Confirmation of the Sigma Meson

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A very general model and an analysis of data on the lightest 0^{++} meson nonet shows that the $f_0(980)$ and $f_0(1300)$ resonance poles are two manifestations of the same $s\bar{s}$ state. On the other hand, the $u\bar{u} + d\bar{d}$ state, when unitarized and strongly distorted by hadronic mass shifts, becomes an extremely broad (880 MeV) and light (860 MeV) resonance, with its pole at $s = 0.158 - i0.235$ GeV^2. This is the σ meson required by models for spontaneous breaking of chiral symmetry. It has been named the Higgs meson of QCD, because it generates most of the light hadron masses. It dominates $\pi\pi$ scattering below 900 MeV and it is also the resonance required by nuclear physics.

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The understanding of the $\pi\pi$ S wave has been controversial for a long time. Before 1974 (see [1]) one believed in the existence of a broad and light isoscalar resonance (then called σ, ϵ, or η_{0+}). After the two heavier resonances $f_0(980)$ and $f_0(1300)$ were established one generally discarded the σ, assuming it could be replaced by the two heavier ones, which could complete a light $q\bar{q}$ nonet.

The lightest scalar-isoscalar meson coupling strongly to $\pi\pi$ is of importance in most models for spontaneous breaking of chiral symmetry, like the linear σ model or the Nambu–Jona-Lasinio model [2], which require a scalar meson of twice the constituent quark mass or $\approx$700 MeV, and a very large $\pi\pi$ width of $\approx$850 MeV. This meson is crucial for our understanding of all hadron masses and is the Higgs boson of QCD. Thus most of the nucleon mass is believed to be generated by its coupling to the σ. However, the lightest well-established mesons with the quantum numbers of the σ, the $f_0(980)$, and $f_0(1300)$ do not have the right properties. They are both too narrow, $f_0(980)$ couples mainly to $K\bar{K}$, and $f_0(1300)$ is too heavy.

Recently one of us [3] showed that one can understand the data on the lightest scalars in a model which includes most well-established theoretical constraints: Adler zeros as required by chiral symmetry, all light two-pseudoscalar (PP) thresholds with flavor symmetric couplings, physically acceptable analyticity, and unitarity. A unique feature of this model is that it simultaneously describes the whole scalar nonet, and one obtains a good representation of a large set of relevant data. Only six parameters, which all have a clear physical interpretation, were needed: an overall coupling constant ($\gamma = 1.14$), the bare mass of the $u\bar{u}$ or $d\bar{d}$ state ($m_0 = 1.42$ GeV), the extra mass for a strange quark ($m_s = 100$ MeV), a cutoff parameter ($k_0 = 0.56$ GeV$/c$), an Adler zero parameter for $K\pi$ ($s_{A_{K\pi}} = -0.42$ GeV^2), and a phenomenological parameter enhancing the $\eta\eta'$ couplings ($\beta = 1.6$). We have now realized that one could discard the β parameter if one also included the next group of important thresholds or pseudoscalar (0^{-+})-axial (1^{+-}) thresholds, since then the $K\bar{K}_{1B}$ + c.c. thresholds give a very similar contribution to the mass matrix as $\eta\eta'$.

As expected, the coupling to PP turns out [3] to be very large, causing the inverse propagators to have very large imaginary parts. Normally, this is expected to result in large widths, but it was shown [3] that because of the many nonlinear effects the $a_0(980)$ and the $f_0(980)$ naturally come out narrow. Furthermore, the large flavor symmetry breaking in the positions of the PP thresholds induce large flavor symmetry breaking in the mass shifts. This makes the physical spectrum quite distorted compared to the simple bare spectrum, which obeys the equal spacing rule and the Okubo-Zweig-Iizuka rule with flavor symmetric couplings.

The analysis [3] yielded resonance parameters and pole positions for the four states $a_0(980)$, $f_0(980)$, $f_0(1300)$, and $K_0^*(1430)$, close to their conventional values [4]. In addition, however, one also expects "image" poles (sometimes called "shadow" or "companion" poles), which normally lie far away and do not play a significant role. Recently Morgan and Pennington [5] showed that for each $q\bar{q}$ state one expects at least one such image pole, which in principle can be used to distinguish a $q\bar{q}$ state from a meson-meson bound state.

In this Letter we report a more detailed search for all relevant poles in the amplitudes of the model of [3]. This reveals interesting and surprising new features, which simultaneously resolve two long-standing puzzles in meson spectroscopy: (i) What is the nature of the $f_0(980)$ and $f_0(1300)$? (ii) Where is the long sought for σ meson?

