Ratcheting Motion of Concentric Rings in Cellular Flames

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Ordered states of cellular flames consist of concentric rings of luminous cells. These states are observed to bifurcate to ratcheting states in which one or more rings drift ($\sim$1 deg/sec) in a circular path, speeding up and slowing down in a characteristic manner. Experimental results from three ratcheting states are presented: one in which two rings of cells ratchet together; one in which an outer ring ratchets, locked with the inner ring through an angle, until the inner ring unlocks and snaps back to its original position; and one in which an outer ring ratchets and an inner ring remains fixed. These results are discussed in the context of bifurcations in symmetric systems.

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A substantial number of recent experimental studies have documented the occurrence of ordered patterns and periodic states in fluid flows and chemically reacting systems [1]. We have previously described observations [2] of ordered states of cellular flames consisting of concentric rings of cells. This paper reports the first observation of ratcheting motion in which the slow ($\sim$1 deg/sec) drifting of a symmetric ring in a circular path is punctuated by periods of abrupt, faster angular displacement.

The appearance of traveling wave states in translationally invariant systems is well understood [3], and such states have been observed in a number of fluid systems [4]. A stationary, symmetric ordered pattern undergoes a symmetry-breaking bifurcation, producing a nonsymmetric pattern which uniformly propagates in either characteristic direction. In circularly symmetric systems such states appear as uniformly rotating states [5]. We have also observed rotating states [6] in isobutane-air cellular flames. One or more rings of cells rotate with a constant angular velocity, and the cells assume an asymmetric shape which depends on the direction of rotation. These rotating states are found in regions in parameter space which are widely separated from those of the ratcheting states.

The ratcheting states differ from uniformly rotating states in at least four significant ways: (1) their motions are not uniform, (2) the spatiotemporal symmetries of these states are more complicated, (3) the types of motion are more varied, and (4) the average angular velocity is 2 orders of magnitude smaller. The *minimum* value of the angular velocity for the rotating states is 100 deg/sec. In contrast, the *maximum* value of the average angular velocity for the ratcheting states is 1 deg/sec.

In this paper the motions of three ratcheting states are described, and measurements of angular displacement and angular velocity are presented. The relevance of these results to dynamical models based on symmetry-breaking arguments is discussed. We distinguish different states of cellular flames by labeling them according to the number of cells in each ring. A cellular pattern with twelve outer cells surrounding six cells which encircle a single central cell is designated as the 12/6/1 state. Ratcheting states are designated by attracting a W to the number of cells, indicating that a particular ring moves. Thus, $12W/6W/1$ designates a state in which the outer two rings ratchet. The examples of ratcheting states are as follows: a state in which two rings ratchet together ($12W/6W/1$); a state in which an inner and an outer ring both ratchet, but the inner ring snaps back after a critical angle is reached ($13W/6W/1$); and a state in which an outer ring ratchets around a stationary inner ring ($12W/5$) [7]. These three states are representative of the more than fifteen ratcheting states we have observed to date.

Cellular flames are formed in rich isobutane-air mixtures due to relative motion of the lighter oxidizer with respect to the heavier fuel, creating hotter (brighter) cells of enhanced chemical reaction due to the excess oxygen and cooler (darker) cusps and folds of diminished chemical reaction due to reduced oxygen. The dynamical variable is the local temperature, whose dynamics can be measured using the intensity of the chemiluminescence at a point in the flame from [8].

In the experiments a 0.5 mm thick premixed flame is stabilized at a distance of 5 mm above a 5.62 cm circular, water-cooled, porous plug burner which is housed in a glass combustion chamber at a typical dynamic pressure of 1/2 atm [9]. The control parameters in the experiment are the total flow rate of the premixed gases (isobutane and air) and the equivalence ratio of the mixture. As these parameters are slowly changed, the ordered states make transitions to other ordered states with different numbers of inner or outer cells or to dynamic states in which individual cells execute a hopping motion [10] or rings of cells rotate [6]. Ratcheting states are observed by abrupt (rather than slow) changes in a control parameter, usually near the values at which the corresponding ordered states is stable.

