Squeezed Phonon States: Modulating Quantum Fluctuations of Atomic Displacements

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We study squeezed quantum states of phonons, which allow the possibility of modulating the quantum fluctuations of atomic displacements below the zero-point quantum noise level of coherent phonon states. We calculate the corresponding expectation values and fluctuations of both the atomic displacement and lattice amplitude operators, and also investigate the possibility of generating squeezed phonon states using a three-phonon parametric amplification process based on phonon-phonon interactions. Furthermore, we also propose a detection scheme based on reflectivity measurements.

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Photon squeezed states have attracted much attention during the past decade [1]. These states are important because they can achieve lower quantum noise than the zero-point fluctuations of the vacuum or coherent states. Thus they provide a way of manipulating quantum fluctuations and have a promising future in different applications ranging from optical communications to gravitational wave detection [1]. Indeed, squeezed states are currently being explored in a variety of non-quantum-optics systems, including classical squeezed states [2]. Here we study the properties of *phonon* squeezed states and explore the possibility of generating these states through phonon-phonon interactions. After briefly presenting the quantum mechanical description of various kinds of phonon states, we study a simple model for generating phonon squeezed states, in which analytical results can be obtained [3]. We also propose a scheme for detecting this squeezing effect.

In most macroscopic situations, a classical description is adequate. However, the quantum fluctuations of a phonon system can be dominant at low enough temperatures. Indeed, a recent study shows that quantum fluctuations in the atomic positions can influence observable quantities (e.g., the Raman line shape) [4] even when temperatures are not very low.

An experimentally observable quantity for a phonon system is the real part of the Fourier transform of the atomic displacement: $\mathrm{Re}[u_\alpha(\mathbf{q})] = \sum_\lambda \sqrt{\hbar/8m\omega_{\mathbf{q}\lambda}}\{U^\lambda_{\mathbf{q}\alpha}(b_{\mathbf{q}\lambda} + b^\dagger_{-\mathbf{q}\lambda}) + U^{\lambda *}_{\mathbf{q}\alpha}(b_{-\mathbf{q}\lambda} + b^\dagger_{\mathbf{q}\lambda})\}$ [5]. For simplicity, hereafter we will drop the branch subscript λ, assume that $U_{\mathbf{q}\alpha}$ is real, and define a $\mathbf{q}$-mode dimensionless lattice amplitude operator: $u(\pm\mathbf{q}) = b_\mathbf{q} + b^\dagger_{-\mathbf{q}} + b_{-\mathbf{q}} + b^\dagger_\mathbf{q}$. This operator contains essential information on the lattice dynamics, including quantum fluctuations. It is the phonon analog of the electric field in the photon case.

Phonon vacuum and number states.—When no phonon is excited, the crystal is in the phonon vacuum state $|0\rangle$. The eigenstates of the harmonic phonon Hamiltonian are number states which satisfy $b_\mathbf{q}|n_\mathbf{q}\rangle = \sqrt{n_\mathbf{q}}\,|n_\mathbf{q} - 1\rangle$. The phonon number and the phase of atomic vibrations are conjugate variables. Thus, due to the uncertainty principle, the phase is arbitrary when the phonon number is certain, as is the case with any number state $|n_\mathbf{q}\rangle$. Therefore, the expectation values of the atomic displacement $\langle n_\mathbf{q}|u_{i\alpha}|n_\mathbf{q}\rangle$ and $\mathbf{q}$-mode lattice amplitude $\langle n_\mathbf{q}|u(\pm\mathbf{q})|n_\mathbf{q}\rangle$ vanish due to the randomness in the phase of the atomic displacements.

