Glueball Spectroscopy in a Relativistic Many-Body Approach to Hadronic Structure

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A comprehensive, relativistic many-body approach to hadron structure is advanced based on the Coulomb gauge QCD Hamiltonian. Dynamical chiral symmetry breaking naturally emerges, and both quarks and gluons acquire constituent masses when standard many-body techniques are employed. Gluonia are studied both in the valence and in the collective, random phase approximations. Calculated quenched glueball masses are found to be in remarkable agreement with lattice gauge theory when using representative values for the strong coupling constant and string tension.

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Our knowledge of the standard model cannot be considered complete until explicit gluonic degrees of freedom are found and understood [1]. In an effort to address this issue we advance a comprehensive framework for consistently describing and understanding hadron structure—including the glueball and hybrid sectors. The model is motivated in part by our previous studies of relativistic [2] and nonrelativistic [3] quark models and by the pioneering work of Le Yaouanc *et al.* [4], who, following the spirit of Nambu and Jona-Lasinio [5], have constructed a quark-based model of the QCD vacuum. Only a brief description of our approach is provided here. A full, detailed treatment will be contained in a separate communication.

The idea is to build on the known successes of the constituent quark model for heavy quarks by considering a many-body relativistic Hamiltonian in a quasiparticle basis, where dynamical chiral symmetry breaking and massive gluon modes are explicit. Such a model incorporates an extensive Fock space but reduces to the simple quark model in the valence approximation. Furthermore, the simultaneous presence of quark and gluon degrees of freedom permits studying their mixture in hybrid and glueball states. This is especially important since glueball searches tend to occur in meson-rich regions of the hadron spectrum and also because it may be years before lattice gauge calculations provide significant insight. This Letter focuses on the gluonic sector of the model Hamiltonian, presenting the glueball spectrum calculation and a discussion of the associated approximation schemes. In the summary we comment on other issues regarding applications to mesons, baryons, and hybrids.

There have been a variety of previous glueball studies: the bag model [6–8], QCD sum rules [9–12], the constituent glue model [13], and the flux tube model [14]. These approaches differ markedly in their mass predictions, sometimes by as much as 1 GeV, and no single approach has consistently reproduced lattice gauge measurements [15–17]. As stated above, our goal is to model QCD in a way which is in accordance with the successes of the constituent quark model. Therefore we start from the Coulomb gauge QCD Hamiltonian H [18] and assume the existence of a set of phenomenological interactions $H_{\rm phen}$ such that H can be written as $H = H_0 + H_I$ where H_0 is defined as $H_0 = K + H_{\rm phen}$, and K stands for the kinetic energy

$$K = \int d\mathbf{x}\Psi_q^\dagger(\mathbf{x})(-i\vec{\alpha}\cdot\vec{\nabla} + \beta m_q)\Psi_q(\mathbf{x}) + \frac{1}{2}\int d\mathbf{x}[|\mathbf{E}^a(\mathbf{x})|^2 + |\mathbf{B}^a(x)|^2], \quad (1)$$

involving current quarks having masses m_q and zero mass transverse gluons. The residual potential $H_I = H_I^{\rm QCD} - H_{\rm phen}$ is given by the difference between the original QCD and phenomenological interactions. The motivation for introducing phenomenological interactions is to generate a much weaker residual potential H_I at all energy scales. Furthermore, as in the phenomenological quark model, we will assume that in the quasiparticle quark and gluon basis this residual interaction can be approximated by the canonical QCD interaction Hamiltonian with a coupling g_s that is small and saturates at low energies. Under this approximation, to any order in g_s the residual interaction can, in principle, be derived using standard methods of time independent perturbation theory. However, since the infrared behavior has already been determined from phenomenology, the residual interaction must be free from infrared divergences to avoid double counting. This can be achieved, for example, by imposing a cutoff, $\Lambda_{\rm IR}$, on H_I that removes coupling between quasiparticle states whose energy difference is smaller than the cutoff scale. The perturbative expansion of H_I then generates an effective Hamiltonian which has nonvanishing matrix elements below the cutoff and can be either added to H_0 and diagonalized nonperturbatively or, because of the small coupling, included in the bound state perturbation theory using the eigenstates of H_0 [19].