These new, important features can appear when the coupling to S-wave thresholds becomes very strong. In particular, two true resonance poles can emerge near the

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physical region, although only one $q\overline{q}$ state is present. If the coupling is reduced one of these poles disappears as a distant image pole far from the physical region.

In order to explain this phenomenon in the simplest possible terms we use two theoretical demonstrations. The first is based on the actual model amplitude in [3], which fits the $K\pi$ S-wave data and the $K_0^*(1430)$. We chose the channel with strangeness, because there one has only one $s\overline{d}$ quark model state, whereas in the flavorless case one has the more complicated situation of at least two resonances, the $u\overline{u} + d\overline{d}$ and $s\overline{s}$, which mix in an energy dependent and complex way. By increasing the overall coupling γ, we show how a second pole appears. In the second demonstration we chose a simple model for the threshold behavior, which still has the desired analytic properties, and from which the poles can be found analytically. This can also be used to demonstrate the phenomenon for the $K\overline{K}$ channel.

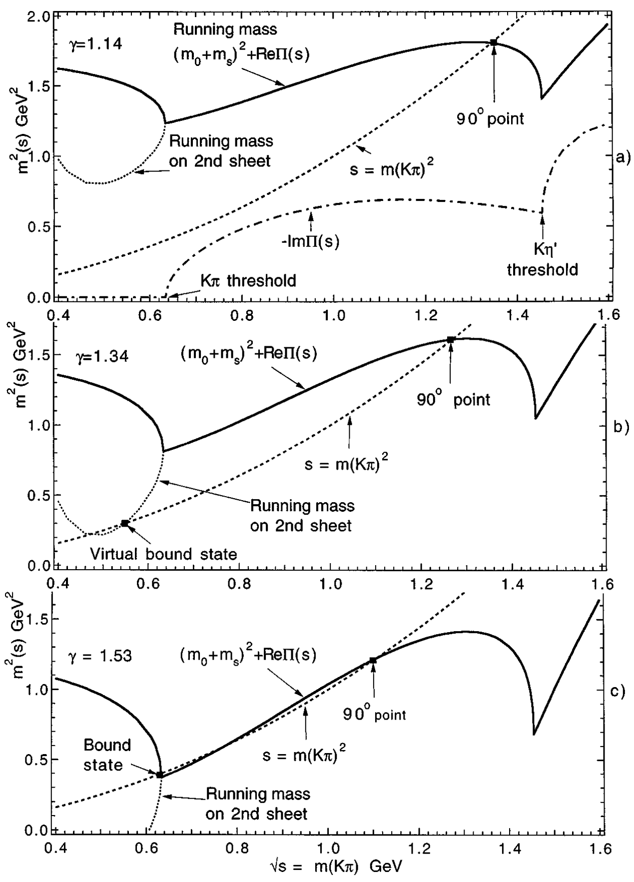

FIG. 1. (a) The running mass and $-\mathrm{Im}\Pi(s)$ which fits the $K\pi$ S-wave data. In (b) and (c) the overall coupling γ is increased, whereby first a virtual bound state (b) and then a bound state (c) appears below the $K\pi$ threshold in addition to the K_0^* resonance whose 90° mass gets shifted to 1100 MeV. The second crossing point of the two curves between the bound state and the 90° point does not correspond to a resonance since it does not satisfy the Wigner condition of anticlockwise motion in the Argand diagram.

In Fig. 1(a) we show the running mass, $m_0^2 + \mathrm{Re}\Pi(s)$ and the widthlike function $-\mathrm{Im}\Pi(s)$ for the $K\pi \to K\pi$ S wave as was found in [3]. The $K\pi$ partial wave amplitude (PWA) is obtained from these functions through

$$A(s) = -\mathrm{Im}\Pi_{K\pi}(s)/[m_0^2 + \mathrm{Re}\Pi(s) - s + i\mathrm{Im}\Pi(s)]. \quad (1)$$

This fits the $K\pi$ data well and one finds the K_0^* resonance parameters listed in Table I. As one increases γ there appears, in addition to the resonance, first a virtual bound state and then a true bound state just below the $K\pi$ threshold [see Figs. 1(b) and 1(c)]. Both poles, the original $K_0^*(1430)$ (now shifted such that $m_{\mathrm{BW}} =$ 1100 MeV and $m_{\mathrm{pole}} = 1496$ MeV) and the new bound state at the $K\pi$ threshold are then two manifestations of the same $s\overline{u}$ state, whose bare mass is kept at 1520 MeV.

