Energy Dissipation in Dynamic Fracture

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Measurements in PMMA of both the energy flux into the tip of a moving crack and the total surface area created via the microbranching instability indicate that the instability is the main mechanism for energy dissipation by a moving crack in brittle, amorphous material. Beyond the instability onset, the rate of fracture surface creation is proportional to the energy flux into the crack. At high velocities microbranches create nearly an order of magnitude larger fracture surface than smooth cracks. This mechanism provides an explanation for why the theoretical limiting velocity of a crack is never realized.

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Although the subject of much research over the past decades, the fracture of brittle amorphous materials remains in many ways not understood. Of particular interest is the mechanism by which energy in the system is dissipated. Experimental measurements of the flow of energy into the tip of a running crack [1] have indicated that the fracture energy (i.e., the energy needed to create a unit extension of a crack) is a strong function of the crack's velocity and that the majority of the energy stored in the system prior to the onset of fracture ends up as heat [2]. In this Letter we present quantitative measurements indicating that this increased dissipation is due entirely to the onset of a microbranching instability [3,4] which occurs at a critical value v_c of the velocity v. As v increases beyond v_c we find that the energy needed to create microbranches is precisely enough to account for the velocity dependence of the fracture energy.

The long-standing problem of the limiting velocity of a crack is also explained by this mechanism. While linear elastic theory predicts that a crack should continuously accelerate up to the Rayleigh wave speed V_R, experiments in a number of brittle materials [5] show that a crack will seldom reach even half of this value. As we will show, the total amount of fracture surface created by both the main crack and the microbranches increases rapidly with v. Thus, rather than acceleration, increased driving results in increased ramification of structure below the fracture surface.

There have been a number of suggestions for the velocity dependence of fracture energy. One view is that the energy flow into the tip of a single moving crack is dissipated by plastic deformation around the crack tip. Depending on the model used to describe the area of deformation around the tip, either a nonmonotonic or monotonically increasing function [6] of the velocity of the crack can result. An alternative view of the dissipation process was suggested by Ravi-Chandar and Knauss [7]. They viewed the fracture process as the coalescence of preexisting microvoids or defects situated in the path of the crack and activated by the intense stress field at the crack tip. An increase in the energy flux to the tip, in this picture, causes an increase in the number of microcracks formed and thereby enhanced dissipation. This picture suggests that crack propagation via interacting microvoids occurs as a randomly activated process.

Recent experiments [4] on brittle PMMA (polymethylmethacrylate) offer a different view. The formation and evolution of microcracks were seen to be the result of a *dynamic instability* of a moving crack. A sharp transition from a single propagating crack to an ensemble of cracks occurs above the critical velocity of $v_c = 0.36V_R$ when a single crack sprouts small microscopic side branches (microbranches). As a function of the velocity v, these branches grow as the mean dynamics of a crack change dramatically; the mean acceleration drops, the velocity develops oscillations, and structure is formed on the fracture surface [3,8].

There have been a number of recent theoretical attempts to model this instability, both on a mesoscopic scale [9] as well as in the context of lattice models [10] and finite element calculations [11], where local crack branching at a critical velocity has been observed. Many of the experimental results have also been observed in molecular dynamic simulations [12].

In this Letter we measure both the mean energy flux into the crack tip and the total surface area formed as a function of the velocity of a crack. We find that the total surface area, resulting from the dynamic generation of microbranches above v_c, increases linearly with the energy flux into the crack tip. This mechanism thus provides a simple dynamical description of the apparent velocity dependence of the fracture energy (or energy dissipation by the crack). We also find that the fluctuations in the crack velocity are proportional to the energy flux into the tip, supporting the view that velocity fluctuations result from the dynamic transfer of energy between the main crack and the frustrated local branches.

The experimental apparatus is similar to that described in [3]. Our experiments were conducted in thin, quasi-2D sheets of brittle, cast PMMA [13] having a thickness of

either 0.8 or 3 mm with vertical (parallel to the direction of applied stress or "*Y*" direction) and horizontal (parallel to the propagation or "*X*" direction) dimensions between 50 and 450 mm and 200 and 400 mm, respectively. The sample geometry was varied to provide either steady-state crack propagation at a given energy density within the sample or continuous acceleration of the crack throughout the experiment.