Phonon coherent states.—A single-mode ($\mathbf{q}$) phonon coherent state is an eigenstate of a phonon annihilation operator: $b_\mathbf{q}|\beta_\mathbf{q}\rangle = \beta_\mathbf{q}|\beta_\mathbf{q}\rangle$ [6]. It can also be generated by applying a phonon displacement operator $D_\mathbf{q}(\beta_\mathbf{q})$ to the phonon vacuum state $|\beta_\mathbf{q}\rangle = D_\mathbf{q}(\beta_\mathbf{q})\,|0\rangle = \exp(\beta_\mathbf{q} b^\dagger_\mathbf{q} - \beta^*_\mathbf{q} b_\mathbf{q})|0\rangle = \exp(-|\beta_\mathbf{q}|^2/2)\sum_{n_\mathbf{q}=0}^\infty \beta_\mathbf{q}^{n_\mathbf{q}}\,|n_\mathbf{q}\rangle/\sqrt{n_\mathbf{q}!}$. Thus it can be seen that a phonon coherent state is a phase coherent superposition of number states. Moreover, coherent states are a set of minimum-uncertainty states which are as noiseless as the vacuum state [7]. Coherent states are also the quantum states that best describe the classical harmonic oscillators [8].

Phonon squeezed states.—In order to reduce quantum noise to a level below the zero-point fluctuation level, we need to consider phonon squeezed states. Quadrature squeezed states are generalized coherent states [9]. Here "quadrature" refers to the dimensionless coordinate and momentum. Compared to coherent states, squeezed ones can achieve smaller variances for one of the quadratures during certain time intervals and are therefore helpful for decreasing quantum noise.

A single-mode quadrature phonon squeezed state is generated from a vacuum state as $|\alpha_\mathbf{q}, \xi\rangle = D_\mathbf{q}(\alpha_\mathbf{q})S_\mathbf{q}(\xi)|0\rangle$; a two-mode quadrature phonon squeezed state is generated as $|\alpha_{\mathbf{q}_1}, \alpha_{\mathbf{q}_2}, \xi\rangle = D_{\mathbf{q}_1}(\alpha_{\mathbf{q}_1})D_{\mathbf{q}_2}(\alpha_{\mathbf{q}_2})S_{\mathbf{q}_1,\mathbf{q}_2}(\xi)|0\rangle$. Here $D_\mathbf{q}(\alpha_\mathbf{q})$ is the coherent state displacement operator with $\alpha_\mathbf{q} = |\alpha_\mathbf{q}|e^{i\phi}$, $S_\mathbf{q}(\xi) = \exp(\xi^* b_\mathbf{q}^2/2 - \xi b_\mathbf{q}^{\dagger 2}/2)$ and $S_{\mathbf{q}_1,\mathbf{q}_2}(\xi) = \exp(\xi^* b_{\mathbf{q}_1} b_{\mathbf{q}_2} - \xi b^\dagger_{\mathbf{q}_1} b^\dagger_{\mathbf{q}_2})$ are the single- and two-mode squeezing operator [10], and $\xi = re^{i\theta}$ is the complex squeezing factor with $r \ge 0$ and $0 \le \theta < 2\pi$. The two-mode phonon quadrature operators have the form $X(\mathbf{q}, -\mathbf{q}) = (b_\mathbf{q} + b^\dagger_\mathbf{q} + b_{-\mathbf{q}} + b^\dagger_{-\mathbf{q}})/2^{3/2} = 2^{-3/2} \times u(\pm\mathbf{q})$ and $P(\mathbf{q}, -\mathbf{q}) = (b_\mathbf{q} - b^\dagger_\mathbf{q} + b_{-\mathbf{q}} - b^\dagger_{-\mathbf{q}})/2^{3/2}i$.

