Linear Temperature Dependence of Electrical Resistivity in a Single-Impurity Model

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Using the Majorana fermion representation, we consider a compactified Anderson impurity model, which has a non-Fermi-liquid weak-coupling fixed point. The impurity free energy, self-energies, and vertex function are perturbatively formulated in terms of Pfaffian determinants. A linear temperature dependence of the electrical resistivity is obtained from the second-order perturbation. In the third order of U, the vertex function is found to be logarithmic divergent. A summation of the leading logarithmic terms gives a new weak-coupling low-temperature energy scale $T_c = \Delta \exp[-\frac{1}{9}(\frac{\pi\Delta}{U})^2]$.

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Strongly correlated electron systems, especially the high-T_c cuprate superconductors, have been the focus of intense investigation. The unusual normal state of these materials has been ascribed to a non-Fermi-liquid (NFL) state. Its microscopic origin remains to be established and may hold the key to understanding the nature of the superconductivity. It has recently been suggested that certain heavy fermion U-based superconductors may also exhibit NFL behavior due to the interaction between the conduction electrons and the localized impurity [1]. One characteristic behavior of the NFL is the linear temperature dependence of the electrical resistivity. Lattice models of these systems are very difficult to solve and so far there is no generally accepted explanation for this behavior. Impurity models which display NFL behavior may provide valuable insights. They are more accessible and most of them have been found to have exact solutions. In this context, the two-channel Kondo model is of particular interest, as it has been shown to have NFL thermodynamic behavior. The exact solution for this model, obtained from conformal field theory [2], gives a resistivity of $\rho(T) = \rho(0)(1 - a\sqrt{T})$ in the low-temperature limit, in contrast to the experimental observations on $Y_{1-x}U_xPd_3$ [3]. The question arises, therefore, whether a linear temperature dependence of resistivity can be found in any single-impurity model.

In this Letter, we consider a compactified Anderson single-impurity model introduced from the usual symmetric Anderson model by breaking down the symmetry from O(4) to O(3) in the hybridization. A new perturbation theory is constructed around the weak-coupling fixed point. In contrast to the two-channel Kondo model, the linear temperature dependence of the electrical resistivity is obtained from second order perturbation theory. The vertex function is logarithmically divergent in the third order in U.

The ordinary symmetric Anderson impurity model can be expressed in the form

$$H = it\sum_{n,\sigma}[C_\sigma^\dagger(n+1)C_\sigma(n) - \text{H.c.}] + iV\sum_\sigma[C_\sigma^\dagger(0)d_\sigma - \text{H.c.}] + U(d_\uparrow^\dagger d_\uparrow - \tfrac{1}{2})(d_\downarrow^\dagger d_\downarrow - \tfrac{1}{2}), \quad (1)$$

where the symmetric condition $\epsilon_d = -U/2$ has been used, and the chemical potential is set to zero. [The factor i can be absorbed in a redefinition of the phase of the conduction electron states but is convenient for the formulation of the Hamiltonian Eq. (2).] The Hamiltonian has O(4) symmetry due to the SU(2) symmetry from the spin rotational invariance and an additional SU(2) from particle-hole symmetry, giving O(4) $\sim$ SU(2) $\otimes$ SU(2). The O(4) symmetry can be displayed explicitly when the fermions of each type of spin are expressed in terms of four Majorana fermions [4]

$$C_\uparrow(n) = \frac{1}{\sqrt{2}}[\Psi_1(n) - i\Psi_2(n)], \quad d_\uparrow = \frac{1}{\sqrt{2}}(d_1 - id_2),$$

$$C_\downarrow(n) = \frac{1}{\sqrt{2}}[-\Psi_3(n) - i\Psi_0(n)], \quad d_\downarrow = \frac{1}{\sqrt{2}}(-d_3 - id_0).$$

These new operators satisfy $\{\Psi_\alpha(n), \Psi_\beta(n')\} = \delta_{\alpha,\beta}\delta_{n,n'}$ and $\{d_\alpha, d_\beta\} = \delta_{\alpha,\beta}$. Breaking down the symmetry from O(4) to O(3) in the hybridization, the model becomes

$$H = it\sum_n\sum_{\alpha=0}^{3}\Psi_\alpha(n+1)\Psi_\alpha(n) + iV_0\Psi_0(0)d_0 + iV\sum_{\alpha=1}^{3}\Psi_\alpha(0)d_\alpha + Ud_1d_2d_3d_0, \quad (2)$$

where $V_0 \neq V$. In the large U limit, a Schrieffer-Wolff transformation can be applied generating an *s-d* type of model: the so-called compactified two-channel Kondo model, where the local impurity spin couples to both the conduction electron spin and the conduction electron "isospin" (charge) density. When $V_0 = 0$, the two exchange couplings are identical and it had been conjectured that this form of the model has the same low-energy excitations as the two-channel Kondo model [5].

