Comment on "Amplification under the Standard Quantum Limit"

Quantum limits on optical amplification have been widely discussed theoretically in the last decade [1], and several experiments [2–5] have crossed the standard quantum limit (SQL). In a recent Letter [6], D'Ariano, Machiavello, and Paris reported on Monte Carlo simulation of a saturated laser amplifier, and claimed to beat the SQL in this system.

This claim, however, is based on a binary "on"-"off" *large-signal* noise-figure calculation in a saturated, *nonlinear* device. Here we point out that the 3 dB SQL for a phase-insensitive amplifier only has a well defined meaning in a *linear* regime [1], i.e., either for a small signal linearized regime or for a truly linear device. For a nonlinear device which modifies the statistics between input and output, the noise figure based on a large signal, as in [6], must be used with caution since the noise figure is no longer easily related to the information degradation as expressed in symbol error probability [or bit error rate (BER)]. For digital coding, BER, not the signal-to-noise ratio (SNR), is the quantity of relevance to evaluate the information transfer efficiency [7].

In Ref. [6], the gain $\mathcal{G}$ and the noise figure $\mathcal{R}$ are, respectively, defined by

$$\mathcal{G} = \frac{S_{\text{out}}}{S_{\text{in}}}, \quad \mathcal{R} = \frac{(S^2/\mathcal{N})_{\text{in}}}{(S^2/\mathcal{N})_{\text{out}}}, \tag{1}$$

where S and $\mathcal{N}$ denote signal and noise at the input (in) and at the output (out) of the amplifier. In [6], the quantities S and $\mathcal{N}$ are defined for on-off modulation as

$$S = \langle \hat{O} \rangle^{\text{on}} - \langle \hat{O} \rangle^{\text{off}}, \quad \mathcal{N} = \tfrac{1}{2}\left(\langle \Delta \hat{O}^2 \rangle^{\text{on}} + \langle \Delta \hat{O}^2 \rangle^{\text{off}}\right), \tag{2}$$

where $\hat{O}$ is the detected observable, and the brackets $\langle \cdots \rangle$ denote an ensemble average.

Let us show by an example that, for a nonlinear device, the noise figure $\mathcal{R}$ defined as above [6] does not account for the efficiency of the information transfer of the system, and therefore is not relevant to the SQL. Let us consider an optical repeater based on a detector plus electronic decision circuitry triggering a powerful laser. The transfer function of this device is described as $\hat{O}_{\text{out}} = \{P_o, \text{ if } \hat{O}_{\text{in}} \geq P_{\text{th}}, \text{ and } 0, \text{ if } \hat{O}_{\text{in}} < P_{\text{th}}\}$, where P_{th} is the decision threshold. Assume now equal probability (equal average numbers of zeros and ones) on-off keying with zero photons in the off state and a coherent state, with $\langle n \rangle$ photons on the average in the on state. The input SNR is $(S^2/\mathcal{N})_{\text{in}} = 2\langle n \rangle$. Setting, for example, the decision threshold P_{th} to one input photon, the output SNR is given by $(S^2/\mathcal{N})_{\text{out}} = 2(e^{\langle n \rangle} - 1)$, leading to a noise figure $\mathcal{R} = \langle n \rangle/(e^{\langle n \rangle} - 1)$. Here, P_o was assumed large enough that the relative output laser fluctuations could be neglected. As shown by this example, $\mathcal{R}$ (as it is defined in [6]) not only can be less than 3 dB, but also less than 0 dB (unity), corresponding to an improvement of SNR. Yet, there is of course no improvement of the BER since symbol errors are introduced when a digital "one" is interpreted as a "zero," which is not reduced by a threshold function. This illustrates that for a nonlinear device the noise figure $\mathcal{R}$ of [6] cannot be used directly to assess the information degradation, and hence $\mathcal{R}$ cannot be compared to the 3 dB SQL for a linear phase-insensitive amplifier. Thus, the conclusion in Ref. [6] on "amplification below the standard quantum limit" is unwarranted.

If the noise figure instead is defined for a small signal input, which should be a small linearizable modulation $S = \delta\langle \hat{O} \rangle$ around a given working point, the model used by the authors of [6] would likely show that a saturated laser amplifier does not go below the SQL.

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