We have considered two cases where squeezed states were involved in modes $\pm\mathbf{q}$. In the first case, the system is in a two-mode ($\pm\mathbf{q}$) squeezed state $|\alpha_\mathbf{q}, \alpha_{-\mathbf{q}}, \xi\rangle$,

($\xi = re^{i\theta}$), with fluctuations $\langle[\Delta u(\pm\mathbf{q})]^2\rangle_{\rm sq} = 2(e^{-2r}\cos^2\frac{\theta}{2} + e^{2r}\sin^2\frac{\theta}{2})$. In the second case, the system is in a single-mode squeezed state $|\alpha_{\mathbf{q}}, \xi\rangle$ ($\alpha_{\mathbf{q}} = |\alpha_{\mathbf{q}}|e^{i\phi}$) in the first mode and an arbitrary coherent state $|\beta_{-\mathbf{q}}\rangle$ in the second mode. The fluctuation is now $1 + e^{-2r}\cos^2(\phi + \frac{\theta}{2}) + e^{2r}\sin^2(\phi + \frac{\theta}{2})$. In both of these cases, $\langle[\Delta u(\pm\mathbf{q})]^2\rangle_{\rm sq}$ can be smaller than in coherent states.

Phonon parametric process.—Now we propose a scheme to generate phonon quadrature squeezed states [11,12]. This scheme is based on a "phonon" parametric amplification process (e.g., the decaying process LO phonon → two LA phonons, where LO refers to longitudinal optical and LA to longitudinal acoustic), which in turn is based on three-phonon interactions. Typically, three-phonon interactions are the dominant anharmonic processes in a phonon system and the lowest order perturbation to the harmonic Hamiltonian. We will neglect all the higher order interactions because they are generally much weaker than the third-order ones. For all parametric processes, the pump wave (of phonons in this case) must be very strong because the generic physical processes inside parametric amplifiers are generally nonlinear and weak. This pumping process can be realized by using two lasers to illuminate a crystal. With appropriate laser frequencies and directions, coherent LO phonons of the pump mode at the Brillouin zone center can be generated through, for example, stimulated Raman scattering (provided that the pump mode is Raman active), as discussed, e.g., in Refs. [13,14].

The Hamiltonian for the whole process initiated by the Raman scattering is (see Fig. 1)

$$H_{\rm param} = H_0 + H_{\rm Raman} + H_{\rm anh}\,, \tag{1}$$

$$H_0 = \hbar\omega_{\mathbf{k}_1} a^\dagger_{\mathbf{k}_1} a_{\mathbf{k}_1} + \hbar\omega_{\mathbf{k}_2} a^\dagger_{\mathbf{k}_2} a_{\mathbf{k}_2} + \sum_{\mathbf{q}} \hbar\omega_{\mathbf{q}} b^\dagger_{\mathbf{q}} b_{\mathbf{q}}\,,$$

$$H_{\rm Raman} = \eta a_{\mathbf{k}_1} a^\dagger_{\mathbf{k}_2} b^\dagger_{\mathbf{q}_p} + \eta^* a^\dagger_{\mathbf{k}_1} a_{\mathbf{k}_2} b_{\mathbf{q}_p}\,,$$

$$H_{\rm anh} = \lambda_{\mathbf{q}_s\mathbf{q}_i} b_{\mathbf{q}_p} b^\dagger_{\mathbf{q}_s} b^\dagger_{\mathbf{q}_i} + \lambda^*_{\mathbf{q}_s\mathbf{q}_i} b^\dagger_{\mathbf{q}_p} b_{\mathbf{q}_s} b_{\mathbf{q}_i} + \sum_{\mathbf{q}'\mathbf{q}''}(\lambda_{\mathbf{q}'\mathbf{q}''} b_{\mathbf{q}_p} b^\dagger_{\mathbf{q}'} b^\dagger_{\mathbf{q}''} + \lambda^*_{\mathbf{q}'\mathbf{q}''} b^\dagger_{\mathbf{q}_p} b_{\mathbf{q}'} b_{\mathbf{q}''})\,.$$

Here a (b) refer to photon (phonon) operators. The higher (lower) energy incident photon mode is labeled by $\mathbf{k}_1$ ($\mathbf{k}_2$). Notice that the lower energy photon mode is generally called the Stokes mode in the context of Raman scattering. The sums over $\mathbf{q}'$ and $\mathbf{q}''$ in $H_{\rm anh}$ represent decay channels other than the special one with acoustic signal and idler modes.

