Spectrum of Light Scattered from a Weakly Interacting Bose-Einstein Condensed Gas

Robert Graham* and Dan Walls

Department of Physics, University of Auckland, Private Bag 92019, Auckland, New Zealand

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Light scattered off a dilute Bose-Einstein condensed gas of atoms at large detuning, besides a broad background, shows two symmetrically placed peaks whose intensities are related by detailed balance and whose frequencies depend on the product of the number of atoms in the condensate and the scattering length.

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The recent achievement of Bose-Einstein (BE) condensation in a gas of atoms confined by a magnetic trap [1] has stimulated renewed interest in the question as to what signatures Bose-Einstein condensation imprints in the spectrum of light scattered from atoms in such a condensate [2–7]. Recently, Javanainen [7] calculated the spectrum of light, in the limit of large detuning, when scattered from atoms in an *ideal* Bose gas of a size much larger than the scattered wavelength. He identified a two-peak structure of the scattering function $S(\mathbf{k},\omega)$ at $\omega = \pm\omega_R$ as an indicator for the presence of a Bose condensate. Here $\hbar\omega$ and $\hbar\mathbf{k}$ are the energy and momentum transfer in the light scattering; $\omega_R = \hbar k^2/2m$ and $k = |\mathbf{k}|$. Similar results for the scattering function are discussed in [8].

The quasiparticle excitation spectrum of a dilute Bose-Einstein condensed gas, according to Bogoliubov's theory [9–12] is given by [13]

$$\omega_{\mathbf{k}} = \frac{\hbar k}{2m}(k^2 + 16\pi naf)^{\frac{1}{2}}, \tag{1}$$

where n is the number density, f the fraction of atoms in the condensate (which may take any value between 1 and 0 if Huang's formulation of the theory is used [11]), and a the scattering length for s-wave scattering. For $k^2 \gg 16\pi naf$ Eq. (1) predicts a frequency shift $\omega_{\mathbf{k}} - \omega_R = \frac{4\hbar}{m}\pi naf$ which is constant, i.e., given sufficient resolution, never becomes negligible. For application to experimentally realized Bose condensates it is therefore necessary to ask how the results of [7] are modified by the two-particle interaction. We extend here Javanainen's treatment [7] to that end. As the detuned light couples to the local number density of the atoms [7] the relevant scattering function is as usual [14] given by

$$S(\mathbf{k},\omega) = \frac{1}{Z}\sum_{\mu,\mu'} e^{-\beta E_\mu} \times |\langle\mu'|\rho(\mathbf{k})|\mu\rangle|^2 \delta(\hbar\omega - E_\mu + E_{\mu'}), \tag{2}$$

where ω and $\mathbf{k}$ denote the change of frequency and wave vector of the photon. The detailed balance condition $S(\mathbf{k},\omega) = e^{-\beta\hbar\omega}S(\mathbf{k},-\omega)$ at finite temperature will automatically give rise to a symmetrically placed two-peak structure if $S(\mathbf{k},\omega)$ develops a sufficiently sharp peak at a nonzero frequency. The density fluctuation operator $\rho(\mathbf{k})$ for $\mathbf{k} \neq 0$ can be expressed in terms of the atom creation and annihilation operators as

$$\rho(\mathbf{k}) = \sqrt{fN}(a_{\mathbf{k}} + a_{-\mathbf{k}}^\dagger) + \sum_{\mathbf{q}\neq 0,-\mathbf{k}} a_{\mathbf{q}}^\dagger a_{\mathbf{k}+\mathbf{q}}, \tag{3}$$

and (2) is most conveniently evaluated after diagonalizing the Hamiltonian by performing the transformation to Bogoliubov's quasiparticles in a standard manner [8,12]. The result takes the following form:

$$S(\mathbf{k},\omega) = \frac{fN(1-\alpha_{\mathbf{k}})}{\hbar(1+\alpha_{\mathbf{k}})}(1+\langle n_{\mathbf{k}}\rangle) \times \left[\delta(\omega+\omega_{\mathbf{k}}) + e^{-\beta\hbar\omega_{\mathbf{k}}}\delta(\omega-\omega_{\mathbf{k}})\right] + S_b(\mathbf{k},\omega), \tag{4}$$

