Controlling the Chaotic Regime of Nonlinear Ionization Waves using the Time-Delay Autosynchronization Method

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The chaotic regime of an unstable glow discharge is controlled using a continuous time-delay autosynchronization method. At a fixed location along the discharge tube, the electric field is chaotic within certain ranges of the discharge current. The control signal, built from the difference between the delayed signal and the real-time chaotic signal, leads to the stabilization of the discharge when applied to the control parameter (the discharge current).

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There are now a large number of experimental works carried out on the control of chaotic regimes of nonlinear dynamical systems, i.e., the stabilization of unstable periodic orbits (UPOs) embedded in a chaotic attractor. These works are most often based on the algorithm proposed by Ott, Grebogi, and Yorke (OGY) [1] allowing the taming of chaos in various systems. Up to now, very few results have been reported on the control of chaos in the field of plasma physics despite the occurrence of numerous chaotic situations often deeply studied during the past few years [2–4]. Moreover, the crucial role of chaos and turbulence in fusion plasmas induces a special interest in controlling chaos in the field of plasma physics. Recently, results obtained in controlling the driven chaotic states in a discharge tube [5] have been published. This paper is a first report on the control of a spontaneously chaotic plasma system.

In a general manner, the chaotic regime of a nonlinear dynamical system is triggered by increasing a sensitive parameter, the control parameter. The main idea in controlling chaos is to use the high sensitivity of the system to small perturbations to change the dynamical state. Correcting the trajectories of the dynamical system in the phase space leads to the attraction of the system by the stable manifold of one of the underlying unstable periodic orbits.

This is achieved in the OGY algorithm through a permanent computer monitoring of the state of the system and a precise characterization of the topology of the attractor near the UPO to be stabilized. The determination of the stable and unstable directions in the chosen Poincaré section leads to the application of small *ad hoc* perturbations to the control parameter in order to stabilize the targeted UPO. Successful developments of this original method and its modifications [6] have been reported [7–10]. However, one disadvantage is the high sensitivity of the chaotic system to noise since only rare and small corrections are applied to the control parameter. This method has been slightly modified, for instance, by Hunt [11] in the case of the chaotic nonlinear diode oscillator. The latter author introduced the OPF (occasional proportional feedback) method, which is easy to develop when the system is periodically externally driven. In this case, the driver allows for a precise timing of the control perturbations and for stroboscopic investigations. However, the applications are restricted to relatively slow phenomena since real-time processing is required. On the other hand, the algorithm proposed by Pyragas [12,13] is simpler in application, and it is well suited for autonomous chaotic systems. The basic idea is to realize an active continuous control of the dynamical system by applying a signal continuously processed from the time series to one of the state variables. In order to stabilize the UPO of period T, the times series has the time delay T, and a signal proportional to the difference between the delayed signal and the actual signal is applied to the nonlinear system. More exactly, when the system exhibits a spatial extent, the local dynamical variable has to be influenced by the control process. This has been achieved recently in electronics and laser systems by applying the difference signal to the control parameter. This is the basic concept of the delayed continuous feedback method (DCF) [14], i.e., the time-delay autosynchronization (TDAS) method [15]. In these two cases, it is important to note that the difference signal is not applied to the state variable, but to the control parameter, thus achieving the stabilization of the targeted UPO with a proper choice of the feedback loop parameters. The UPO can be stabilized for a small level of the control signal.

However, the specific case of the ionization waves depicted here is associated with wave propagation phenomena. It does not allow for a direct local influence of the dynamical variable, e.g., the local electron density or the local total light emission. Hence only a perturbation of the control parameter is possible, inducing a global response. In fact, as will be shown hereafter, the control information is transmitted to the probed region modulating the amplitude of the ionization waves. Consequently, an additional delay occurs between the moment of the application of the control information and the effective action in the probed section of the tube. This problem of delay due to propagation has been underestimated up to now [5].