For the applications considered in this work the phenomenological Hamiltonian is taken to be

$$H_{\rm phen} = -\frac{1}{2}\int d\mathbf{x}d\mathbf{y}\rho^a(\mathbf{x})V_L(|\mathbf{x}-\mathbf{y}|)\rho^a(\mathbf{y}), \quad (2)$$

with color charge density $\rho^a(\mathbf{x}) = \Psi_q^\dagger(\mathbf{x})T^a\Psi_q(\mathbf{x}) + f^{abc}\mathbf{A}^b(\mathbf{x})\mathbf{E}^c(\mathbf{x})$; V_L is a linear confining potential,

$$V_L(|\mathbf{x} - \mathbf{y}|) = \frac{2N_c b}{N_c^2 - 1}|\mathbf{x} - \mathbf{y}|\,(1 - e^{-\Lambda_{\rm phen}|\mathbf{x}-\mathbf{y}|}), \quad (3)$$

and $N_c = 3$ is the number of colors. The string tension will be fixed at $b = 0.18\ \mathrm{GeV}^2$, commensurate with Regge phenomenology and the naive quark model. Since this interaction is meant to represent soft physics, we have introduced an ultraviolet cutoff directly into the potential. Note that the approach advocated here requires that $\Lambda_{\rm phen} \sim \Lambda_{\rm IR}$, and hence to completely describe low energy phenomena we must add the QCD interactions below the cutoff $\Lambda_{\rm phen}$ to this phenomenological Hamiltonian. To order α_s we only keep the Coulomb potential and ignore self-energies, hyperfine interactions, and vacuum corrections. This will be discussed in more detail below. The complete interaction in the Hamiltonian H, which we diagonalize nonperturbatively, is thus given by Eq. (2) with V_L replaced by

$$V_L(r) \to V(r) = V_L(r) + V_C(r), \quad (4)$$

where

$$V_C(|\mathbf{x} - \mathbf{y}|) = -\frac{\alpha_s}{|\mathbf{x} - \mathbf{y}|} \quad (5)$$

is the Coulomb potential. Finally, we note that both V_C and H_I need to be ultraviolet regulated. The standard perturbative renormalization procedure may then be followed for these terms [20].

Regarding the structure of $H_{\rm phen}$, we have assumed that the bulk of the low energy dynamics of the $q\bar{q}$ and gluon-gluon systems may be described by a two-body interaction as shown and have employed a linear confinement potential in keeping with the constituent quark model and lattice gauge theory. Also, note that the confining interaction is between color densities rather than scalar currents as is usually assumed in the constituent quark model. This is discussed below. Note that Eq. (2) implies that gluons may be confined into gluon-gluon bound states which form the basis of our glueball investigation. However, lattice gauge results [21] indicate that static color octets become screened at very large distances, and Eq. (2) does not reflect this. Physically, one expects that the screening of the gluonic confinement potential is due to the creation of low-lying glueballs at large valence gluon separation. Thus the model as presented here is similar in spirit to the naive quark model, where linear confinement in the $q\bar{q}$ sector is absolute and it is mixing to other Fock sectors which is responsible for the screening. Furthermore, we note that constituent gluons are not static and that lattice gauge calculations indicate that low-lying glueballs tend to be compact and well separated in mass.

It should be noted that the structure of $H_{\rm phen}$ is specific to the Coulomb gauge. One could construct a similar model in a different gauge but this would require a different form for $H_{\rm phen}$. The model is gauge covariant only in this restricted sense. Furthermore, while the model is relativistic, Lorentz covariance has been lost. The severity of this may be easily tested by computing observables such as the pion decay constant or hadronic dispersion relations. This will be done in a future publication. Finally, one should not think of the confinement potential as a flux tube because this quickly leads to conundrums about double counting gluonic degrees of freedom. The Coulomb gauge Hamiltonian makes a clear distinction between the gluonic color density and the Coulomb interaction—one which applies equally well to the confinement interaction.