To distinguish the model in the form of Eq. (2) from others, we will refer to it as the compactified Anderson impurity model. Here we concentrate on the $V_0 = 0$ case. Fourier transforms for the conduction electrons can be introduced as usual

$$\Psi_\alpha(n) = \frac{1}{\sqrt{N}} \sum_k \Psi_\alpha(k) e^{ikx_n}, \quad \alpha = 0,1,2,3, \tag{3}$$

where N is the total number of the sites. The anticommutation relation for the conduction electrons becomes $\{\Psi_\alpha(k), \Psi_\beta(-k')\} = \delta_{\alpha,\beta}\delta_{k,k'}$. The new symmetry in hybridization is the key feature of this model. Since the scalar field ($\alpha = 0$) of the conduction electrons decouples from the local impurity, its propagator defined by retarded double-time correlation function is easily found to be a free propagator: $\langle\langle \Psi_0(k) | \Psi_0(-k') \rangle\rangle = \frac{\delta_{k,k'}}{i\omega_n - \epsilon_k}$, where $\epsilon_k = 2t \sin(ka)$ is the dispersion relation of the conduction electrons, a is the lattice spacing, $\omega_n = (2n + 1)\pi/\beta$, and β is the inverse of the temperature. The vector field $\Psi_\alpha(k)$ ($\alpha = 1,2,3$) hybridizes with the impurity vector field d_α, and the scattering of the conduction electrons from the local impurity is given by the following relation:

$$\langle\langle \Psi_\alpha(k) | \Psi_\alpha(-k') \rangle\rangle = \frac{\delta_{k,k'}}{i\omega_n - \epsilon_k} + \frac{V^2}{N} \frac{G_{\rm vec}(\omega_n)}{(i\omega_n - \epsilon_k)(i\omega_n - \epsilon_{-k'})}, \tag{4}$$

where $G_{\rm vec}(\omega_n)$ is the Fourier transform of the vector propagator $-\langle T_\tau d_\alpha(\tau) d_\alpha(\tau')\rangle_H$. The conduction electron t matrix is thus expressed as $t(\omega_n) = \frac{V^2}{N} G_{\rm vec}(\omega_n)$, and *the electrical resistivity will be determined by the impurity vector field propagator only*.

Before considering the effects of interactions, it is useful to examine the unperturbed part H_0 ($U = 0$). The impurity Green functions are easily obtained

$$G_0(\omega_n) = \frac{1}{i\omega_n}, \quad G_\alpha(\omega_n) = \frac{1}{i\omega_n + i\Delta \,{\rm sgn}\,\omega_n},$$

where $\Delta = \pi\rho V^2$ is the hybridization width, $\rho = (hv_f)^{-1}$ is the conduction electron density of states, and $\omega_n = (2n + 1)\pi/\beta$. Here we find that the impurity scalar propagator is a fermionic zero mode with $G_0(\tau) = -{\rm sgn}\,\tau/2$, and both propagators are odd in their arguments. The local impurity spectral function is given by $A_d(\omega) = \frac{3}{2\pi}\frac{\Delta}{\omega^2 + \Delta^2} + \frac{1}{2}\delta(\omega)$, and reveals the basic physics of the unperturbed Hamiltonian. The change of conduction electron's free energy due to the hybridization is $F_{\rm imp}^{(0)} = \frac{3}{2\pi}\int_{-\infty}^{\infty} f(\omega) \tan^{-1}(\frac{\Delta}{\omega})\, d\omega - \frac{T}{2}\ln 2$, where $f(\omega)$ is Fermi distribution function, and the impurity residue entropy is $\ln\sqrt{2}$, reflecting the unusual impurity spectral function. Since the impurity spin and charge density operators can be defined

$$S_d^z = \frac{1}{2}(d_\uparrow^\dagger d_\uparrow - d_\downarrow^\dagger d_\downarrow) = -\frac{i}{2}(d_1 d_2 - d_0 d_3),$$
$$n_d = \frac{1}{2}(d_\uparrow^\dagger d_\uparrow + d_\downarrow^\dagger d_\downarrow - 1) = -\frac{i}{2}(d_1 d_2 + d_0 d_3), \tag{5}$$