For our second way to demonstrate this phenomenon we chose the form factor such that $\Pi(s)$ takes a simple analytic form

$$\Pi(s) = \overline{\gamma}^2 s_{\mathrm{th}}\{[(s_{\mathrm{th}} - s)s_{\mathrm{th}}]^{1/2} - s_{\mathrm{th}}\}/s. \quad (2)$$

See Fig. 2. This still has the desired analytic form and satisfies the dispersion relation (see discussion in Sec. 2.7 of [3]). The condition for the poles, $m_0^2 = \Pi(s) - s = 0$, now gives an equation of third degree. If $m_0^2 = s_{\mathrm{th}}(1 + \overline{\gamma}^2)$ one has one bound state at threshold $s = s_{\mathrm{th}}$ and (for $\overline{\gamma} > 1/2$) a complex conjugate pair at $s = s_{\mathrm{th}}[\overline{\gamma}^2 + 1/2 \pm i(\overline{\gamma}^2 - 1/4)^{1/2}]$. For $\overline{\gamma}^2 \gtrsim 1$ one has a running mass (cf. Fig. 2) quite similar to the one in Fig. 1(c) for the $K\pi$ threshold and to the actual one fitting the data at the $K\overline{K}$ threshold [Figs. 2(b) and 9(a) of Ref. [3]]. If $\overline{\gamma} > 1$ the phase shift passes through 90° at $s = \overline{\gamma}^2 s_{\mathrm{th}} = m_{\mathrm{BW}}^2$. This simplified model is also similar to the actual situation in [3] for the $s\overline{s}$ channel and the $K\overline{K}$ threshold. The $\overline{\gamma}$ of the simplified model is comparable in magnitude to the γ used in [3], such that with $\gamma = \overline{\gamma}$ both models give similar $\mathrm{Im}\Pi_{K\overline{K}}(s)$ near the $K\overline{K}$ threshold for $s\overline{s}$. Thus the fact that $\gamma = 1.14 > 1$ actually shows that the real world is not too far from our simplified model. Using $\overline{\gamma} = 1.14$ one would predict $(\mathrm{Re}s_{\mathrm{pole}})^{1/2} = 2m_K(\overline{\gamma}^2 + 1/2)^{1/2} = 1329$ MeV and $m_{\mathrm{BW}} = 2m_K\overline{\gamma} =$ 1129 MeV, which is not far from what was actually obtained for the $f_0(1300)$: 1202 and 1186, respectively. In reality, of course, other thresholds and mixing with $u\overline{u} + d\overline{d}$ complicate the picture.

The situation is also similar for the $a_0(980)$ and $a_0(1450)$ in the $I = 1$ channel, although now the Clebsch-Gordan coefficient reduces the effective $\overline{\gamma}$ by $1/\sqrt{2}$. However, the fact that the $\pi\eta$ channel is already open at the $K\overline{K}$ threshold helps in creating a similar situation of a running mass rising fast enough after threshold.

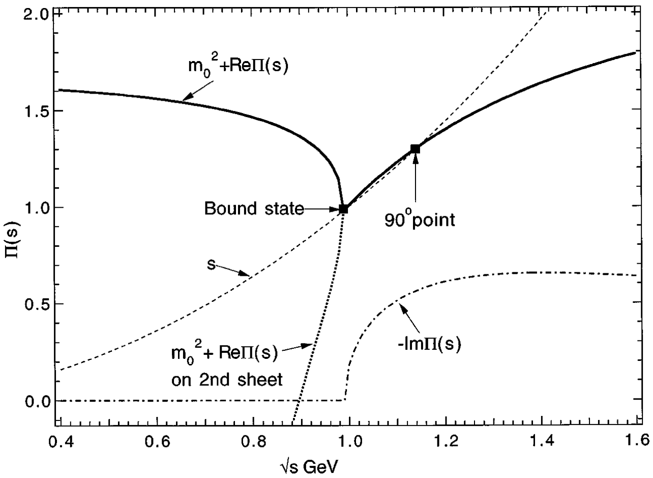

FIG. 2. The running mass and $-\mathrm{Im}\Pi(s)$ for the simple model function (2) and the $K\overline{K}$ threshold using $\overline{\gamma} = 1.14$. Choosing m_0 such that there is a bound state at the threshold, there is another resonance as seen from the second crossing of s with the running mass. This crossing point gives the 90° or Breit-Wigner mass of the second state.

