Luminosity-Limiting Coherent Phenomena in Electron-Positron Colliders

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We have developed a new simulation program that correctly models the transverse beam-beam dynamics of beams with arbitrary distribution and ellipticity, in e^+e^- colliders. We find that the dynamics, and hence the achievable luminosity, is limited by two kinds of coherent phenomena— the flip-flop effect and period-n beam-size oscillations. While both solutions coexist, the former are typically stronger and occur at lower currents than the latter. These results are in broad agreement with experimental observations, and suggest that greater care needs to be taken in the choice of operating parameters of the high-luminosity B factories that are presently under construction.

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The event rate per unit cross section of a high-energy reaction is defined as the *luminosity* of a storage-ring collider. In order to effectively study rare events, such as *CP* violation in *B*-meson systems, it is necessary to maximize the luminosity of the collider. It is widely believed that the most important factor limiting the luminosity of e^+e^- colliders is the beam-beam interaction—the effect of the electromagnetic fields of one beam on the particles of the other. Although the beam-beam interaction has been studied with a wide variety of theoretical, experimental, and computational techniques, the dynamical reason for this beam-beam limit is not well understood.

One potential source of this limitation is coherent (or collective) beam-beam phenomena that act to distort the beam shape. In one such effect, beams that start out with equal sizes end up in a steady state with very unequal sizes: one of the beams gets blown up transversely to a very large size, while the other remains small. Consequently, the overlap integral is small, and the luminosity is limited. This effect is widely observed in operating colliders and is called the *flip-flop instability* or pitchfork bifurcation. It may be looked upon as one possible solution to the coherent nonlinear dynamics, and we henceforth refer to it as the “*F* solution.”

In another possible solution, predicted earlier for beams with a round transverse profile (i.e., axisymmetric beams) [1], there are coherent oscillations in the sizes of the two beams: the beam sizes vary from turn to turn in a fixed n-fold pattern (where n is a small integer). On any given turn, typically one beam is dense while the other is hollow, so that the overlap between the beams is again poor, and the luminosity is limited. These are described as *period-n anticorrelated oscillations*, and we henceforth refer to them as the “*O* solution.” This kind of behavior has recently been observed for flat beams at the CERN LEP [2]. It is not clear if the same phenomenon occurs in other colliders since the requisite diagnostics are not available.

Coherent beam-beam effects have been analyzed with two different types of models. In the first, of Hirata [3] and of Chao, Furman, and Ng [4], nonlinear maps for the colliding beam system are developed in the moments of the distributions. Working under different approximations they find either flip-flop solutions (*F* solutions) or period-n solutions (*O* solutions), respectively. In the second type of model, of Dikansky and Pestrikov [5] and of Chao and Ruth [6], modes develop in the phase-space distributions of the two beams. The stability of these modes is analyzed with the linearized Vlasov equation, assuming small perturbations from equilibrium. Their results are characterized by the appearance of even-order nonlinear coherent resonances that correspond to the *O* solutions. It should be emphasized that no single model predicts the existence of both the experimentally observed phenomena.

Computer simulations are an important tool in the study of beam-beam phenomena. Conventionally these have assumed that the beams always have a Gaussian distribution, in order to be able to employ analytical formulas in the calculation of the electromagnetic fields [7]. However, the study of coherent phenomena requires that both beams be allowed to influence each other, and though a beam may start out Gaussian, it cannot retain that shape after experiencing the nonlinear force that the opposing Gaussian beam produces. A fully self-consistent calculation therefore requires a numerical algorithm for calculating the beam-beam force from non-Gaussian distributions. Earlier we have developed such an algorithm to predict *O* solutions for the dynamics of beams with arbitrary distributions but nearly round profiles [1]. However, that algorithm fails for the flat beams that coast in all operating e^+e^- colliders. In this Letter we report results using a new field-calculation algorithm that does away with the constraint of nearly round beams, and is valid for beams of arbitrary ellipticity. We find, for the first time, that the dynamics of flat beams allows for both *O* solutions as well as *F* solutions.

We assume that the beams collide once per turn at the interaction point (IP). We only model the dynamics in the two transverse dimensions *X* and *Y*; longitudinal

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dynamics is not included. Particles comprising both beams are initialized in a Gaussian distribution in all four phase-space dimensions, with any chosen ellipticity κ (defined as the ratio of the horizontal to the vertical beam size; by convention $\kappa \geq 1$). They are then transported from the IP once around the ring using maps for the betatron transport, the radiation damping and fluctuations, and the beam-beam interaction. The maps are iterated for a large number of turns, typically until the beams achieve equilibrium. Let the initial phase-space variables be $(X_0, Y_0) = (x_0, p_{x0}, y_0, p_{y0})$. Then a single turn around the ring may be represented as

$$(X_0, Y_0) \xrightarrow{\text{tpt}} (X', Y') \xrightarrow{\text{rad}} (X'', Y'') \xrightarrow{\text{BBI}} (X_1, Y_1). \quad (1)$$

We now describe each of these elements briefly.

