Nucleation and Growth of the Normal Phase in Thin Superconducting Strips

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We investigated the kinetics of normal phase nucleation and flux line condensation in the type-II superconductors by numerical study of the time-dependent Ginzburg-Landau equation. We have shown that under a sufficient transport current the normal phase nucleates in superconducting strips in the form of the macroscopic droplets having multiple topological charge. We discussed the stability and dynamics of the droplets. We have found that pinning suppresses the droplet formation.

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The study of magnetic flux penetration in type-II superconductors has attracted wide interest both in view of important technological questions and as a prototype of a general class of problems of nonlinear dynamics. Observations showed that the flux dynamics exhibits features that are similar to the viscous-fingering growth phenomenon in liquid-solid systems [1–5]. In particular, recent experiments revealed dendritic flux penetration and the fingering of the remagnetization front [3–5]. The formation of vortex structure is traditionally viewed as the sequential penetration of vortices through the Bean-Livingstone surface barrier [6]. It was found recently that flux penetration may also occur via dynamic instabilities of the order parameter caused by the applied current and/or magnetic field. Numerical simulations revealed the invasion of extended macroscopic normal areas (droplets) carrying flux into the superconducting sample [7,8].

While the formation of normal areas looks natural for type-I superconductors with the positive surface energy of the normal-superconductor (NS) interface, it seems surprising at first sight that such an interface, having in the static case a *negative* surface energy, persists in type-II superconductors. We see the explanation of this phenomenon in the fact that the transport current or alternating magnetic field drives the superconductor into a strongly nonequilibrium state, where the *moving* interface becomes stable. The idea that free energy considerations do not apply to nonstationary processes in superconductors was put forward by Anderson [9] in the context of the phase-slips phenomenon.

In this Letter we report on our investigation of the kinetics of *normal* phase nucleation and flux line condensation in type-II superconductors. We present the results of a numerical study of the dynamics of the flux penetration into strips with transverse dimensions less than the effective penetration length $\lambda_{\text{eff}} = \lambda^2/h$, where h is the thickness of the strip and λ is the London penetration depth. We propose that the existence of the macroscopic normal regions is a direct consequence of their motion under the transport current. A current cannot penetrate the *immobile* compact normal zone immersed in a superconducting state [10], therefore the NS interface moves towards the normal phase with the velocity going to infinity [11,12], and the normal droplet disappears. At the same time, the penetration of the current into the normal phase makes it stable with respect to small fluctuations [13]; i.e., the transport current drives the system into a *bistable* state. Since the expulsion of the current from the normal regions requires a finite time, the current penetrates the moving normal droplet. The normal state develops and invades the superconducting region provided the current j flowing through the interface exceeds the stall current j^* [14,15]. Thus a sufficient transport current stabilizes the moving nuclei of the normal state in type-II superconductors.

The process of flux penetration occurs via the suppression of the order parameter on the macroscopic scale and can be viewed as the nucleation of the extended droplets of the normal phase in the superconducting sample. An adequate description of such a process involving fast variations of the order parameter on the relevant spatial scale is given by the time-dependent Ginzburg-Landau equation (TDGLE) completed by the appropriate Maxwell equations:

$$u(\partial_t + i\mu)\Psi = (\nabla - i\mathbf{A})^2\Psi + (1 - |\Psi|^2)\Psi\,, \quad (1)$$

$$\mathbf{j} = (\Psi)^2(\nabla\varphi - \mathbf{A}) - (\nabla\mu + \partial_t\mathbf{A})\,, \quad (2)$$

$$\nabla\cdot\mathbf{j} = , \qquad \nabla\cdot\mathbf{A} = 0\,, \quad (3)$$

$$\Delta\mathbf{A} = -\frac{1}{\lambda_{\text{eff}}}\mathbf{j}\delta(z)\,, \quad (4)$$

where Ψ is the (complex) order parameter, $\varphi = \arg\Psi$, $\mathbf{A}$ and μ are vector and scalar potentials, and $\mathbf{j}$ is the current density. The value of the dimensionless material parameter u is obtained from the microscopic theory [13]. The unit of length is the coherence length ξ, the unit of time is $t_0 = \xi^2/Du$, $D = v_F l/3$ is the diffusion constant, l is a mean free path, v_F is a Fermi velocity, the field is measured in units of the upper critical field $H_{c2} = \Phi_0/2\pi\xi^2$, and Φ_0 is the flux quantum. The unit of current