We now consider two mean field averages in order to simplify an otherwise analytically intractable problem. The first mean field is over the photons. The photons in the incident modes $\mathbf{k}_1$ and $\mathbf{k}_2$ (often denoted by "laser" and "Stokes" light) originate from two lasers. As long as these two incident laser modes are not strongly

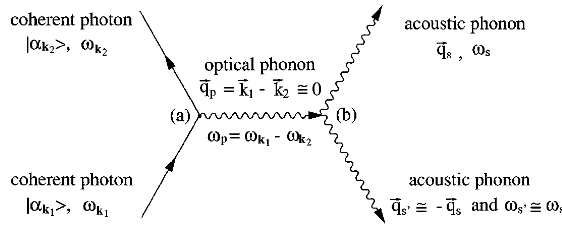

FIG. 1. A schematic diagram of a three-phonon parametric process. Here (a) refers to a stimulated Raman scattering and (b) to a three-phonon anharmonic scattering process. The subscript $\mathbf{k}_1$ ($\mathbf{k}_2$) refers to the higher (lower) energy incident coherent photons. The arrows in the diagram illustrate the directions of the photon and phonon momentum vectors. A typical process is as follows: A photon in mode $\mathbf{k}_1$ interacts with the phonon system and emits one LO phonon in the pump mode of frequency ω_p, while the photon itself is scattered into mode $\mathbf{k}_2$; the generated pump mode LO phonon proceeds in the crystal, interacts with the lower energy phonon modes through the three-phonon interaction, and eventually splits into two LA phonons in modes s and s'. The latter can be squeezed for appropriate initial states. Notice that the pump mode LO phonons have an almost-zero wave vector, so that the two lower energy LA phonon modes have nearly opposite wave vectors $\pm\mathbf{q}_s$. Also notice that this figure is not to scale. For the sake of clarity, we have increased the angle between the lines and drawn a longer line for the pump-mode phonon.

perturbed by the Raman scattering process, we can treat both of these incoming photon states as coherent states $|\alpha_{\mathbf{k}_1} e^{-i\omega_{\mathbf{k}_1}t}\rangle$ and $|\alpha_{\mathbf{k}_2} e^{-i\omega_{\mathbf{k}_2}t}\rangle$, and perform a mean field average over them. The second mean field average is over the LO pump-mode phonons. Since phonons produced by coherent or stimulated Raman scattering are initially in coherent states, we denote this pump-mode phonon coherent state as $|\beta_0(t)\rangle$, with $\langle\beta_0(t)|b_{\mathbf{q}_p}|\beta_0(t)\rangle = \beta_0(t)$. Since these LO phonons are in coherent states, the results from the average over the pump-mode phonons are c-numbers with a well-behaved time dependence. Now we drop all the c-number terms because they will not affect our results. In addition, we will also drop all the phonon modes involved in the decay channels other than the special one consisting of the signal modes, considering them only weakly coupled to the pump mode; i.e., we assume $\lambda_{\mathbf{q}'\mathbf{q}''} \ll \lambda_{\mathbf{q}_s\mathbf{q}_i}$. The Hamiltonian now becomes

$$H'_{\rm param} = \hbar\omega_{\mathbf{q}_s} b^\dagger_{\mathbf{q}_s} b_{\mathbf{q}_s} + \hbar\omega_{-\mathbf{q}_s} b^\dagger_{-\mathbf{q}_s} b_{-\mathbf{q}_s} + \lambda_{\mathbf{q}_s,-\mathbf{q}_s}\,\beta_0(t)\,b^\dagger_{\mathbf{q}_s} b^\dagger_{-\mathbf{q}_s} + \lambda^*_{\mathbf{q}_s,-\mathbf{q}_s}\,\beta^*_0(t)\,b_{\mathbf{q}_s} b_{-\mathbf{q}_s}\,, \tag{2}$$