Betatron transport.—We assume that the magnetic lattice is linear and the horizontal and vertical motions are uncoupled, so that the (transverse) transport of particles around the ring can be described by two 2×2 rotation matrices, M_x and M_y, such that $X' = M_x X_0$ and $Y' = M_y Y_0$, where

$$M_{x,y} = \begin{pmatrix} \cos(2\pi Q_{x,y}) & \beta_{x,y} \sin(2\pi Q_{x,y}) \\ \frac{1}{\beta_{x,y}} \sin(2\pi Q_{x,y}) & \cos(2\pi Q_{x,y}) \end{pmatrix}. \quad (2)$$

Here $Q_{x,y}$ are the *tunes* (the frequencies of oscillation normalized to the revolution frequency) in the X and Y dimensions, and $\beta_{x,y}$ are the *amplitude functions* that characterize the magnetic lattice. Both are inputs to the simulation.

Radiation damping and fluctuations.—In a real storage ring, an electron emits many synchrotron radiation photons in a single turn, causing fluctuations in its energy. In its journey through an rf cavity it gains energy, leading to the phenomenon of radiation damping [8]. In a computer simulation it is neither practical nor necessary to model these distributed phenomena. Instead, one calculates their average effect, over one turn, and puts this in at a single point in the ring. We can write [8,9]

$$X'' = M_r\, X' + X_f\,, \quad (3)$$

where

$$M_r = \begin{pmatrix} e^{-\delta/2} & 0 \\ 0 & e^{-\delta/2} \end{pmatrix} \quad (4)$$

and

$$X_f = \begin{pmatrix} \hat{r}[\beta_x \epsilon_{0x}(1 - e^{-\delta})]^{1/2} \\ \hat{r}\left[\frac{\epsilon_{0x}}{\beta_x}(1 - e^{-\delta})\right]^{1/2} \end{pmatrix}, \quad (5)$$

with a similar equation for Y''. Here δ is the average fractional energy radiated by a particle per turn, ϵ_{0x} is the horizontal emittance of the beams in the absence of the beam-beam interaction, and $\hat{r}$ is a Gaussian random number with zero mean and unit standard deviation.

Beam-beam interaction.—The model for the beam-beam interaction assumes that the beams are ultrarelativistic. In this case the force due to the magnetic field has the same magnitude and direction as that due to the electric field. One can therefore ignore the magnetic field and solve the corresponding electrostatic problem (by Lorentz transforming to the rest frame of the bunch and solving numerically for the electrostatic field from the coordinates of the test particles comprising the beam). The actual force on a particle is then twice that given by the electrostatic calculation.

The electrostatic field calculation is done on a two-dimensional Cartesian grid. Particles are cast onto the grid using second-order weighting (quadratic spline). The Poisson solver is based on the Fourier analysis by cyclic reduction (FACR) method developed by Christiansen and Hockney and implemented in the code DELSQPHI [10]. It uses a five-point stencil for the ∇^2 operator. To calculate the field from the potential, we use a six-point differencing scheme for the gradient operator. For interpolating from the field at the grid points to any arbitrary point, we must again use second-order weighting in order to conserve momentum. Further details of the algorithm are found in Ref. [11].

We have made extensive efforts to test the algorithm. We have checked that for sample Gaussian distributions our numerical solutions for the fields agree with the corresponding analytical expressions [7]. In addition, the following diagnostic is built into the code: Every so many turns (100 at present) the code takes the calculated potential, differentiates it to get the density along the X and Y axes, and compares this derived density with the original density. They are required to agree, at every grid point, to within a specified tolerance (10% presently). In the results presented below, this diagnostic did not turn up any problems with the algorithm; details are in Ref. [11]. In addition, the code was able to reproduce the results obtained in Ref. [1] for round beams, and some of the results reported below (for $\kappa = 2$) have been confirmed independently, using our earlier field-calculation algorithm [12]. We have also checked that our results are independent of the granularity and size of the grid.