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is $j_0 = \sigma\hbar/2et_0$, where σ is the normal conductivity. In these units the depairing current $j_p = 2/3\sqrt{3} \approx 0.3875$. The condition $\lambda_{\mathrm{eff}} \gg 1$ enables us to neglect the magnetic field created by currents [16] and, therefore, drop Eq. (4). We choose the origin of the coordinate frame at the midpoint of the strip with the x axis lengthwise and the y axis in the lateral direction, so that the edges are located at $(x, -d/2)$ and $(x, d/2)$. The perpendicular to the strip magnetic field B is associated with the vector potential $\mathbf{A} = (By, 0, 0)$ (see Ref. [7] for details).

We performed numerical simulations of TDGLE. We took the homogeneous superconducting state as initial condition ($\Psi = 1$, i.e., the state without magnetic field) perturbed by a small amplitude noise. We used the no-flux boundary conditions, $\partial_y \Psi = 0$ (i.e., the boundary with the vacuum) in the transverse direction and the NS boundary conditions in the longitudinal direction $(\Psi(x, y) \to 0$ for $x \to 0, L$, where L is the strip length$)$. We apply the split-step method described in Refs. [7,17]; the number of grid points was 256×256 and the time step was $0.05 - 0.1$. Results of the simulations are shown in Fig. 1. The simulations were performed for $j = 0.25$, $B = 0.0175$, where, as has been shown in Ref. [7], the pure superconducting state is unstable with respect to vortex nucleation (note that our equations do not contain fluctuations). The integration domain was 120×60.

In Fig. 1 the large dark droplets (for $t = 50, 80$) represent the normal phase emerging at one side of the strip and traversing toward the opposite edge. The droplets are long-lived objects and, as well as the vortices, play a crucial role in dissipative processes. In our simulations the topological charge of these droplets would become as big as 5–7 and even greater. The droplets possess long tails (due to the finite relaxation time of the order parameter at the superconducting areas swept by the droplet). Our simulations show that new vortices appear at the edge just at the tail and then get sucked into the droplet. This can easily be understood since the formation of new vortices is favored in the regions with suppressed order parameter. The normal phase areas can evolve in two different ways. First, the normal droplet emerges at the edge, passes through the sample, and vanishes at the opposite edge of the strip. In the second scenario, which occurs under elevated currents, the droplet traverses a strip, leaving a channel (wake) of the normal phase behind.

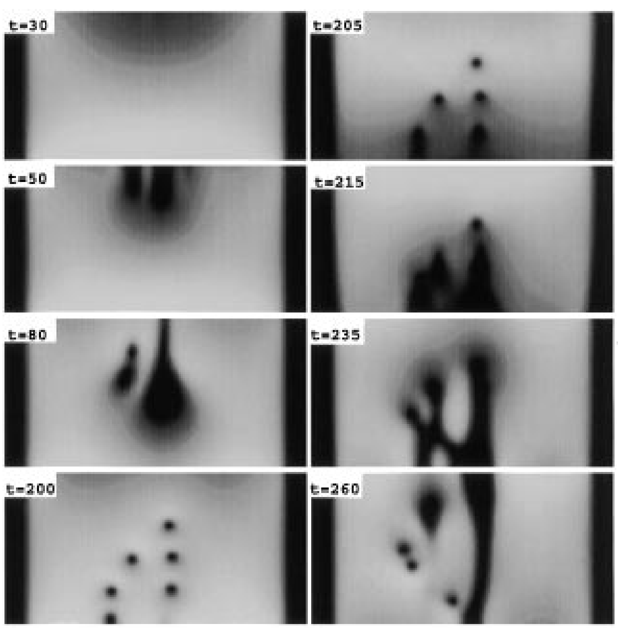

FIG. 1. Dynamics of the normal phase. The current is applied along the x axis and the magnetic field is perpendicular to the strip. Gray-coded images show $|\Psi(x,y)|$ ($|\Psi| = 0$ is shown in black and $|\Psi| = 1$ is shown in white). The field is reversed at $t = 200$.