As in Ref. [4] the Fock space in which we diagonalize H_0 is constructed from the variational, BCS vacuum $|\Omega\rangle$ by an application of the constituent quark (antiquark) $B^\dagger$ ($D^\dagger$) and gluon $\mathbf{a}^\dagger$ creation operators with

$$\Psi_q(\mathbf{x}) = \sum_\lambda \int \frac{d\mathbf{k}}{(2\pi)^3}[U(\mathbf{k},\lambda)B(\mathbf{k},\lambda) + V(-\mathbf{k},\lambda)D^\dagger(-\mathbf{k},\lambda)]e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (6)$$

$$\mathbf{A}^a(\mathbf{x}) = \int \frac{d\mathbf{k}}{(2\pi)^3}\frac{1}{\sqrt{2\omega(|\mathbf{k}|)}}[\mathbf{a}^a(\mathbf{k}) + \mathbf{a}^a(-\mathbf{k})^\dagger]e^{i\mathbf{k}\cdot\mathbf{x}},$$

and with the vacuum defined by $B|\Omega\rangle = D|\Omega\rangle = \mathbf{a}|\Omega\rangle = 0$. We note that the gluon operators are transverse so that one has

$$[a_i^a(\mathbf{k}), a_j^b(\mathbf{q})^\dagger] = \delta_{ab}\,(2\pi)^3\delta(\mathbf{k} - \mathbf{q})\,(\delta_{ij} - \hat{k}_i\hat{k}_j)\,. \quad (7)$$

The extra term in the final factor complicates the calculations of the glueball spectrum but is crucial to maintaining the correct gluonic degrees of freedom.

The variational gap functions U, V, and ω are obtained by minimizing the ground-state (vacuum) energy $E_0 = \langle\Omega|H_0|\Omega\rangle$. This leads to gap equations which are equivalent to the Schwinger-Dyson equations for the quark or gluon self-energies in the rainbow approximation. For the gluon spectral function ω the gap equation is given by

$$\omega(q)^2 \equiv \omega(|\mathbf{q}|)^2 = q^2 + \frac{N_c}{4}\int\frac{d\mathbf{k}}{(2\pi)^3}\tilde{V}(\mathbf{k} + \mathbf{q})[1 + (\hat{\mathbf{k}}\cdot\hat{\mathbf{q}})^2] \times \frac{\omega(|\mathbf{k}|)^2 - \omega(|\mathbf{q}|)^2}{\omega(|\mathbf{k}|)}\,. \quad (8)$$

A similar variational treatment of the Hamiltonian in the quark sector results in the well known realization of dynamical chiral symmetry breaking by the BCS vacuum.

The presence of the Coulomb term in V introduces a quadratic cutoff dependence, which can be removed by including the neglected terms in H_I (in particular, the self-energy corrections resulting from the expansion of the Faddeev-Popov determinant and the transverse gluon exchange calculated to order α_s). However, the net effect of the order α_s terms in H_I is expected to be small, and we simply ignore them when solving the gap equation. Further investigations are in progress.

A good fit to the numerical solution of Eq. (8) is obtained with

$$\omega(k) = \sqrt{k^2 + m_g^2 e^{-k/\kappa}}. \tag{9}$$

If one defines the gluon mass in terms of the effective mass as $m_g = m_g(0)$ where $m_g(k) = \sqrt{\omega(k)^2 - k^2}$ then the proceding fit yields $m_g = 0.8$ GeV (and $\kappa = 6.5$ GeV).

The gluon condensate may be simply calculated within the context of the pairing ansatz. The result is

$$\left\langle \frac{\alpha}{\pi} G_a^{\mu\nu} G_{\mu\nu}^a \right\rangle = \frac{N_c^2 - 1}{\pi^3} \int_0^\infty dk\, k^2 \alpha_s(k) \frac{[\omega(k) - k]^2}{\omega(k)}. \tag{10}$$

We have allowed for the possibility that α_s runs above the cutoff although this is not crucial. The calculated condensate agrees well with the QCD sum rule value of 0.012 GeV^4 [22] and is only weakly sensitive to the cutoff above $\Lambda_{\text{phen}} \sim 4$ GeV.