Steady-state propagation is achieved by using a thin strip configuration with the ratio of the vertical to horizontal dimensions of the sample typically between 0.25 and 0.5. Stress is applied via a uniform displacement of the vertical boundaries of the plate. If the crack tip is sufficiently far from the horizontal boundaries of the system this geometry approximates an infinitely long strip with "translational invariance" in the direction of propagation. This invariance is realized when the crack reaches a length of about half of the vertical size of the system. At this point, advance of the crack by a unit length frees an amount of energy equal to the (constant) energy per unit length stored in the plate far ahead of the crack. Under these conditions, a crack arrives at a state of constant mean velocity with the energy flux into the tip per unit crack extension G, given by $\sigma^2 L/(2E)$. Here σ is the applied stress at the vertical boundaries, L the vertical size of the system, and E the Young's modulus. Using this sample geometry we have a direct measurement of G with an 8% accuracy. In the steady-state experiments described, G was varied between 800 and 5000 $\mathrm{J/m^2}$. Experiments were also performed with vertical to horizontal sample size ratios of up to 2 to obtain a continuously accelerating crack. In this configuration G is an increasing function of the crack's length although we do not have a direct measure of G as the crack progresses across the plate.

Upon fracture initiation, the location of the crack tip was measured as in [3] at 0.1 μsec intervals with a 0.1 mm spatial resolution yielding a velocity resolution of better than 25 m/s. After fracture, the crack profile in the *X*-*Y* plane was measured optically with a spatial resolution of 1–5 μm. The optical measurements were then correlated with the velocity and energy flux measurements.

In Fig. 1 we show examples of steady-state running cracks at velocities both above and below the instability onset together with magnified pictures in the *X*-*Y* plane of the fracture surface created at these velocities. As in [4] we see that above v_c local crack branching occurs with the mean length of the branches increasing with v. For $v < v_c$ the fracture surface created by a crack is just twice the length of the crack times the sample width but as microbranches develop, the total fracture surface increases with the velocity of the crack as shown in Fig. 2. Figure 2 indicates that the total fracture surface [14] formed is a well-defined function of the mean velocity of the crack, independent of either the crack acceleration or the experimental conditions. The amount of surface created by the microbranches can be considerable, reaching velocities approaching 600 m/s,

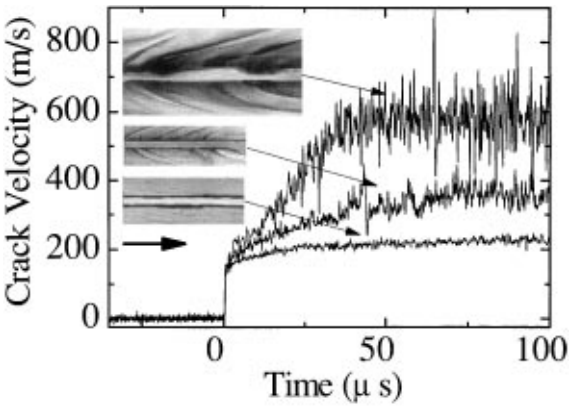

FIG. 1. Velocity profiles of cracks in the strip geometry, together with the corresponding images of the fracture surface in the *X*-*Y* plane. The thick arrow, of length 0.25 mm, indicates the direction of propagation. For $v < v_c = 340$ m/s (bottom) neither velocity oscillations nor microbranches are observed. Above v_c the microbranching instability is indicated by the appearance of local side branches from the main crack accompanied by velocity fluctuations (middle). These effects increase with velocity (top).

over five times the surface formed by a single crack. This additional surface is over an order of magnitude larger than the surface increase due to "roughness" of the fracture surface.