We chose storage-ring parameters corresponding to those of the Cornell Electron Storage Ring (CESR). Our first application was to study the dynamics at a tune of $Q_\beta (= Q_x = Q_y) = 0.79$, with $\delta = 1 \times 10^{-3}$. We have seen in our earlier work that period-3 coherent oscillations (O solutions) appear in this region [1]. We studied the change in the nature of the dynamics as the ellipticity of the beams was varied—in this case from $\kappa = 1$ (round) to $\kappa = 6$. The parameters for the flat beams were derived from those of the round by requiring that the nominal luminosity and *tune-shift parameters* be identical in the two cases [9]. We looked at five different current values between 20 and 40 mA, in steps of 5 mA. Results are

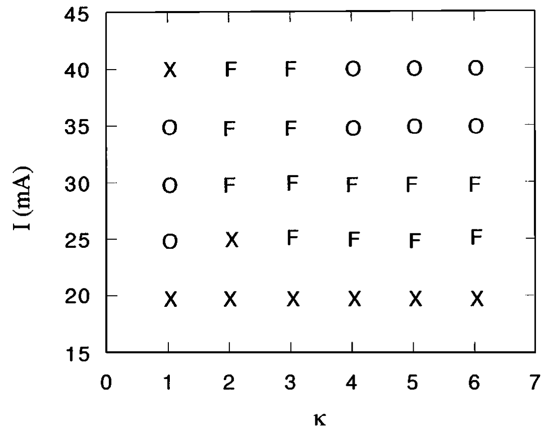

FIG. 1. Change in the nature of coherent motion as a function of the beam ellipticity κ, for different currents. $X \rightarrow$ no coherent motion; $O \rightarrow O$ solutions (period-3 coherent oscillations); $F \rightarrow F$ solutions (flip-flop). $Q_\beta = 0.79$ and $\delta = 1 \times 10^{-3}$.

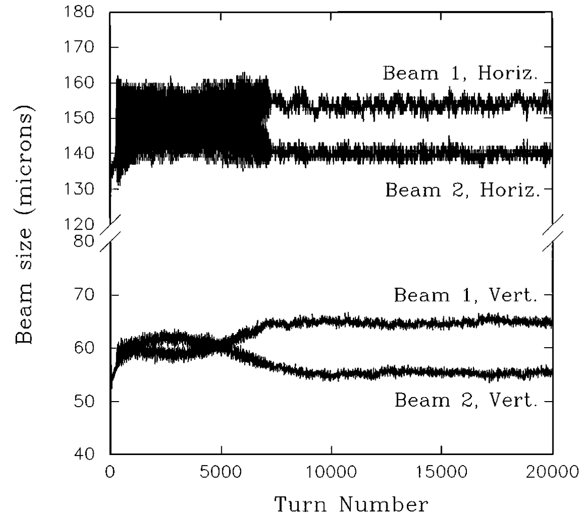

FIG. 2. rms beam size as a function of turn number, for $Q_\beta = 0.79$, $\kappa = 3$, $I = 30$ mA, and $\delta = 1 \times 10^{-3}$. Initially, period-3 oscillations do set in, but ultimately the F solution dominates and is the equilibrium solution.

shown in Fig. 1. For round beams only O solutions are seen, and the region of coherent activity is restricted to the range 25–35 mA. As soon as one gets away from round beams, F solutions make their appearance. In fact, for lower κ these are the only solutions, but at higher values of κ both kinds of solutions are observed. However, the F solutions always occur at lower currents, and therefore in an actual collider they are more likely to be the luminosity-limiting factor.

Figure 2 shows a typical plot of the evolution of the horizontal and vertical beam sizes with time (or turn number), here for $\kappa = 3$, $Q_\beta = 0.79$, and $I = 30$ mA. In the first 5000 turns or so the two solutions compete. The large beam-size oscillations of the O solutions are clearly seen in the horizontal dimension, while the vertical shows an interweaving of the two beam sizes. Ultimately, it is the F solution that is dominant, and hence the equilibrium state is a flip-flop. However, it takes time for the system to reach this equilibrium, and experience shows that one must run the simulation for at least 10 transverse damping times (20 000 turns in the present case) in order to be confident that the beams have indeed settled into equilibrium. In particular cases we have run for as long as 25 damping times to confirm that there is no further change in the equilibrium state. The initial onset of coherent oscillations that later die out to leave behind a flip-flop is commonly observed, but we have never observed the reverse situation, i.e., the beams finding a transient F solution which dies out to leave behind an O solution as the equilibrium state. This suggests that the F solution is typically stronger than the O solution.

To look more closely at the difference between flat and round beams, we explored the structure of the coherent resonances in the I vs Q_β plane for both cases. Results are shown in Fig. 3. For round beams we find, consistent with our earlier work [1], that coherent behavior consists solely of O solutions, and the domain of coherent behavior spans a finite, tubular region in the plane. In Fig. 3 the circles mark the onset and offset currents for this coherent behavior, and outside the solid lines there is no coherent motion. For flat beams (here with $\kappa = 4$), the rectangles and the dotted line mark the threshold for the onset of coherent motion. At $Q_\beta = 0.79$ and 0.80 there is no offset threshold—at least up to the maximum current of 50 mA that we investigated. At $Q_\beta = 0.81$, however, there is a clear offset threshold at 22 mA, marked in the figure by the solitary rectangle. The precise nature of coherent activity is more complicated for flat beams. Near the onset threshold and at very high currents, the equilibrium state is always an F solution. At $Q_\beta = 0.81$ this is the only solution. However, at the other two tunes there is embedded a region where the equilibrium solution is an O solution. At $Q_\beta = 0.80$ this region is between 28 and 33 mA, and at $Q_\beta = 0.79$ between 33 and 42 mA; in Fig. 3 these onset and offset currents are marked by solid triangles.