where $\beta_0(t)$ is the coherent amplitude of the pump-mode phonons. We use $H_0 + H_{\rm Raman}$ to determine $\beta_0(t)$, and then substitute it back into $H_{\rm param}$ to obtain $H'_{\rm param}$. Here we have implicitly assumed that the Raman scattering process is stronger than the anharmonic scattering. According to our previous discussion [10], the two-mode LA phonon system will evolve into a two-mode squeezed state $|\alpha_{\mathbf{q}_s}, \alpha_{-\mathbf{q}_s}, \xi(t)\rangle$ from an initial coherent or vacuum state, with a squeezing factor of

$$\xi(t) = \frac{i}{\hbar} \int_{-\infty}^{t} \lambda_{\mathbf{q}_s,-\mathbf{q}_s} \beta_0(\tau) e^{2i\omega_{\mathbf{q}_s}\tau} d\tau , \qquad (3)$$

which is valid in the short-time limit. This condition can be relaxed if the incident photons are in an ultrashort pulse with duration less than the optical phonon period.

In summary, we have just considered generating two-mode LA phonon squeezed states $|\alpha_{\mathbf{q}_s}, \alpha_{-\mathbf{q}_s}, \xi(t)\rangle$ by using the three-phonon anharmonic interaction [15]. The higher energy LO phonon mode, which is called the "pump" mode, is driven into a coherent state through stimulated Raman scattering. This mode in turn is used as a pump in the parametric amplification process involving itself and the two lower energy LA phonon modes ($\pm\mathbf{q}_s$), the signal and the idler. Both of these modes can here be called "signal" because the "idler" mode is not really "idle"; indeed, it is actively involved in the squeezing process. In conclusion, we have shown that the LA phonons in the two signal modes ($\pm\mathbf{q}_s$) are in a two-mode squeezed state if (i) the LO pump mode is in a coherent state and (ii) we can neglect the other decay channels.

Detection schemes.—It is possible to directly detect a single-mode phonon squeezed state with phonon counters [16] such as superconducting tunnel junction bolometers and vibronic detectors. The signature of a single-mode squeezed state is a sub-Poissonian phonon number distribution in that mode. However, these phonon counters are either wide band or have low efficiency. Therefore, direct detection might not be the best method to detect squeezing effect.

Phase-sensitive schemes such as homodyne and heterodyne detectors are most often used to detect photon squeezed states because of their ability to lock phase with the electric field of the measured state [9]. There appears to be no available phase-sensitive detection method for phonons. A promising candidate might be measuring the intensity of a reflected probe light [14]. This method has already been used to detect phonon amplitudes, since the reflectivity is linearly related to the atomic displacements in a crystal. The value of the lattice amplitude operator can be extracted by making a Fourier analysis on the sample reflectivity. If squeezing should happen, its effect will be contained in the Fourier components of the intensity of the reflected light. In this manner the information on the squeezing effect in the phonons is also carried by the reflected light in the form of squeezing of the photon intensity. We can then use a standard optical detection method to determine whether the related light is squeezed or not. One shortcoming of this method is that it is not direct. In the measurement there can be noise added into the signal, such as the intensity fluctuation of the original probe light, the efficiency for the reflected light to pick up the signals in the phonons, etc. Needless to say, further research needs to be done on how to realize this phase-sensitive detection scheme, and we hope that our initial proposals stimulate further theoretical and experimental work on this problem.