Just as in conventional nuclear structure theory, our BCS many-body vacuum state can be systematically improved by utilizing the Tamm-Dancoff (TDA), random phase (RPA), or even more accurate approximations involving exact diagonalization in an extensive multiparticle-hole model space. In the glueball case we have performed both TDA and RPA calculations, however, because of the large constituent gluon mass we expect the Tamm-Dancoff approximation to be a reasonable one. Indeed the 0^{++} glueball mass is shifted by less than 2% in going to the random phase approximation. In the glueball rest frame, the TDA gluon-gluon bound states are given by

$$|J^{PC}\rangle = \int \frac{d\mathbf{p}}{(2\pi)^3} \chi_{ij}^{JPC}(\mathbf{p}) a_i^b(\mathbf{p})^\dagger a_j^b(-\mathbf{p})^\dagger |\Omega\rangle, \tag{11}$$

with the glueball wave function χ_{ij}^{JPC} satisfying

$$E^{JPC} \mathcal{G}_{ij}^{JPC} \chi_{ij}^{JPC}(\mathbf{q}) = -\frac{N_c}{4} \int \frac{d\mathbf{k}}{(2\pi)^3} \tilde{V}(|\mathbf{k} + \mathbf{q}|) \frac{[\omega(q) + \omega(k)]^2}{2\omega(q)\omega(k)} \mathcal{F}_{ij}^{JPC}(\mathbf{k}, \mathbf{q}) \chi_{ij}^{JPC}(\mathbf{k}) + \left[\left(\omega(q) + \frac{q^2}{\omega(q)}\right) \mathcal{G}_{ij}^{JPC} + \frac{N_c}{4} \int \frac{d\mathbf{k}}{(2\pi)^3} \tilde{V}(|\mathbf{k} + \mathbf{q}|) \frac{\omega(q)^2 + \omega(k)^2}{\omega(q)\omega(k)} \mathcal{F}_{ij}^{JPC}(\mathbf{k}, \mathbf{q})\right] \chi_{ij}^{JPC}(\mathbf{q}). \tag{12}$$

Here the $\mathcal{F}_{ij}^{JPC}$'s are determined from coupling the two transverse gluons labeled by the Cartesian indices i, j to a state with total angular momentum J, parity P, and charge conjugation C. For example,

$$\mathcal{F}_{ij}^{0++}(\mathbf{k}, \mathbf{q}) = [1 + (\hat{\mathbf{k}} \cdot \hat{\mathbf{q}})^2] \delta_{ij}, \tag{13}$$

$$\mathcal{F}_{ij}^{0-+}(\mathbf{k}, \mathbf{q}) = \hat{k}_i \hat{q}_j, \tag{14}$$

with more complicated expressions for $J \geq 2$. The functions $\mathcal{G}_{ij}^{JPC}$ are normalization matrices which arise from mixing between different LS states induced by the transverse nature of the gluon. We use the Coulomb and linear contributions to $\tilde{V}$ in Eq. (12). There are no Faddeev-Popov terms, and transverse gluon exchange is treated as a perturbation (it is of order $1/m_g^2$). We note that it is not possible to make a two-gluon $J = 1$ state as is consistent with Yang's theorem. Such spurious states exist in models with explicitly massive gluons.

There is an interesting property associated with divergences in the bound state equation. The linear potential is infrared divergent; however, this potentially problematic divergence is canceled by the self-energy term in the kinetic energy. This cancellation appears to be a property of a density-density interaction. For example, the cancellation does not occur for scalar quark currents (and, indeed, a stable vacuum cannot be obtained with an interaction between scalar currents). Furthermore, the cancellation appears to occur only in bound state equations for color singlet objects. This has been observed previously in the context of the Bethe-Salpeter approximation to the $q\bar{q}$ bound state problem [20].