The state of a cellular flame can be characterized by either the positions of the luminous cells or by the locations of the three-dimensional structures—cusps and folds—which separate the cells. A line between two cells appears to be a ridge or fold in the flame surface, protruding upward away (~5 mm) from the surface of the burner. Three or more folds intersect to form a cusp. Figure 1(a) shows top and side views of the $12W/6W/1$ state. There is an optical illusion associated with the side view. Many observers see a pattern similar to water drops on the underside of a plate. The correct view is that the dark cusps and folds point away from the burner surface. They are not visible in the top view of the flame because of the limited dynamic range of $1/2''$ videotape (40 db).

Figures 1(a) and 1(c) depict the motion of the three states. These pictures were obtained from the output of a Silicon Intensified Target camera which views the flame from above through a mirror mounted at the top of the combustion chamber. The motion of the flame was recorded on videotape along with time code for later frame-by-frame analysis. Five sequential frames of digitized videotape taken at equal intervals are shown for each state. Because the effects of the motion are subtle, arrows have been drawn in order to guide the eye, as if they were attached to a particular cell. In Figure 1(a) top and side views of the $12W/6W/1$ ratcheting state are shown. In this state both rings move together. The top view emphasizes the motion of the luminous concentric rings of cells. The corresponding frames of the side view (from a low angle looking down onto the burner) show an ordered array of the darker cusps and folds at various angles. At most viewing angles, bright cells in the background shine through darker cusps and folds in the foreground along the line of sight, chopping off the tops of the dark cusps.

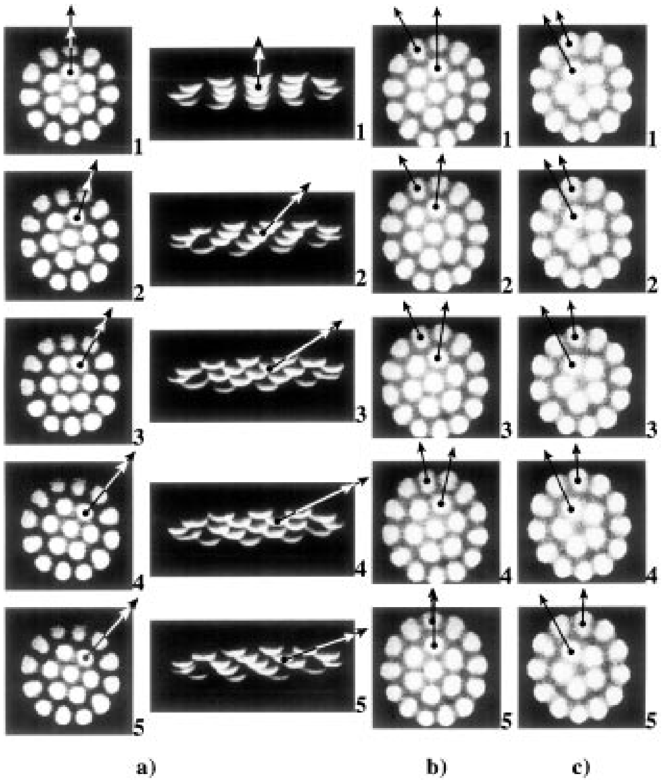

FIG. 1. Frames of the video of three ratcheting states taken at equal intervals of time: (a) top and side views of the $12W/6W/1$ state; (b) top view of the $13W/6W/1$ state, and (c) top view of the $12W/5$ state.

The printed frames in Fig. 1 suggest a rigid pattern; however, the ordered patterns are not steady. The cells make small changes in their size and shape; or, equivalently, the cusps and folds execute small amplitude oscillations about their equilibrium positions. This chaotic motion gives the drifting of the cells a shimmering quality in direct visual observation.

The motion of the $13W/6W/1$ ratcheting state of Fig. 1(b) most closely resembles that of a mechanical ratcheting device. In frames 1–4 both the outer ring and the inner ring rotate together until they reach an angle of approximately sixteen degrees. In frame 5 they separate, and the outer ring slides over the inner ring as the inner ring (quickly) returns to its original position. The locking and slipping then reoccurs with the next outer cell. The motion of the $12W/5$ ratcheting state is depicted in Fig. 1(c). While the inner ring remains fixed [11], the outer ring moves very slowly in frames 1 and 2, and significantly faster in frames 3–5. The relative motion of the two rings in the $13W/6W/1$ and $12W/5$ states produces a more complicated motion of the cusps and folds than in the $12W/6W/1$ state.