Phonon squeezing depends on the absolute value r and also on the phase θ of the squeezing factor $\xi(t) = re^{i\theta}$. More explicitly, $\langle[\Delta u(\pm\mathbf{q})]^2\rangle_{\mathrm{sq}} = 2(e^{-2r}\cos^2\frac{\theta}{2} + e^{2r}\sin^2\frac{\theta}{2})$. Only when θ is close to 0 is noise suppressed in the lattice amplitude operator. This means that, in order to suppress the noise, the squeezing factor $\xi(t)$ has to have a dominant positive real part so that $\cos\theta > \tanh r$. The squeezing factor obtained from the three-phonon process is $\xi(t) = \frac{i}{\hbar}\int_0^t \lambda\, \alpha(\tau)\, e^{i(\omega_s+\omega_i)\tau} d\tau$, where the real number λ is the strength of the interaction and α is the amplitude of the phonon coherent state in the pump mode. From this expression for $\xi(t)$, we can see that the squeezing effect only appears during certain time intervals. If $\alpha(t)$ does not depend on time or has a periodic dependence on time, squeezing will be periodic in time, which makes phase-sensitive detection easier to achieve.

To make the above schemes work, some noise problems have to be overcome. First, any attempt to generate or detect squeezed states should be at low temperatures to avoid thermal noise in the crystal. For instance, the excitation energy of a 10 THz optical phonon corresponds to a temperature of about 100 K. Therefore, the experiment might have to be carried out at a temperature well below 100 K, such as 10 K or lower. Second, the fluctuations in the laser intensity and in the interaction between the laser and the crystal has to be very small, so that they will not suppress the noise reduction process in the squeezing effect. Indeed, one of the possible ways to reduce the noise coming from the laser beam is to use a beam of squeezed photons. Finally, the incoherence in the procedure itself has to be minimized. For example, the finite lifetime of pump-mode phonons does not favor the generation of squeezed states because it gives rise to an additional noise in the intensity of the mode. Therefore, we need long lifetime LO phonons, which can be realized in, for instance, materials with weak anharmonic interactions and low concentration of isotopic defects (e.g., diamond).

In conclusion, we have investigated the dynamics and quantum fluctuation properties of phonon quadrature squeezed states. In particular, we calculate the experimentally observable time evolution and fluctuation of the lattice amplitude operator $u(\pm\mathbf{q})$, and show that $\langle u(\pm\mathbf{q})\rangle_{\mathrm{sq}}$ is a sinusoidal function of time, while $\langle[\Delta u(\pm\mathbf{q})]^2\rangle_{\mathrm{sq}}$ is periodically smaller than the vacuum and coherent state value 2. In other words, phonon squeezed states are periodically quieter than the vacuum state. We have discussed one particular approach to generate phonon squeezed states. This approach is based on a three-phonon process where the higher energy optical phonon mode is coherently pumped. We show that the two lower energy acoustic phonon modes can be in a two-mode phonon quadrature squeezed state given appropriate initial conditions. We achieve this by dealing separately with (i) the optical excitation of the pump mode optical phonons and (ii) the anharmonic scattering of the pump-mode phonons into the lower energy acoustic phonons. We have also briefly analyzed a potential detection method of phonon squeezed states. Experiments in quantum optics indicate that phase-sensitive methods—such as homodyne detection—are the best in detecting photon squeezed states. Therefore, we have proposed a detection scheme based on a reflected probe light and an ordinary phase-sensitive optical detector.

As in the photon case [1], the experimental realization of phonon squeezed states might require years of work after its initial proposal. We hope that our effort will lead to more theoretical and experimental explorations in the area of phonon quantum noise modulation.