Therefore, one can expect a repetition of the phenomenon, such that there could exist a second manifestation of the $I = 1$ state, somewhere in the 1.5 GeV region, in addition to $a_0(980)$. This could be the $a_0(1450)$ seen by the crystal barrel [6]. And as we shall see below, the model of [3] actually has an image pole near this mass, which in an improved model and fit could emerge as the $a_0(1450)$. On the other hand, in the strange channel, there is only one important channel open, the $K\pi$ with a Clebsch-Gordan coefficient reducing the coupling compared to $s\overline{s}$-$K\overline{K}$ by $(3/4)^{1/2}$. This, together with the fact that the $K\pi$ threshold involves two unequal mass mesons, implies that the resonance doubling phenomenon does not appear in the strange sector.

We now look for the actual pole positions in the model of [3], and list the significant ones in Table I. Four of these, which are near the physical region, were already given in [3]: the $f_0(980)$, $f_0(1300)$, $a_0(980)$, and $K_0^*(1430)$. However, we now find two new poles. (There are of course more image poles, which we do not list, since these are very far from the physical region.) Note that all the poles in Table I are manifestations of *the same nonet.* In [3] the Breit-Wigner parameters of the σ meson were also given, but it was not specified which pole should be associated with it. We list in Table I the pole positions both in the usual variable $\sqrt{s}_{\mathrm{pole}}$ and in $s = m_{\mathrm{pole}}^2 - i\mathrm{Im}s_{\mathrm{pole}}$, which is more natural in a relativistic theory.

We now unambiguously find that the $f_0(980)$ and $f_0(1300)$ are two manifestations of the same $s\overline{s}$ state. The only important pole in $u\overline{u} + d\overline{d}$ is the first pole in Table I which is the long sought for σ. Its pole is at $s = 0.158 - i0.235\ \mathrm{GeV}^2$, and it gives rise to a very broad, 880 MeV, Breit-Wigner-like background with $m_{\mathrm{BW}} = 860$ MeV. One can convince oneself that this is the right conclusion by decoupling the two channels $s\overline{s}$ and $u\overline{u} + d\overline{d}$. This can be done within the model, maintaining unitarity, etc., by sending m_0 or $m_0 + 2m_s$ (gradually) to infinity. (One may if one wishes also increase together with m_s the K, η, and η' masses in the thresholds.) The σ pole remains always almost at the same position as in Table I even when $m_s \to \infty$, while there is no trace of $f_0(980)$ nor $f_0(1300)$ in $u\overline{u} + d\overline{d}$. The two latter poles remain, however, in the $s\overline{s}$ channel even when this is completely decoupled from $u\overline{u} + d\overline{d}$. The $f_0(980)$ remains near the $K\overline{K}$ threshold whereas $f_0(1300)$ is shifted to somewhat higher values.

A posteriori, this result is natural also in light of the mixing angles δ_S found for the physical states (see Table I). At the σ pole, as well as for energies $\lesssim$900 MeV, δ_S is small along the real s axis. Thus the $\pi\pi$ amplitudes below this energy are dominated by the σ and only slightly perturbed by the $s\overline{s}$ and $K\overline{K}$ channels. The $f_0(980)$ and the near octet $f_0(1300)$ owe their existence to the $s\overline{s} \to K\overline{K}$ channel dynamics and have a comparatively small mixing with the σ, also evident from the rather small mixing angle δ_S of these states. The near octet nature of the $f_0(1300)$ is supported by the small branching ratio of 0.02 to $\eta\eta$ found by GAMS2000 [7], since the 8-$\eta\eta$ coupling nearly vanishes for the conventional pseudoscalar mixing angle.

TABLE I. The $^3P_0$ resonance parameters in units of MeV. The first resonance is the σ. The two following are both manifestations of the same $s\overline{s}$ state. The $f_0(980)$ and $a_0(980)$ have no Breit-Wigner-like description, and the Γ_{BW} for the latter is rather the peak width. The last entry is an image pole to the $a_0(980)$, which in an improved fit could represent the $a_0(1450)$. The $f_0(1300)$ and $K_0^*(1430)$ poles appear simultaneously on two sheets since the $\eta\eta$ and the $K\eta$ couplings, respectively, nearly vanish. The mixing angle δ_S for the σ is with respect to $u\overline{u} + d\overline{d}$, while for the two heavier f_0's it is with respect to $s\overline{s}$. Pure SU3_f states have $\delta_S = -35.3°$.