Quantitative measurements of angular cell positions were obtained from the video recordings by evaluation of frames at one second intervals. An angular grid with one degree of resolution was superimposed on each frame as it was displayed on a large monitor. The leading edge of a single cell boundary in the direction of motion was taken as a measure of the cell position. The average angular displacement was obtained by applying Simpson's rule to approximate the area under the curve formed by the point and its two neighbors. This averaging procedure reduces the effects of the faster (~5 Hz) chaotic motion which underlies the slow ratcheting. The average velocity was computed using a centered difference of the average displacement. Negative velocities are real. They arise because the cells change their size and shape due to the chaotic motion of their boundaries. At times, the amplitude of this motion is larger than the angular motion due to ratcheting, and the angular position of a cell decreases in sequential frames.

The angular displacement and angular velocity of a cell in the outer ring of the $12W/6W/1$ ratcheting state are shown in Figs. 2(a) and 2(b) for a 710 sec run. The angular displacement plot shows intervals of abrupt motion superposed on an average angular velocity of 0.6 deg/sec. The computed velocity shows sharp maxima which occur in a definite sequence. These velocity

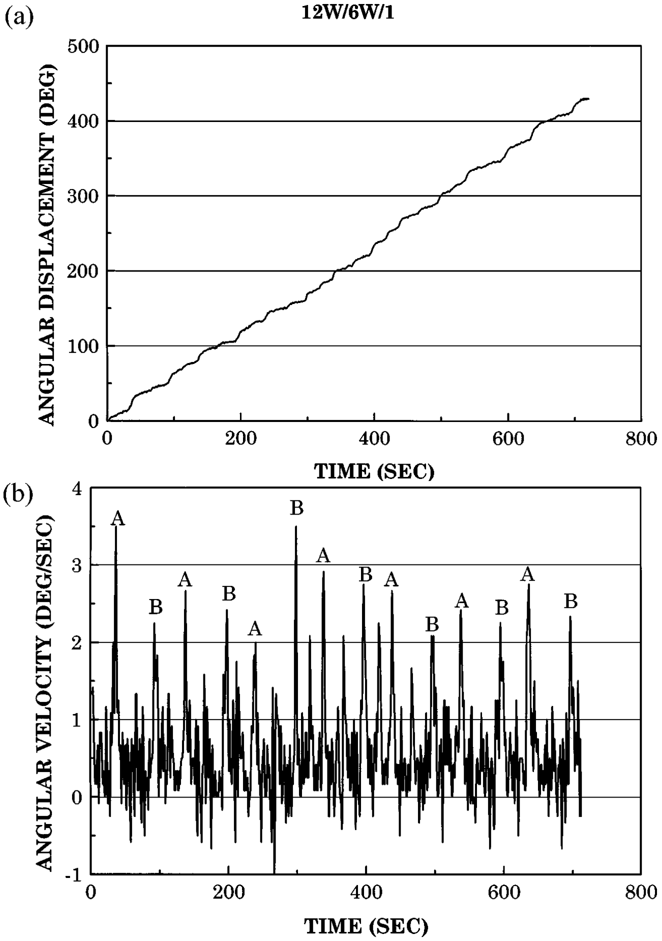

FIG. 2. (a) The average angular displacement and (b) the angular velocity of a cell in the outer ring of the $12W/6W/1$ ratcheting state in a 710 sec run. The letters A and B designate alternate velocity maxima.

maxima are doubly periodic with measured values of the period (A–B) at 59.0 ± 1.0 sec and the period (A–A) at 100 ± 1.5 sec. The angular velocity plot also suggests an additional substructure of peaks which will require an analysis at a higher temporal resolution than the 1 Hz used in these plots.

The motion of the $13W/6W/1$ ratcheting state involves the relative motion of the inner and outer rings. In order to demonstrate the variations of the angular velocity, Fig. 3 compares the *angular displacement* of a cell in the inner ring (solid line) with the *deviation in displacement from uniform rotation* [12] (of 1.1 deg/sec) of a cell in the outer ring (dotted line) for 150 sec of a 470 sec run. The motions of the two rings are highly correlated. Both rings move together until the inner ring approaches its maximum angular displacement of approximately sixteen degrees. The rapid snap back of the inner ring is followed by a significant decceleration of the outer cells as the two rings slide over each other.