the spin and charge density-density correlation functions are equal and their Fourier transforms are

$$\chi_{\rho,\sigma}(\omega_n) = \frac{1}{4\beta}\sum_{\omega_{n'}} [G_\alpha(\omega_{n'}) G_\alpha(\omega_n - \omega_{n'}) + G_\alpha(\omega_{n'}) G_0(\omega_n - \omega_{n'})], \tag{6}$$

where $\omega_n = 2n\pi/\beta$ is the bosonic Matsubara frequency. The first term in the brackets corresponds to the normal FL-like density-density spectrum, but the second term is anomalous. As far as the singularity is concerned, the imaginary part of the spectral functions is obtained: $\chi''_{\rho,\sigma}(\omega, T) = -\frac{1}{8}\frac{\Delta}{\omega^2 + \Delta^2}\tanh(\frac{\omega}{2T})$. When $\omega \ll T$, $\chi''_{\rho,\sigma} \sim \omega/T$, while for $\omega \gg T$, $\chi''_{\rho,\sigma} \sim$ constant. Such a behavior was assumed by the marginal FL phenomenology [6], and it is believed the ordinary FL theory has broken down.

Now we consider the perturbed Hamiltonian. The partition function is expressed as a power series of U:

$$Z/Z_0 = \sum_{n=0}^{\infty} U^n \int_0^\beta d\tau_n \int_0^{\tau_n} d\tau_{n-1} \cdots \int_0^{\tau_2} d\tau_1 \times F_n(\tau_n, \ldots, \tau_1),$$

where Z_0 denotes the partition function for the unperturbed Hamiltonian, and the thermodynamic average is carried out over the unperturbed part H_0, expressed as $\langle \cdots \rangle$. Noting that in H_0 the four Majorana components of the local impurity decouple completely. $F_n(\tau_n, \tau_{n-1}, \ldots, \tau_1)$ can be factorized as

$$\langle d_0(\tau_n) \cdots d_0(\tau_1)\rangle \prod_{\alpha=1}^{3} \langle d_\alpha(\tau_n) \cdots d_\alpha(\tau_1)\rangle .$$

When the Wick theorem is implemented, it can be verified order by order that each Majorana expectation average can be represented by a Pfaffian determinant. For the expectations of impurity vector operators, the Pfaffian determinant [7] is defined by the square root of an antisymmetric determinant composed of the impurity vector propagator $G_\alpha(\tau)$,

$$\begin{vmatrix} 0, & G_\alpha(\tau_1 - \tau_2), & G_\alpha(\tau_1 - \tau_3), & \ldots, & G_\alpha(\tau_1 - \tau_n) \\ G_\alpha(\tau_2 - \tau_1), & 0, & G_\alpha(\tau_2 - \tau_3), & \ldots, & G_\alpha(\tau_2 - \tau_n) \\ \vdots & \vdots & \vdots & & \vdots \\ G_\alpha(\tau_n - \tau_1), & G_\alpha(\tau_n - \tau_2), & G_\alpha(\tau_n - \tau_3), & \ldots, & 0 \end{vmatrix} = \backslash D_n(\tau_1, \tau_2, \ldots., \tau_n)|^2.$$

Since the impurity vector propagators are odd in its argument, $G_\alpha(\tau_r - \tau_s) = -G_\alpha(\tau_s - \tau_r)$ for $r > s$ and $G_\alpha(\tau_r = \tau_s) = 0$. Then, $\backslash D_n(\tau_1, \tau_2, \ldots, \tau_n)|$ is given by

$$\left| \begin{array}{ccc} G_\alpha(\tau_1 - \tau_3), & \ldots, & G_\alpha(\tau_1 - \tau_n) \\ G_\alpha(\tau_2 - \tau_3), & \ldots, & G_\alpha(\tau_2 - \tau_n) \\ & & \vdots \\ & & G_\alpha(\tau_{n-1} - \tau_n) \end{array} \right|$$
$$= \sum \pm G_\alpha(\tau_1 - \tau_a) G_\alpha(\tau_b - \tau_c) \ldots G_\alpha(\tau_l - \tau_m),$$