where the background is given by

$$S_b(\mathbf{k},\omega) = \sum_{\mathbf{q}\neq 0,-\mathbf{k}} \frac{(1+\langle n_{\mathbf{q}}\rangle)(1+\langle n_{\mathbf{q}+\mathbf{k}}\rangle)(\alpha_{\mathbf{q}}+\alpha_{\mathbf{q}+\mathbf{k}})^2}{2\hbar(1-\alpha_{\mathbf{q}}^2)(1-\alpha_{\mathbf{q}+\mathbf{k}}^2)} \Bigg[2\left(\frac{1+\alpha_{\mathbf{q}}\alpha_{\mathbf{q}+\mathbf{k}}}{\alpha_{\mathbf{q}}+\alpha_{\mathbf{q}+\mathbf{k}}}\right)^2 e^{-\beta\hbar\omega_{\mathbf{q}+\mathbf{k}}}\delta(\omega+\omega_{\mathbf{q}}-\omega_{\mathbf{q}+\mathbf{k}}) + \delta(\omega+\omega_{\mathbf{q}}+\omega_{\mathbf{q}+\mathbf{k}}) + e^{-\beta\hbar(\omega_{\mathbf{q}}+\omega_{\mathbf{q}+\mathbf{k}})}\delta(\omega-\omega_{\mathbf{q}}-\omega_{\mathbf{q}+\mathbf{k}})\Bigg]. \tag{5}$$

Here,

$$\langle n_{\mathbf{k}}\rangle = [e^{\beta\hbar\omega_{\mathbf{k}}} - 1]^{-1},$$
$$\alpha_{\mathbf{k}} = 1 + \lambda^2k^2 - \lambda k\sqrt{2+\lambda^2k^2},$$
$$\lambda = (8\pi naf)^{-\frac{1}{2}}.$$

For temperatures above the critical temperature for BE condensation these formulas apply with

$$\alpha_{\mathbf{k}} = 0, \quad \omega_{\mathbf{k}} = \frac{\hbar k^2}{2m} - \frac{\mu}{\hbar}, \quad f = 0,$$

μ (< 0) is the chemical potential. The two peaks (4) above the broad background generalize the result of [7]

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and go over into that result if $k^2 \gg 16\pi naf$ is satisfied, apart from the constant frequency shift already mentioned. Therefore, given sufficient resolution in frequency, the important parameter naf can be determined from the position of the sharp line. For $T \rightarrow 0$ the double peak structure, required by detailed balance for finite T, disappears. Then $f \rightarrow 1 - 8/3\sqrt{na^3/\pi}$ [11] and the spectrum (4) has a single sharp line at $\omega = -\omega_{\mathbf{k}}$ from the excitation of single quasiparticles, and a broad background from the excitation of pairs of quasiparticles.

The present results apply to an infinitely extended condensate. A condensate in a trap differs from this idealized situation at least in two respects:

The presence of a trapping potential leads to a condensate which is (i) spatially inhomogeneous and (ii) has a finite size $L \simeq \sqrt{\hbar/m\omega}$ where ω is a typical oscillation frequency in a harmonic trap. In order to be able to still treat the condensate as homogeneous in regard to its elementary excitations, at least in a rough approximation, the superfluid coherence length ξ [15] must satisfy $\xi \ll L$. As ξ is given, from Eq. (1), by $\xi^{-2} = 16\pi naf$ we obtain the condition

$$\sqrt{16\pi nfa}\, L \gg 1\,. \tag{6}$$

In order to be able to still use the dispersion relation (1) of elementary excitations in the bulk, one must also satisfy $k \gg 2\pi/L$ for the transferred wave number. If these conditions are satisfied, one may estimate the effect of the finite size L of the condensate in stating that it blurs the transferred momentum $\hbar k$, and hence the scattered frequency by $\sim\frac{\partial\omega_k}{\partial k}\frac{2\pi}{L}$. To resolve the predicted shift it would therefore be preferable to have

$$k \leq 2nafL\,, \tag{7}$$

which is compatible with $k \gg 2\pi/L$ if (6) is satisfied. It would seem possible to satisfy the condition (6) already in the present generation of experiments and in the next generation the measurement should become feasible. This will be of great interest, because then light scattering could provide, via the temperature dependence of f, unambiguous evidence for the presence of a Bose condensate.

In summary, we have shown that the frequency of the sharp line which signals the presence of a Bose condensate in light scattering [7] contains essential information about the condensate and the interaction.

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*Permanent address: Fachbereich Physik, Universität-GH Essen, Essen, Germany.

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