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We are reporting here on the stabilization of a spontaneously chaotic glow discharge in the case of strongly nonlinear ionization waves by means of the modified TDAS method. However, as pointed out by So and Ott [16] in their numerical and theoretical investigations, this method is certainly far from *a priori* efficient in any nonlinear dynamical system. In fact, the necessary detailed analysis of the relevance of this method in the case of the chaotic regimes of the discharge tube is beyond the scope of this Letter.

Ionization waves have been extensively studied for 30 years and recent investigations [17,18] have shown that the glow discharge can easily exhibit chaotic behaviors. The neon discharge under pressures between 1 and 4 mbar is unstable for ionization waves, which are related to the modulation of the electron temperature and to the subsequent modulation of the ionization rate. The existence of several eigenmodes with strong couplings leads to the chaotic dynamics [19].

In our experimental situation, the proper choice of the discharge conditions leads to the easy onset of the chaotic regime of the ionization waves. The cold cathode discharge tube, 4 cm in diameter and 60 cm in length, is filled with neon gas (about 3 mbar). The discharge voltage is applied between the end electrodes, and a tiny radially movable Langmuir probe measures the floating potential fluctuations at the edge of the plasma and at a fixed axial position along the positive column. Several axially movable collimated photodetectors collect the light emitted from a 6 mm section of the positive column, without monitoring selective light emission. The current (i.e., the control parameter) can be modulated by applying a voltage modulation to the anode.

The probe potential fluctuations and the optical signals are recorded by digital storage oscilloscopes and applied to a high resolution spectrum analyzer. The time series is then sent to the computer for further processing, especially in order to reconstruct the phase-space trajectories, to estimate the dimensionality of the attractor, and to compute the spectrum of the Lyapunov exponents. Two movable optical detectors are used with a digital correlator for determining the wave numbers along the tube.

The discharge is stable in most cases at high discharge current (20 to 60 mA, depending on the gas pressure) and exhibits an unstable coherent mode decreasing the discharge current. These oscillations have been identified though interferometric measurements as typical p waves propagating from the anode to the cathode [17]. At the working pressure, the typical collision lengths are below 1 mm ensuring that there is no global effect that could interfere with the local disturbance of the electron density associated with the ionization waves. The coherent regime obtained at high discharge current switches suddenly to a chaotic state when the discharge current is sufficiently reduced.

At the same axial location, the recorded optical signal (total light emission) is very similar to the electric signal of the Langmuir probe, though the mechanisms involved in building the signal are very different. Anyway, the recorded axial phase is exactly the same using the electric or optical detector. Hence, no information is lost using only the optical detectors during the controlling process in order to have the minimum disturbance of the discharge.

A typical evolution of the power spectra is shown in Fig. 1. The first spectrum (a) depicts the periodic regime of the discharge. The multipeaked feature is characteristic of a strongly nonlinear ionization wave. Starting from this situation, a slight decrease of the discharge current induces the chaotic regime: Curve (b) depicts a broad spectrum (logarithmic vertical units, 20 dB/division). The main peak corresponds to the fundamental mode of the stable system, and the first period doubling is distinguishable.

The recorded time series (about 103 pseudoperiods) is numerically analyzed. Several methods are available for the reconstruction of the attractor, e.g., the time delays method of Takens [20]. The delay coordinate vector is computed with the properly chosen delay τ and integer N, the embedding dimension. The result is a global representation of the system. Here, the delay τ is given by the calculation of the mutual information between the points of the time series [21]. The correlation dimension of typical time series was estimated to be $D = 3.1$. The Poincaré section of the attractor gives useful information on the dynamical state of the system. The determination of the Lyapunov exponents spectrum was performed using the computation program of Kruel and Eiswirth [22], based on the algorithm of Sano and Sawada [23] and led to a positive value $\lambda = 0.02$ in typical chaotic situations.