where the subscripts $1, a, b, c, \ldots, l, m$ of each term under the summation are a permutation of the first n integers, each Green function $G_\alpha(\tau_r - \tau_s)$ has $s > r$, all different terms of this type are included, and the total number of terms is $(n-1)!!$. The sign attached to each term is positive or negative according to whether the permutation is even or odd. The basic property of the Pfaffian determinant is that all odd-order determinants identically vanish. On the other hand, the impurity scalar propagator has a special form $G_0(\tau) = -\mathrm{sgn}\,\tau/2$, and the corresponding expectation is $\langle d_0(\tau_{2n}) d_0(\tau_{2n-1}) \cdots d_0(\tau_1) \rangle = (\frac{1}{2})^n$ because the imaginary time sequence has been assumed $\beta > \tau_{2n} > \tau_{2n-1} > \cdots > \tau_2 > \tau_1 > 0$. Therefore, the partition function for H is formulated by the cube of the Pfaffian determinant. According to the linked cluster theorem, the free energy associated with the local impurity is given by

$$F_{\mathrm{imp}} = F_{\mathrm{imp}}^{(0)} - \sum_{n=1}^{\infty} \left(\frac{U}{\sqrt{2}} \right)^{2n} \frac{1}{\beta} \int_0^\beta d\tau_{2n} \int_0^{\tau_{2n}} d\tau_{2n-1} \times \cdots \int_0^{\tau_2} d\tau_1 \{ \backslash D_2 n(\tau_1, \ldots, \tau_{2n})|\}_l^3 , \quad (7)$$

where the subscript l on the bracket indicated that only linked diagrams are to be considered. For the ordinary symmetric Anderson model with O(4) symmetry ($V_0 = V$), the power of the Pfaffian determinant in the free energy is four rather than three [8], thus one power corresponds to each of the Majorana fermions involved in the hybridization. The first singular contribution to the free energy comes from the second order of U, and the impurity specific heat is singular $C_{\mathrm{imp}} \approx \frac{\pi^2}{2} (\frac{U}{\pi \Delta})^2 \frac{T}{\pi \Delta} \ln(\frac{\Delta}{T})$ in the limit $T \ll \Delta$.

The perturbed propagator for the impurity scalar field is defined as $G_{\mathrm{sc}}(\tau, \tau') = -\langle T_\tau d_0(\tau) d_0(\tau') \rangle_H$. As we treated the partition function, the perturbed scalar propagator in its Fourier transform can be expanded in powers of U:

$$G_{\mathrm{sc}}(\omega_n) = G_0(\omega_n) + \sum_{n=1}^{\infty} \frac{U^{2n}}{\beta} \int_0^\beta d\tau \int_0^\beta d\tau' \int_0^\beta d\tau_{2n} \int_0^{\tau_{2n}} d\tau_{2n-1} \cdots \int_0^{\tau_2} d\tau_1 e^{i\omega_n(\tau - \tau')} \times \{ \langle T_\tau d_0(\tau) d_0(\tau') d_0(\tau_{2n}) \cdots d_0(\tau_1) \rangle \backslash D_2 n(\tau_1, \ldots, \tau_{2n})|^3 \}_l . \quad (8)$$

Recall $G_0(\tau) = -\mathrm{sgn}\,\tau/2$, the expectation value $-\langle T_\tau d_0(\tau) d_0(\tau') d_0(\tau_{2n}) \cdots d_0(\tau_1) \rangle$ can be calculated as

$$\left(\frac{1}{2} \right)^{n-1} \sum_{i<j} (-1)^{i+j} \{ G_0(\tau - \tau_i) G_0(\tau' - \tau_j) - G_0(\tau - \tau_j) G_0(\tau' - \tau_i) \} + \left(\frac{1}{2} \right)^n G_0(\tau - \tau').$$

Then completing the integrals over τ and τ', we obtain $G_{\mathrm{sc}}(\omega_n) = G_0(\omega_n) + G_0(\omega_n) \Sigma'_{\mathrm{sc}}(\omega_n) G_0(\omega_n)$, where an improper self-energy is represented as

$$\Sigma'_{\mathrm{sc}}(\omega_n) = \sum_{n=1}^{\infty} \frac{4i}{\beta} \left(\frac{U}{\sqrt{2}} \right)^{2n} \int_0^\beta d\tau_{2n} \cdots \int_0^{\tau_2} d\tau_1 \sum_{i<j} \{ (-1)^{i+j} [\sin \omega_n(\tau_i - \tau_j)] \backslash D_2 n(\tau_1, \ldots, \tau_{2n})|^3 \}_l . \quad (9)$$

In the second order of U, the self-energy for the impurity scalar field can be obtained $\Sigma'_{\mathrm{sc}}(\tau) = -U^2 G^3(\tau)$ corresponding to a diagram (a) in Fig. 1. The imaginary part of its retarded Fourier transform is

$$\mathrm{Im}\Sigma'_{\mathrm{sc}}(\omega, T) \approx -\frac{\pi U^2}{2(\pi \Delta)^3} [\omega^2 + (\pi T)^2].$$

Such a self-energy ensures the fermionic zero mode of the scalar field in H_0 is preserved.

