative with respect to $\tilde{b}$, using $\partial_{\tilde{b}}\chi_k[b_k] = \tilde{X}[\tilde{b}]\partial_{\tilde{b}}b_k$, dividing by $\tilde{X}[\tilde{b}]$ and integrating over $\tilde{b}$, one gets

$$b_k = b_k^1 + \int_{\tilde{b}}^{\tilde{b}^1} d\tilde{b}'\frac{4\mathcal{V}''(\tilde{b}')}{[k^2 + \overline{M} - \overline{M}(\tilde{b}')]^2}. \quad (16)$$

This implies the self-consistency condition $1 = 4\mathcal{V}''(\tilde{b})\int_k[k^2 + \overline{M} - \overline{M}(\tilde{b})]^{-2}$ which coincides with the marginality condition for the replicon. One obtains

$$\tilde{X}(\tilde{b}) = -a_d\mathcal{V}'''(\tilde{b})[-\mathcal{V}''(\tilde{b})]^{2(d-3)/(4-d)} \quad (17)$$

with $a_d = (4 - d)(4c_d)^{2/(4-d)}/2$. This result, derived in a rather simple way, is formally identical to the result of the statics [4] and is here *shown to apply directly to the nonequilibrium dynamics*. The self-energy of the statics $[\sigma](u)$ is thus formally identified with $\overline{M} - \overline{M}(\tilde{b})$.

One general prediction for the aging regime of manifolds is the scaling form

$$b(x,t,t') = b_{x=0}(t,t')H[x/b_{x=0}(t,t)^{1/2\zeta}], \quad (18)$$

which holds at large x and large, widely separated times. The Hartree method yields a remarkably simple analytical form. Using (16) one finds [15] $\partial_{\tilde{b}}b_k = -4\mathcal{V}''(\tilde{b})/\{k^2 + [-4\mathcal{V}''(\tilde{b})c_d]^{2/4-d}\}^2$, e.g., in real space in $d = 3$, $\partial_{\tilde{b}}b_x = \exp[\mathcal{V}''(\tilde{b})x/2\pi]$. One speculates that similar scaling properties of finite N-nonequilibrium dynamics—and possibly aging effects—be captured by extensions of the RG [9] for the statics, going beyond the FDT usually *assumed* in dynamic RG.

In conclusion, we described, using the Hartree approximation, the features of aging of a manifold in a random medium. This provides a frame of reference for the analysis of data from experiments and simulations on realistic systems and will allow one to determine in each case whether such a regime exists and up to what times.

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