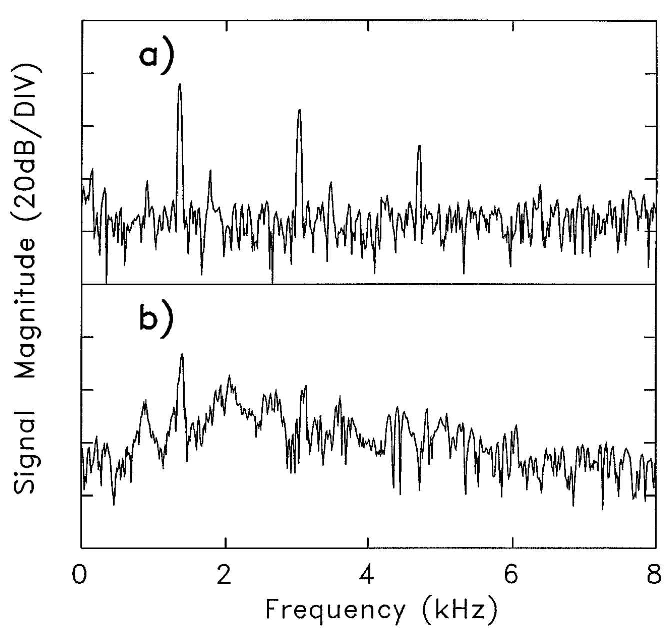

FIG. 1. Power spectra of the optical detector signal: (a) stable regime, (b) chaotic regime after several bifurcations.

For the application of the DCF method, the delay has to vary between 0.05 and 2 ms. This can be achieved only through an analog/digital (A/D) conversion of the signal and an electronic delay generated by the transit time of the data inside the memories. More exactly, the signal is digitized by means of a 12 bit A/D converter and stored in FIFO (first-in/first-out) memories whose length is numerically addressable. The transit time of the signal inside the memory is determined by the adjustable memory length and by the fixed clock frequency shifting the data inside the FIFO memory. The signal is then converted back into an analog signal.

The discharge current is adjusted in order to exhibit a chaotic response of the probe. The corresponding spectrum is similar to that displayed in Fig. 1(b). After amplification, the signal of the optical detector is delayed by a time T using the digital system described above. The delayed signal is leveled to the real-time signal and the difference signal is obtained through a conventional differential amplifier. The signal is then amplified again up to a level of a few volts and then applied to the anode. The main result is that, choosing a delay time of one period of the UPO corresponding to the highest frequency in the spectrum (fundamental mode), the control is not achieved for an arbitrary position of the detector when the signal is applied to the anode. Indeed, only two experimental situations bring out the efficiency of the control: when either the detector is very close to the anode or the detector is at a precise location along the tube, 12 cm away from the anode in the case depicted here. In the former case the propagation phenomena are of no influence, and in the latter the propagation time of the control information is exactly one period of the mode to be stabilized. The axial distance of the detector to the anode is then exactly one wavelength of the ionization wave. The DCF method is modified because the application of the control information is made one period later, due to the propagation phenomena.

Minute adjustment of the phase (polarity) and level of the control signal is required: Control is not achieved if the level is too high. The voltage perturbation is typically 5% to 10% of the discharge voltage when the control is switched on. It is below 2% when the UPO is stabilized.

It is important to note that the control is global along the tube though the detection is local. After the action of the control, the plasma is in the regular state at every location along the positive column. The detailed action of the control is depicted in Fig. 2 showing the time series of the optical signal and the control signal during the transition from the uncontrolled to the controlled state: The transient high level of the control signal corresponds to the progressive stabilization of the chosen UPO, and the remaining noncoherent control signal during the stabilized state is typical for a control scheme. A blowup of the signal is depicted in Fig. 3 before the onset of the control (a) and after the transient control sequence (b). The fundamental mode is clearly selected during the controlling process. This is also evident in Fig. 4 depicting a Poincaré section of the attractor before (a) and after (b) switching on the control.