The impurity vector propagator $G_{\mathrm{vec}}(\tau, \tau')$ is defined as similar way, and $G_{\mathrm{vec}}(\omega_n)$ can be written as

$$G_{\mathrm{vec}}(\omega_n) = G_\alpha(\omega_n) + \sum_{n=1}^{\infty} \left(\frac{U}{\sqrt{2}} \right)^{2n} \frac{1}{\beta} \int_0^\beta d\tau \int_0^\beta d\tau' \int_0^\beta d\tau_{2n} \int_0^{\tau_{2n}} d\tau_{2n-1} \cdots \int_0^{\tau_2} d\tau_1 e^{i\omega_n(\tau - \tau')} \times \{ \langle T_\tau d_\alpha(\tau) d_\alpha(\tau') d_\alpha(\tau_{2n}) \cdots d_\alpha(\tau_1) \rangle \backslash D_2 n(\tau_1, \ldots, \tau_{2n})|^2 \}_l .$$

The average $-\langle T_\tau d_\alpha(\tau) d_\alpha(\tau') d_\alpha(\tau_{2n}) \cdots d_\alpha(\tau_1) \rangle$ is evaluated as follows:

$$\sum_{i<j} (-1)^{i+j} \{ G_\alpha(\tau - \tau_i) G_\alpha(\tau' - \tau_j) - G_\alpha(\tau - \tau_j) G_\alpha(\tau' - \tau_i) \} \backslash D_{2n}^{ij}| + G_\alpha(\tau - \tau') \backslash D_{2n}| ,$$

where $\backslash D_{2n}^{ij}|$ is the so-called *cofactor* of Pfaffian determinant $\backslash D_{2n}|$. Then we perform the integrals over τ and τ', and get the equation of the impurity vector propagator, $G_{\mathrm{vec}}(\omega_n) = G_\alpha(\omega_n) + G_\alpha(\omega_n) \Sigma'_{\mathrm{vec}}(\omega_n) G_\alpha(\omega_n)$,

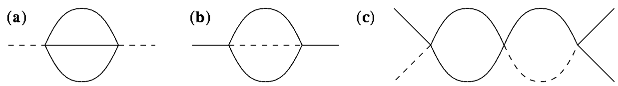

FIG 1. Diagrams (a) and (b) are the second order corrections for impurity scalar and vector self-energies, respectively. Diagram (c) is the logarithmic contribution to the vertex function in the order of U^3. The dotted lines describe $G_0(\tau)$ and the solid lines are $G_\alpha(\tau)$.

where the improper self-energy is

$$\Sigma'_{\rm vec}(\omega_n) = \sum_{n=1}^{\infty} \frac{2i}{\beta}\left(\frac{U}{\sqrt{2}}\right)^{2n}\int_0^{\beta} d\tau_{2n}\cdots\int_0^{\tau_2} d\tau_1 \times \sum_{i<j}\{(-1)^{i+j}[\sin\omega_n(\tau_i-\tau_j)]\backslash D^{ij}_{2n}(\tau_1,\ldots,\tau_{2n})| \times \backslash D_{2n}(\tau_1,\ldots,\tau_{2n})|^2\}_l . \tag{11}$$

In the second order of U, the self-energy is found to be $\Sigma'_{\rm vec}(\tau) = -U^2G^2(\tau)G_0(\tau)$ corresponding to a diagram (b) in Fig. 1. We have to calculate its spectral function carefully because there is a singularity in the $\omega = 0$ limit, and the result is

$$\mathrm{Im}\,\Sigma'_{\rm vec}(\omega, T) = -\frac{\pi}{2}\left(\frac{U}{\pi\Delta}\right)^2|\omega|\coth\left(\frac{|\omega|}{2T}\right).$$

In the case of $|\omega| \ll T$, the imaginary part of the retarded self-energy is $\mathrm{Im}\,\Sigma'_{\rm vec} \sim -(\frac{U}{\pi\Delta})^2(\pi T)$, while for $|\omega| \gg T$, it becomes $\mathrm{Im}\,\Sigma'_{\rm vec} \sim -\frac{\pi}{2}(\frac{U}{\pi\Delta})^2|\omega|$. Such a self-energy greatly differs from the form given by the FL theory.