Thus, for flat beams coherent motion sets in at slightly lower currents than in the case of round beams, and extends out to much higher currents. Both kinds of solutions are supported, but the F solution seems to be the dominant solution, in that it is found to occur more often and it sets in at lower currents than the O solution.

A couple of points need emphasis. First, these coherent phenomena are *not* seen in simulations that assume Gaussian distributions for the beam; they are solely a consequence of the generalized calculations described here. We have confirmed this explicitly for round beams as well as for flat beams with $\kappa = 4$: neither the F solutions nor the O solutions are seen.

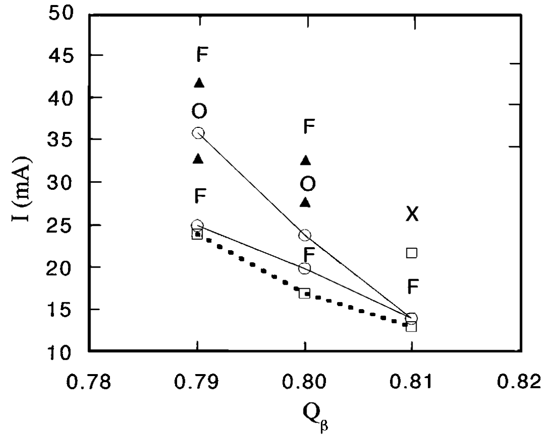

FIG. 3. Stability diagram in the I vs Q_β plane, for round ($\kappa = 1$) and flat ($\kappa = 4$) beams. For round beams the circles indicate thresholds for the onset and offset of coherent behavior. Only O solutions are seen, and the domain of coherent activity is restricted to the tubular region within the solid lines. For flat beams the squares and the dotted line indicate the onset threshold, where F solutions are found. For $Q_\beta = 0.79$ and 0.80 there is no offset threshold (at least up to the maximum current of 50 mA that we investigated). At $Q_\beta = 0.81$ the offset threshold is at 22 mA, indicated by the solitary rectangle. At this tune the equilibrium state is always a F solution, but at the other two tunes a region of O solutions can also be found, the onset and offset currents for which are indicated by solid triangles. The equilibrium state in the various regions (*for the flat beam only*) is indicated by F, O, and X for flip-flop solutions, period-n oscillations, and no coherent motion, respectively.

Second, these simulations show, for the first time, the simultaneous existence of both kinds of coherent phenomena that have been experimentally observed. The fact that the flip-flop is the dominant solution is in qualitative agreement with the general observation of flip-flops in many operating colliders. Nonetheless, period-n oscillations are observed in our simulations and have also been observed at LEP, and we recommend a stronger effort to look for these coherent resonances at other colliders.

Because of constraints in the available computing resources, our results are limited to $\kappa \leq 6$, while typical values for e^+e^- colliders are admittedly larger: for example, $\kappa = 33$ for the SLAC/LBNL/LLNL asymmetric B factory under construction. Nonetheless, we believe our results are valid for larger κ. For a couple of currents we have checked this explicitly at $\kappa = 16$ using the Cray computers of the National Energy Research Supercomputer Center (NERSC).

The observation of these coherent phenomena could have important implications for the B factories that are presently under construction. Our work has been performed with a damping of $\delta = 1 \times 10^{-3}$, and the lowest threshold we find for the onset of coherent motion is 13 mA (at $Q_\beta = 0.81$). For our parameters this corresponds to a nominal tune-shift parameter of $\xi_0 \sim 0.04$. However, for the B factories under construction, $\delta \sim 10^{-4}$, and at this lower damping the deleterious effects of these coherent phenomena are only expected to worsen; in particular, the onset threshold will be lower. Since these B factories have been designed assuming $\xi_0 \sim 0.03-0.05$, care will need to be taken in the choice of operating parameters if they are not to be limited in luminosity by the coherent phenomena described here [13].

In conclusion, our simulations show, for the first time, that the dynamics of flat beams allows for flip-flops (F solutions) as well as period-n beam-size oscillations (O solutions), in broad agreement with experimental observations. The former are typically stronger and set in at lower currents. In the B factories under construction these could limit the luminosity, and therefore care needs to be exercised in the choice of operating parameters.

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