Resonance	m_{BW}	Γ_{BW}	$\delta_{S,\mathrm{BW}}$	$[\mathrm{Re}s_{\mathrm{pole}}]^{1/2}$	$\frac{-\mathrm{Im}s_{\mathrm{pole}}}{m_{\mathrm{pole}}}$	$s_{\mathrm{pole}}^{1/2}$	$\delta_{S,\mathrm{pole}}$	Sheet	Comment
$f_0(400-900)$	860	880	$(-9 + i8.5)°$	397	590	$470 - i250$	$(-3.4 + i1.5)°$	II	The σ meson; near $u\overline{u} + d\overline{d}$ state
$f_0(980)$	⋯	⋯	⋯	1006	34	$1006 - i17$	$(0.4 + i39)°$	II	First near $s\overline{s}$ state
$f_0(1300)$	1186	350	$(-32 + i1)°$	1202	338	$1214 - i168$	$(-36 + i2)°$	III,V	Second near $s\overline{s}$ state
$K_0^*(1430)$	1349	498	⋯	1441	320	$1450 - i160$	⋯	II,III	The $s\overline{d}$ state
$a_0(980)$	987	≈100	⋯	1084	270	$1094 - i145$	⋯	II	First $u\overline{d}$ state
$a_0(1450)$	⋯	⋯	⋯	1566	578	$1592 - i284$	⋯	III	Second $u\overline{d}$ state?

Also, the large ratio of squared couplings for $8 \to \pi\pi/K\overline{K} = 3$ explains why the $f_0(1300)$ has the small inelasticity required by the data.

In conclusion, the σ meson definitely exists in the $u\overline{u} + d\overline{d}$ channel, and we tentatively name it $f_0(400 - 900)$, which emphasizes its broad structure. This is the scalar meson required by models for dynamical breaking of chiral symmetry and by nuclear physics. It has the right mass and width. This is the meson which dominates $\pi\pi$ scattering below 900 MeV.

The very large width of the σ explains why it has been difficult to find in the data. There are, however, several published analyses [8], which with less sophisticated models and many parameters have claimed a light and broad σ pole. But, because alternative fits without this broad and light σ were also possible, these analyses did not convince the community of the necessary existence of the light σ. Also, from the theoretical point of view chiral perturbation theory and the nonlinear version of the sigma model do not necessarily need the σ resonance.

The $f_0(980)$ and $f_0(1300)$ are both manifestations of the same $s\overline{s}$ quark model state. Similarly, we believe that the $a_0(980)$ and $a_0(1450)$ are two manifestations of the $u\overline{d}$ quark model state. In the $s\overline{u}$ and $u\overline{u} + d\overline{d}$ systems the image poles of the $K_0^*(1430)$ and σ, respectively, are sufficiently far from the physical region and therefore do not give rise to additional resonances. We emphasize again that all the states discussed in this paper are manifestations of the same quark model nonet, which naturally can be assumed to be the $^3P_0$ nonet. When unitarized, the $^3P_0$ naive quark model nonet spectrum is strongly distorted, and results not in four, but in six different physical resonances of different isospin.

One could argue that the two states $f_0(980)$ and $a_0(980)$ are kinds of $K\overline{K}$ molecules, since these have a large component of $K\overline{K}$ in their wave function. However, the dynamics of these states is quite different from that of normal two-hadron bound states. In particular, it is very different from the hyperfine interaction suggested by Weinstein and Isgur [9]. If one wants to consider them as molecules, it is the $K\overline{K} \to s\overline{s} \to K\overline{K}$ interaction which creates their binding energy. Although they may spend most of their time as $K\overline{K}$, they owe their existence to the $s\overline{s}$ state. And in general, it is not obvious which of the two states, $f_0(980)$ or $f_0(1300)$, would be the extra state and which is $q\overline{q}$, since one can well imagine situations (with different m_0 and γ) where either one of these is removed to being a distant image pole. Therefore, one should rather consider both as two different manifestations of the same $q\overline{q}$ state.

It would be of great interest to apply our coupled channel model to data on $p\overline{p} \to \pi\pi\pi$, $\pi\eta\eta$, $\pi K\overline{K}$, $\pi\eta\eta'$, $\eta\eta\eta$, etc., which would provide ample tests of the model. Also, one should extend the model to higher energies, where there may be gluonium states. Then it would be necessary to include $\sigma\sigma$, $\rho\rho$, πb_1, etc., channels, and all thresholds related to these by flavor symmetry. Such generalizations, although straightforward, would complicate the numerical calculations, but would certainly be rewarding in terms of a better understanding of the hadron spectrum.

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