The self-energy is usually referred to the proper self-energy, which is related to the improper one by $\Sigma(\omega_n) = \Sigma'(\omega_n)[1 + G_0(\omega_n)\Sigma'(\omega_n)]^{-1}$. As far as the second order contributions are concerned, it is not necessary to distinguish the improper self-energy from the proper one. Thus there is also a temperature dependent contribution to $\mathrm{Im}\,\Sigma_{\rm vec}(0, T)$, and the conduction electron t matrix includes this temperature dependence as well. Assuming the conduction electrons incoherently scatter from the dilute magnetic impurities, the total number is assumed to be $N_{\rm imp}$, and the linear response theory allows the electrical conductivity to be expressed as

$$\sigma(T) = -\frac{2}{3}e^2v_f^2\rho\int_{-\infty}^{\infty}\tau(\omega, T)\frac{\partial f}{\partial\omega}\,d\omega\,, \tag{12}$$

here v_f is the Fermi velocity of the conduction electrons with charge e and density of states ρ, and $\tau(\omega, T)$ is the electron relaxation time, which is related to the t matrix $\tau^{-1} = -2N_{\rm imp}\,\mathrm{Im}\,t(\omega, T)$. On substituting the second order result for the impurity vector self-energy, the electrical resistivity is found to be

$$\rho(T) \approx \frac{3\pi n_{\rm imp}}{e^2}\left[1 + \left(\frac{U}{\pi\Delta}\right)^2\left(\frac{\pi T}{\Delta}\right)\right],$$

where $n_{\rm imp}$ is impurity concentration. The linear temperature dependence is the consequence of the anomalous $\Sigma_{\rm vec}(\omega_n)$, and *such a resistivity makes the weak coupling fixed point of the present model differ from the strong coupling fixed point of the two-channel Kondo model*.

In fact the method of evaluating the single-particle correlation functions can be applied further to the two-particle vertex functions. The nontrivial two-particle correlation function is $\langle T_\tau d_0(\tau)d_1(\tau')d_2(\tau'')d_3(\tau''')\rangle_H$. Following similar treatments, we find the expression for $\Gamma'_{0,1,2,3}(\omega, \omega', \omega'', \omega''')$, and when all the frequencies are set to zero, the improper vertex function is given by

$$\Gamma'_{0,1,2,3}(0) = \sum_{n=1}^{\infty}\frac{\sqrt{2}}{\beta}\left(\frac{U}{\sqrt{2}}\right)^{2n-1}\int_0^{\beta} d\tau_{2n-1}\cdots\int_0^{\tau_2} d\tau_1 \times \left\{\sum_i(-1)^i\backslash D^i_{2n-1}(\tau_1,\ldots,\tau_{2n-1})|\right\}^3_l . \tag{14}$$

To first order in U, $\Gamma'^{(1)}_{0,1,2,3}(0) = -U$, while to order U^3, a logarithmic singularity appears and its leading contribution given by a diagram (c) in Fig. 1 is found to be $\Gamma'^{(3)}_{0,1,2,3}(0) \approx -3U(\frac{U}{\pi\Delta})^2\ln(\frac{\Delta}{T})$ in the limit $T \ll \Delta$. The proper vertex function has the same singularity as well, which implies that the higher order terms in the perturbation expansion have important contributions to the low-temperature behavior, so the marginal FL behavior must break down at very low temperatures. When all the leading logarithmic terms are summed, a characteristic temperature $T_c = \Delta\exp[-\frac{1}{9}(\frac{\pi\Delta}{U})^2]$ is found [9], below which the NFL state of the weak coupling fixed point is unstable; while for $T > T_c$, the low order perturbation results can be justified. In this respect, like the ordinary Kondo problem, the logarithmic divergent vertex poses a new problem as to the nature of the stable ground state of this compactified Anderson model and its low-temperature behavior. In addition, so far it is not clear under what circumstances one can expect the symmetry breaking in the hybridization to occur. O(4) symmetry in the symmetric Hubbard model at half filling is known to be reduced to O(3) upon doping. These problems are currently under investigation.

In conclusion, we have developed a Pfaffian perturbation expansion appropriate for the Majorana formalism of the compactified Anderson impurity model, and the linear temperature dependence of the electrical resistivity is obtained in the second order theory.

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