Before the onset of fracture a large amount of potential energy is stored in the elastic field of the material. The sink for this energy is at the tip of the crack where the stress field is high enough to separate atomic bonds creating new surface as the crack advances. Besides creating new surface, the energy can also either excite motion (kinetic energy) or cause plastic deformation of the material. The large scale motion of the medium is seen as acoustic waves in the material. The small scale motion, together with part of the energy used in plastic deformation, generates heat along the crack face. The remainder of the plastic energy acts to distort the material in the vicinity of the crack. Thus the total fracture energy

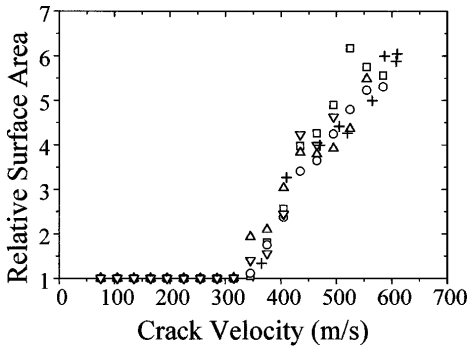

FIG. 2. The surface area formed per unit crack extension as a function of the mean crack velocity (smoothed over 5 mm) for steady-state velocity (cross) and accelerating cracks driven by stored energies of 3.2×10^6 (inverted triangles), 4.7×10^6 (circles), 4.8×10^6 (squares), and 5.1×10^6 $\mathrm{erg/cm^2}$ (upright triangles).

of the material is divided into the generation of heat, acoustic waves, and surface energy (where by surface energy we mean both the energy needed to break bonds together with that stored in any plastic deformation of the material around the crack faces). Doll [2] found that most of the stored energy ends up as heat. This is supported by Gross *et al.* [15] who observed that a maximum of about 3% of G appeared in acoustic emission.

How much energy is needed to cause a crack to propagate at a given velocity? G, as determined from steady-state experiments in strip geometries, is presented in Fig. 3 as a function of the mean crack velocity. The strong velocity dependence of G is evident. If we were to imagine that the amount of energy necessary to break bonds is more or less independent of the rate at which they are broken and we assume that a *single* fracture surface is created by a moving crack, we would naturally conclude that dissipation in fracture is mainly due to plastic deformation of the material as assumed in [6]. The data presented in Fig. 2 suggest an alternative explanation. In Fig. 4 we plot the total surface area formed, normalized by the area that would be formed by a single crack, as a function of G. The data plotted were obtained from 3 mm wide sheets for samples in the strip geometry where the cracks propagated at steady-state mean velocities. After an initial jump near v_c, the total amount of surface created is linearly dependent on the amount of energy flowing into the crack tip. The inverse slope of the line (1.0×10^6 erg/cm^2), equal to twice the fracture energy, indicates that nearly all of the total stored energy simply goes into creating new surface. This value agrees well with the value of G (1.1×10^6 erg/cm^2) immediately preceding v_c. Mechanisms such as plastic deformation may indeed play a role in determining the basic cost in energy needed to form a unit surface, but the *enhanced* dissipation observed as crack velocities increase beyond v_c is of dynamic origin. This dissipation is the direct result of combining a fixed amount of energy expended per unit surface with the large increase in fracture surface production caused by the branching instability. The jump in Fig. 4 near v_c may reflect an inhomogeneous stress distribution along the sample thickness (see [14]) or hysteretic behavior resulting from the fact that microbranches have a *finite* minimum size.

At first glance a constant value for the fracture energy may seem rather obvious as, naively, there is no *a priori* reason that the energy needed to "break bonds" should be strongly dependent on the velocity of the crack. Upon a closer look at the processes that contribute to the fracture energy in PMMA, the constant value for the fracture energy is not at all trivial. The value of 5×10^5 erg/cm^2 obtained for the fracture energy [16] is 3 orders of magnitude larger than the value of the energy expended in breaking bonds in the material. Most of the fracture energy [17] is consumed in complex, rate dependent processes such as the shearing of the long molecules of which PMMA is composed. The fracture energy is *not* constant for $v < v_c$. Although dwarfed by the increase resulting from the microbranching instability, there is an increase of approximately 30% in G as a result of a different (velocity-dependent) dissipative process for velocities between 0.2 and $0.35V_R$.