The spectrum which results from numerically solving Eq. (12) is presented in Fig. 1 along with recent results from lattice gauge calculations. The agreement is remarkably good, especially when it is recalled that the model has been completely fixed from $q\bar{q}$ phenomenology. The spectrum corresponds to the ansatz for a running $\alpha_s(k)$

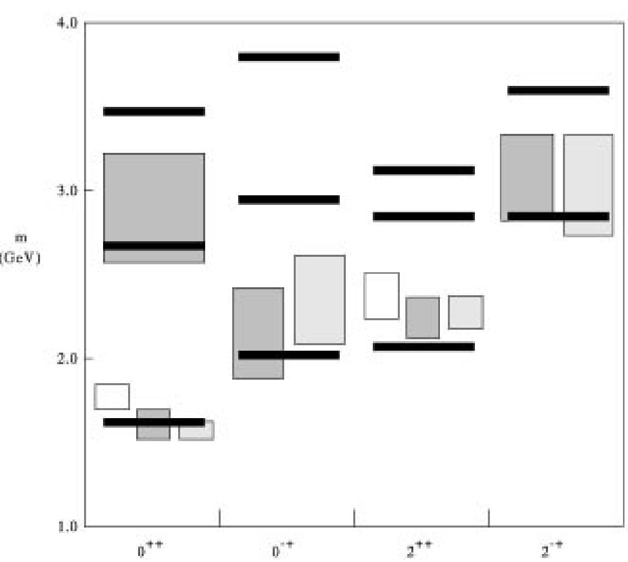

FIG. 1. Lattice gauge and model glueball spectra.

from Ref. [23]. Employing a fixed value for the strong coupling, $\alpha_s = 0.4$, produced no significant difference. Furthermore, the spectrum is essentially independent of the cutoff for $\Lambda_{\rm phen} \gtrsim 4$ GeV. We conclude that the model captures the essential features of glueballs. To our knowledge, this is the only model of gluonia which successfully reproduces lattice data, and therefore it may provide important insight into glueball structure.

Future, more comprehensive, studies may dictate phenomenological potentials beyond the two-body form used here. For example, the phenomenological $^3P_0$ decay vertex cannot be obtained from the density-density confining potential. Reconciling the naive quark model phenomenology with the model presented here should prove very instructive. Another issue is that the dynamical breaking of chiral symmetry should lead to a massless pion solution. Recall, however, that the BCS vacuum is not a true eigenstate of the Hamiltonian. Therefore when diagonalizing the Hamiltonian in a truncated Fock space a chiral pion solution may not necessarily appear. However, in the random phase approximation which builds upon the BCS vacuum, the Nambu-Goldstone realization of chiral symmetry is preserved. In particular the Gell-Mann–Oaks–Renner [24] relation,

$$f_\pi^2 m_\pi^2 = -2m_q\langle\bar{q}q\rangle\,, \qquad (15)$$

follows from Thouless' theorem [25] applied to the chiral charge operator, $Q_5 = \int d\mathbf{x}\Psi^\dagger(\mathbf{x})\gamma_5\Psi(\mathbf{x})$, via the expression

$$2\sum_n |\langle n|Q_5|\Omega\rangle_{\rm RPA}|^2(E_n - E_0)_{\rm RPA} = \langle\Omega|[Q_5,[Q_5,H]]|\Omega\rangle\,. \qquad (16)$$

Examining the interplay of these issues will be instructive, especially as it has a bearing on the rather mysterious nature of the hyperfine splitting and the ultimate utility of the potential quark model.

Future work will also focus on baryon and hybrid structure. Towards this end, we have performed initial, but preliminary, calculations in the quark sector finding $m_q \sim 180$ MeV and $\langle\bar{q}q\rangle \sim -(100\ \text{MeV})^3$ in agreement with Refs. [4,20]. While m_q is in rough agreement with phenomenology the low condensate value may be due to truncation of $H_{\rm phen}$ to two-body form. We plan to extend this to higher terms and also incorporate the above mentioned many-body treatments. Of special interest will be an ambitious multiparticle-multihole diagonalization which will involve higher quark Fock state components. In particular, the importance of such states as $|qqq(q\bar{q})\rangle$ for the proton will directly address the role of sea quarks and hidden flavor. Related to this is the insight this model provides concerning the proton spin.