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[5] A phonon with quasimomentum $\mathbf{p} = \hbar\mathbf{q}$ and branch subscript λ has energy $\epsilon_{\mathbf{q}\lambda} = \hbar\omega_{\mathbf{q}\lambda}$; the corresponding creation and annihilation operators satisfy the boson commutation relations $[b_{\mathbf{q}'\lambda'}, b^{\dagger}_{\mathbf{q}\lambda}] = \delta_{\mathbf{q}\mathbf{q}'}\delta_{\lambda\lambda'}$, $[b_{\mathbf{q}\lambda}, b_{\mathbf{q}'\lambda'}] = 0$. The atomic displacements $u_{i\alpha}$ of a crystal lattice are given by $u_{i\alpha} = (1/\sqrt{Nm})\sum_{\mathbf{q}\lambda}^{N} U^{\lambda}_{\mathbf{q}\alpha} Q^{\lambda}_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{R}_i}$. Here $\mathbf{R}_i$ refer to the equilibrium lattice positions, α to a particular direction, and $Q^{\lambda}_{\mathbf{q}} = \sqrt{\hbar/2\omega_{\mathbf{q}\lambda}}\,(b_{\mathbf{q}\lambda} + b^{\dagger}_{-\mathbf{q}\lambda})$ is the normal-mode amplitude operator.

[6] A single-mode phonon coherent state can be generated by the Hamiltonian $H = \hbar\omega_{\mathbf{q}}(b^{\dagger}_{\mathbf{q}}b_{\mathbf{q}} + 1/2) + \lambda^{*}_{\mathbf{q}}(t)\,b_{\mathbf{q}} + \lambda_{\mathbf{q}}(t)\,b^{\dagger}_{\mathbf{q}}$ and an appropriate initial state. Here $\lambda_{\mathbf{q}}(t)$ represents the interaction strength between phonons and the external source. More specifically, if the initial state is a vacuum state, $|\psi(0)\rangle = |0\rangle$, then the state vector $|\psi\rangle$ becomes a single-mode phonon coherent state thereafter $|\psi(t)\rangle = |\Lambda_{\mathbf{q}}(t)\,e^{-i\omega_{\mathbf{q}}t}\rangle$, where $\Lambda_{\mathbf{q}}(t) = -(i/\hbar)\int_{-\infty}^{t}\lambda_{\mathbf{q}}(\tau)\,e^{i\omega_{\mathbf{q}}\tau}d\tau$ is the coherent amplitude of mode $\mathbf{q}$. If the initial state is a single-mode coherent state $|\psi(0)\rangle = |\alpha_{\mathbf{q}}\rangle$, then the state vector at time t takes the form $|\psi(t)\rangle = |\{\Lambda_{\mathbf{q}}(t) + \alpha_{\mathbf{q}}\}e^{-i\omega_{\mathbf{q}}t}\rangle$, which is still coherent.

[7] In the phonon vacuum state, the fluctuations of the atomic displacement operator are identical for all atoms in the lattice $\langle(\Delta u_{\alpha})^2\rangle_{\rm vac} \equiv \langle(u_{\alpha})^2\rangle_{\rm vac} - \langle u_{\alpha}\rangle^2_{\rm vac} = \sum_{\mathbf{q}}^{N}\hbar|U_{\mathbf{q}\alpha}|^2/2Nm\omega_{\mathbf{q}\alpha}$ and $\langle[\Delta u(\pm\mathbf{q})]^2\rangle_{\rm vac} = 2$. The fluctuations in a phonon number state are larger than in the vacuum state $\langle(\Delta u_{i\alpha})^2\rangle_{\rm num} = \hbar|U_{\mathbf{q}\alpha}|^2 n_{\mathbf{q}}/Nm\omega_{\mathbf{q}\alpha} + \sum_{\mathbf{q}'\neq\mathbf{q}}^{N}\hbar|U_{\mathbf{q}'\alpha}|^2/2Nm\omega_{\mathbf{q}'\alpha}$ and $\langle[\Delta u(\pm\mathbf{q})]^2\rangle_{\rm num} = 2 + 2n_{\mathbf{q}}$. In a single-mode ($\mathbf{q}$) phonon coherent state $|\Lambda_{\mathbf{q}}(t)\,e^{-i\omega\mathbf{q}t}\rangle$, the fluctuation in the atomic displacements is $\langle(\Delta u_{i\alpha})^2\rangle_{\rm coh} = \sum_{\mathbf{q}}^{N}\hbar|U_{\mathbf{q}\alpha}|^2/2Nm\omega_{\mathbf{q}\alpha}$. The unexcited modes are in the vacuum state and thus all contribute to the noise in the form of zero-point fluctuations. Furthermore, $\langle[\Delta u(\pm\mathbf{q})]^2\rangle_{\rm coh} = 2$, which is exactly the same as in the phonon vacuum state.