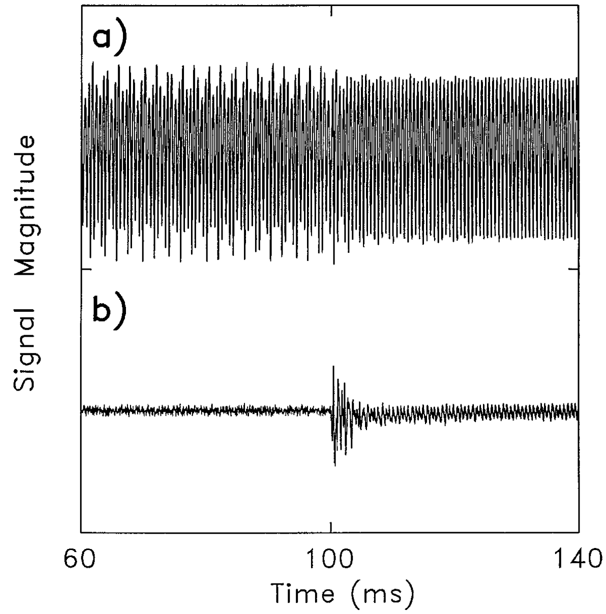

FIG. 2. Time series of the detector signal before and after switching on the control. The second trace (b) is the control signal. The fundamental mode is chosen as the UPO to be stabilized.

As mentioned above, the controllability of the present plasma waves system is very sensitive to the position of the optical probe. Indeed, a 3 mm change in the position of the detector induces the destabilization of the controlled UPO. As a consequence, the controllability of higher order UPOs is not possible at this axial position because the propagation time of the control signal from the anode is not matching the period of the chosen

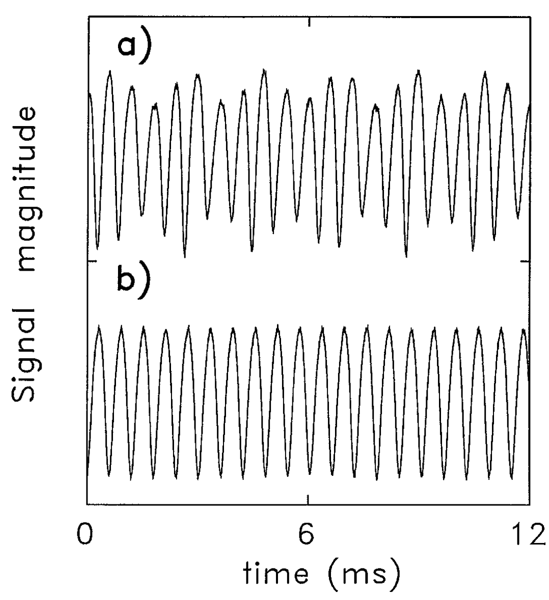

FIG. 3. Blowup of the times series before (a) and after (b) the onset of control. The fundamental mode is clearly recovered.

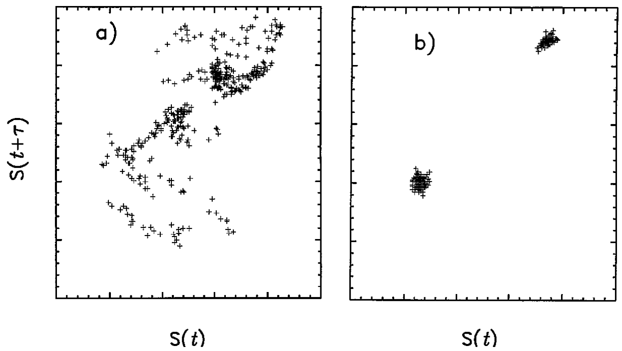

FIG. 4. Poincaré sections of the attractor corresponding to the time series before (a) and after (b) the onset of the control, exhibiting the fundamental mode selection.

subharmonics. The control is possible only when the detector is very close to the anode, but this situation is then similar to the control of a given nonlinear oscillator involving no propagation phenomena. In this latter case, we have obtained results in accordance with earlier works on the control of autonomous chaotic systems. The control of the driven chaotic discharge tube reported recently [5] was indeed obtained with the detecting probes very close to the anode [24] leading to no influence of the propagation phenomena.

To conclude, we have reported here a detailed study of the control of chaotic nonlinear ionization waves using the time delay autosynchronization method. The controllability of the system has been shown to be very sensitive to the position of the detector along the plasma column due to the intrinsic propagating character of the ionization waves.

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