In summary, we have presented a unified, comprehensive approach to hadron structure based on nonperturbative relativistic field theory and the QCD Hamiltonian. The model provides both the appealing physical insight associated with the phenomenologically successful quark model and a consistent unified framework for studying issues such as chiral pions and quark-glue mixing. With the advent of CEBAF, a challenging opportunity is at hand to confront new precision data and to thoroughly investigate a wide variety of issues in hadronic physics.

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[1] C. Amsler and F.E. Close, Report No. RAL-TR-95-003 (unpublished); Report No. RAL-TR-95-036, hep-ph/9505219 (unpublished).
[2] A. Szczepaniak, C.-R. Ji, and S.R. Cotanch, Phys. Rev. D **49**, 3466 (1994); **52**, 5284 (1995); Phys. Rev. C **52**, 2738 (1995); C.-R. Ji and S.R. Cotanch, Phys. Rev. D **41**, 2319 (1990); C.-R. Ji, P.L. Chung, and S.R. Cotanch, *ibid.* **45**, 4214 (1992).
[3] See, e.g., the discussion in T. Barnes and E.S. Swanson, Phys. Rev. D **46**, 131 (1992); P. Geiger and E.S. Swanson, Phys. Rev. D **50**, 6855 (1994).
[4] A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Phys. Rev. D **29**, 1233 (1984); A. Le Yaouanc, L. Oliver, S. Ono, O. Pène, and J.-C. Raynal, Phys. Rev. D **31**, 137 (1985).
[5] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).
[6] R.L. Jaffe and K. Johnson, Phys. Lett. **60B**, 201 (1976).
[7] T. Barnes, F.E. Close, and S. Monaghan, Nucl. Phys. **B198**, 380 (1982); Phys. Lett. **110B**, 159 (1982).
[8] C.E. Carlson, T.H. Hanson, and C. Peterson, Phys. Rev. D **27**, 1556 (1983); **28**, 2895 (1983).
[9] M. Shifman, Z. Phys. C **9**, 347 (1981).
[10] P. Pascual and R. Tarrach, Phys. Lett. **113B**, 495 (1982).
[11] S. Narison, Z. Phys. C **26**, 209 (1984).
[12] C.A. Dominguez and N. Power, Z. Phys. C **31**, 591 (1988); J. Bordes, V. Gimènez, and J.A. Peñarrocha, Phys. Lett. B **223**, 251 (1989).
[13] T. Barnes, Z. Phys. C **10**, 275 (1981).
[14] N. Isgur and J. Paton, Phys. Rev. D **31**, 2910 (1985).
[15] UKQCD Collaboration, G.S. Bali *et al.*, Phys. Lett. B **309**, 378 (1993); Report No. hep-lat/9304012 (unpublished).
[16] H. Chen, J. Sexton, A. Vaccarino, and D. Weingarten, Nucl. Phys. (Proc. Suppl.) **B34**, 357 (1994); Report No. hep-lat/9401020 (unpublished).
[17] M. Teper, Report No. OUTP-95-06P (to be published).
[18] T.D. Lee, *Particle Physics and Introduction to Field Theory* (Harwood Academic Publishers, New York, 1981).
[19] St.D. Glazek and K.G. Wilson, Phys. Rev. D **48**, 5863 (1993); **49**, 4214 (1994).
[20] S. Adler and P. Davis, Nucl. Phys. **B244**, 469 (1984).
[21] C. Michael, Nucl. Phys. (Proc. Suppl.) **B26**, 417 (1992).
[22] M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Nucl. Phys. **B147**, 385,448 (1980).
[23] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[24] M.R. Gell-Mann, R.J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).
[25] D.J. Thouless, Nucl. Phys. **22**, 78 (1961).