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[10] The squeezing operator $S_{\mathbf{q}_1,\mathbf{q}_2}(\xi)$ can be produced by the following Hamiltonian: $H_{\mathbf{q}_1,\mathbf{q}_2} = \hbar\omega_{\mathbf{q}_1}\times b^{\dagger}_{\mathbf{q}_1}b_{\mathbf{q}_1} + \hbar\omega_{\mathbf{q}_2}b^{\dagger}_{\mathbf{q}_2}b_{\mathbf{q}_2} + \zeta(t)b^{\dagger}_{\mathbf{q}_1}b^{\dagger}_{\mathbf{q}_2} + \zeta^{*}(t)b_{\mathbf{q}_1}b_{\mathbf{q}_2}$. The time-evolution operator has the form $U(t) = \exp(-iH_0t/\hbar)\exp[\xi^{*}(t)b_{\mathbf{q}_1}b_{\mathbf{q}_2} - \xi(t)b^{\dagger}_{\mathbf{q}_1}b^{\dagger}_{\mathbf{q}_2}]$, where $H_0 = \hbar\omega_{\mathbf{q}_1}b^{\dagger}_{\mathbf{q}_1}b_{\mathbf{q}_1} + \hbar\omega_{\mathbf{q}_2}b^{\dagger}_{\mathbf{q}_2}b_{\mathbf{q}_2}$ and $\xi(t) = \frac{i}{\hbar}\times\int_{-\infty}^{t}\zeta(\tau)\,e^{i(\omega_{\mathbf{q}_1}+\omega_{\mathbf{q}_2})\tau}d\tau$. Here $\xi(t)$ is the squeezing factor and $\zeta(t)$ is the strength of the interaction between the phonon system and the external source; this interaction allows the generation and absorption of two phonons at a time.

[11] It is important to point out that the squeezing effect is quite general. In fact, any diagonalizable system can be "squeezed" in some manner by an appropriate choice of initial states. For example, consider a free oscillator $H = a^{\dagger}a$, and define $d = \mu a + \nu a^{\dagger}$, where μ and ν are complex scalars. In order for d to be a boson operator, μ and ν must satisfy the relation $|\mu|^2 - |\nu|^2 = 1$. Then the Hamiltonian becomes $H = (|\mu|^2 + |\nu|^2)d^{\dagger}d - (\mu\nu d^{\dagger\,2} + \mu^{*}\nu^{*}d^2) + |\nu|^2$. The generalized "displacement" $d + d^{\dagger}$ is a mixture of the original displacement $(a + a^{\dagger})/\sqrt{2}$ and momentum $-i(a - a^{\dagger})/\sqrt{2}$. If we are interested in a quantity such as $d + d^{\dagger}$, and it is observable, the squeezing effect can be obtained when the initial state is a coherent state of mode "d," i.e., $d|\delta\rangle = \delta|\delta\rangle$. On the other hand, such a hybrid quantity $d + d^{\dagger}$ might not be physically interesting and/or observable. In other words, the squeezing effect is only relevant when it is related to the observables.

[12] For very short times, the following single-mode anharmonic Hamiltonian $H = \hbar\omega_{\mathbf{q}}b^{\dagger}_{\mathbf{q}}b_{\mathbf{q}} + \hbar\lambda_{\mathbf{q}}(b^{\dagger}_{\mathbf{q}}b_{\mathbf{q}})^2$ also produces squeezed phonons in mode $\mathbf{q}$ when the initial state is coherent.

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