How do the instantaneous dynamics of a crack depend on G? Let us now consider the fluctuating component of the propagation velocity. In [4] the velocity oscillations were described by a dynamic exchange of energy between the main crack and the locally branching cracks. In this view the main crack accelerates until arriving at v_c where branching then occurs. At this time, energy is diverted to the "daughter" cracks causing the main crack to slow. When the daughter crack dies, the energy is rediverted to the main crack which consequently accelerates until the occurrence of the next branching event. In this picture we might expect that the amount of energy diverted to the daughter cracks should scale with the total energy G. The fluctuations of the velocity should mirror this and increase with G. In Fig. 5 we plot the measured rms velocity fluctuations of cracks propagating in the strip geometry as a function of G. The expected increase in velocity

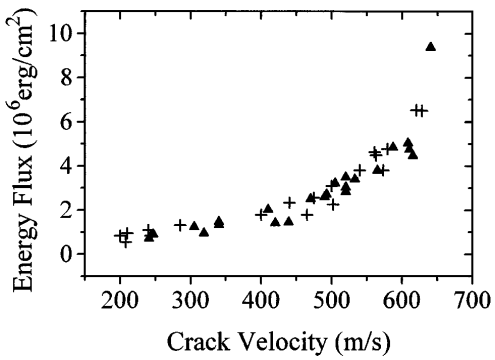

FIG. 3. The energy flux into the crack tip (G) as a function of steady-state crack velocity. The data plotted were obtained from both 0.8 (cross) and 3 mm (triangle) thick plates, in strip geometry.

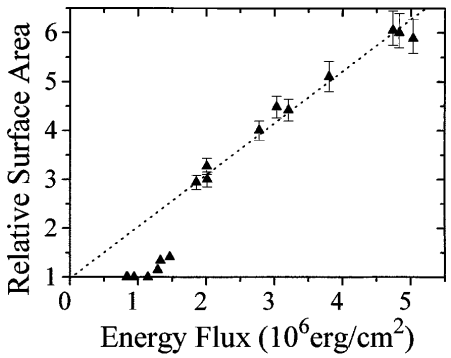

FIG. 4. The relative surface area (A) created by a crack as a function of the energy flux G into the crack tip. The linear dependence above v_c indicates that nearly all of the energy goes into creating new surface, while the energy cost per unit surface area is unchanged. A linear fit (line) to the data yields $A = 1.0 + 1.05 \times 10^{-6}$ G.

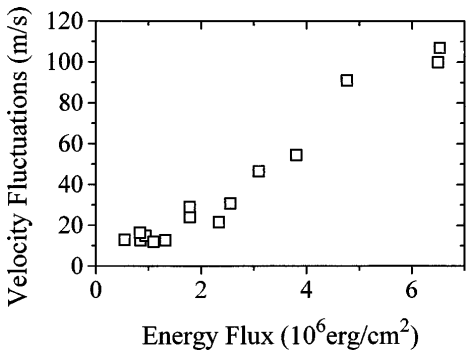

FIG. 5. The rms velocity fluctuations of a running crack as a function of the energy flux G into the crack tip. The linear increase of the velocity fluctuations results from enhanced crack branching at high energies. The data shown are from samples of thickness 0.8 mm.

fluctuations is apparent, and, to experimental accuracy, linear in G.

In conclusion, both the single crack and multiple microcrack pictures are fused into a single dynamic picture where the fracture energy above v_c is constant, its value determined by complex nonlinear processes, and the *apparent* large increase in fracture energy with increasing crack velocity is determined by the number and length of locally bifurcating microcracks. The same mechanism explains why a crack will not reach its limiting velocity; rather than accelerating, a crack will prefer to dissipate energy by creating surface via multiple parallel cracks.

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[14] The fracture surface created is a decreasing function of the distance of the measurement plane from the sample surface. This effect may be caused by nonuniformity of the stress field (as well as the energy stored) across the plate thickness as the external stress, loading the sample, is directly applied only to the faces of the plate. The effect is greater for crack velocities near v_c (60% less area at midsample than at sample edges for $v = 400$ m/s) than for larger velocities (10% less fracture surface at midsample for $v = 600$ m/s). The relative surface area used is the average value across the sample thickness. This is determined by fitting the measured values of the surface areas in different X-Y planes to an exponential function. The value for the relative surface area obtained was nearly independent of the fitting function used, inducing at most a 5% error.
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