The angular displacement of the outer ring in the $12W/5$ ratcheting state is shown in Fig. 4 for a 560 sec run. The outer ring spends substantial time with very low angular velocity, creating a staircaselike structure in the angular displacement plot. However, both the size of the steps (net angular displacement) and duration of the faster motion show substantial (∼30%) variations, indicating that the motion is not simply periodic. The angular velocity (not shown) has a complicated structure of velocity maxima which implies that the space-time symmetry of twelve cells moving around five cells may be more intricate than the periodic-looking plateaus in Fig. 4(a) would imply.

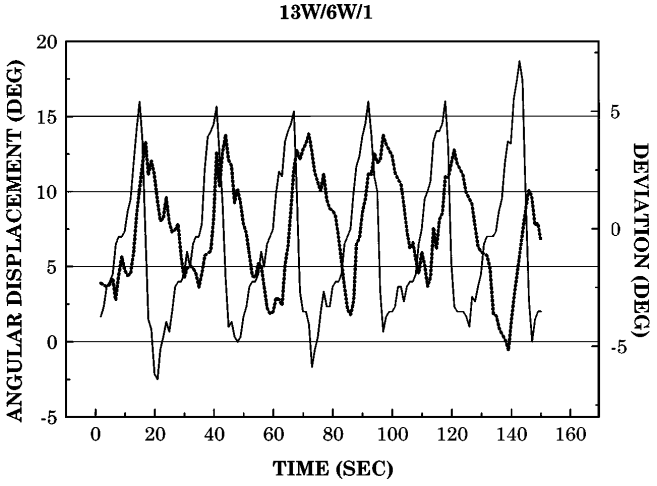

FIG. 3. A comparison of the average angular displacement of a cell in the *inner* ring (solid line) with the deviation in displacement from uniform rotation of a cell in the *outer* ring (dotted line) in the $13W/6W/1$ ratcheting state for 150 sec of a 450 sec run.

In the asymmetric periodic states, such as the rotating states, which arise as a result of secondary bifurcations, the vector field moving along the group orbit in phase space is forced to remain constant by the symmetry of the system, producing uniform rotation of the pattern in physical space. Knobloch, Hettel, and Dangelmayr [13] have presented one model which demonstrates how dynamics more complicated than uniform rotation can arise in a system with periodic boundary conditions. They break the translational symmetry of the system by invoking the

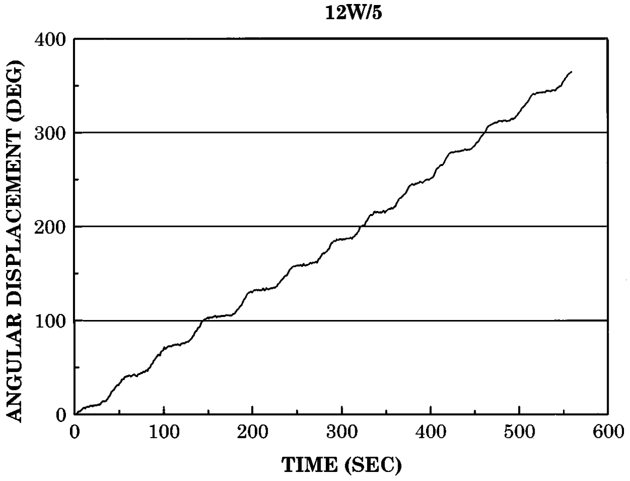

FIG. 4. The average angular displacement of a cell in the outer ring in the $12W/5$ ratcheting state for a 560 sec run.

effects of "inhomogeneities or spatial perturbations." Their inclusion into the normal form of the bifurcation equation leads to a "rich assortment of bifurcations." In their model, secondary bifurcations from the primary state of an ordered pattern produce rotating waves (RW) in which the phase of the pattern increases monotonically but nonuniformly with time. Some RW states describe patterns which propogate in either direction. As the driving parameter is changed, more complicated states are found: RW states "which travel first to the left then to the right" (motion similar to that of the inner ring of the $13W/6W/1$ state) and RW states "which propagate nonuniformly though always in the same direction undergoing regular intervals of stasis followed by fast propagation" (motion similar to that of the outer ring in the $12W/5$ state).

Their model captures some of the elements of observed ratcheting motions, demonstrating that relatively simple considerations based on symmetry can lead to complex motions similar to those in our experiments. However, other mechanisms may produce similar results. The application of any model to explain our data requires explicit consideration of the circular symmetry of the experiment. The dynamics of most ratcheting states are further complicated by interactions between noncommensurate rings of cells, a situation not present in the model system.

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