This scenario is shown in Fig. 1, $t = 260$. This channel traversing the sample then breaks into a sequence of vortices (vortex street), which then propagates across the strip and annihilates at the edge. The nucleation and the propagation of the droplets and the vortices give rise to nonperiodic (probably chaotic) voltage oscillations along the strip. The motion of the vortices and the droplets is also nonmonotonic and can be viewed as "turbulent" flow in contrast to the "laminar flow" of the ordered vortex lattice observed just at the threshold of instability [7].

The droplets possess a topological charge n proportional to the gain in the superconducting phase along the loop enclosing the normal area. A relationship between the characteristic size R of the nucleus and n is then determined from the condition that the supercurrent encircling the nucleus ($\sim n/R$) becomes equal to j_p giving $R \sim n/j_p$. The size of the droplets in the strip can be estimated from the condition that the total transport current at a distance R from the edge $j(R) \approx j - B(R - d/2)$ is equal to j_p. It gives $R = d/2 + (j - j_p)/B$. For the chosen parameters we obtain $R \approx 20$ and $n \approx 6–7$, which is in qualitative agreement with the results of simulations. We expect that the above consideration holds also for the large $d \gg \lambda$ samples, where λ takes the role of the characteristic length. We observed that droplets move much faster than single vortices. Simple analysis shows that the Magnus force exerted on the droplet grows linearly with n whereas the mobility saturates for large n, resulting in this velocity increase.

Shown in Fig. 1 is a sequence of snapshots demonstrating a remagnetization process (we reversed the direction of the magnetic field at $t = 200$). At the first stage of remagnetization large normal phase areas develop at the edge of the strip. These areas swallow vortices corresponding to the previous direction of the magnetic field. The normal areas assume more complicated form and then break up into smaller droplets. At zero applied current, the Abrikosov vortex lattice is formed in the external field. In contrast to the case with nonzero applied current, vortices penetrate from both edges of the strip. When the direction of the field is reversed, large normal areas develop at both edges and swallow vortices corresponding to the initial direction of the field. After awhile, the new Abrikosov lattice forms with vortices along the reversed direction of the field.

To include the Hall effect in our simulations we introduce the complex material parameter $u = 5.79 + i$. The imaginary correction to u describes the effect of the transverse Hall force on the vortex drift [18]. This gives rise to the Hall voltage. Moreover, we observe the turn of the droplet tail. We suggest that the rotation of the droplet's tail in the experimental work [3] is caused by a significant Hall contribution.

To summarize, we have found long-lived droplets of the normal phase inside a superconducting phase, and observed that they may possess a topological charge that can significantly exceed unity. Note that the droplets must be distinguished from the Abrikosov vortices with multiple charge. The linear stability analysis shows that under zero transport current, multicharged vortices are unstable with respect to splitting into singly charged vortices. The characteristic time of the splitting is about 10−15 dimensionless units and, therefore, cannot explain the existence of long-lived droplets (of the order of 100 and more dimensionless units of time). Note that these droplets may be viewed as the result of the "fusion" of the separate vortices.

The qualitative arguments describing the droplet dynamics can be put on a more rigorous basis for the droplets with size well in excess of the coherence length ξ. In this case the boundary of the droplet can be considered locally as a slightly curved NS interface. Inside the droplet the order parameter Ψ vanishes and the field is described entirely by the Laplace equation

$$\Delta\mu = 0\,. \tag{5}$$

Equation (5) has to be completed by the boundary conditions at theinterface, deduced from the continuity equation $\nabla\mathbf{j} = 0$. This gives the relation between the components of currents normal to the interface $j_n^{(n)} = j_n^{(s)}$, where the superscripts s, n denote currents in the normal and superconducting regions, respectively. Using Eq. (2), we arrive at the first boundary condition $-\nabla_n\mu^{(n)} = |\Psi|^2(\nabla_n\varphi - A_n) - \nabla_n\mu^{(s)}$ (here ∇_n means normal projection of the gradient). The order parameter in the superconducting region near the slightly curved interface is given in the "adiabatic approximation" by $|\Psi|^2 = 1 - (\nabla\varphi_0 - A)^2$.

The phase φ of the superconducting order parameter to the leading order is described by the Laplace equation

$$\Delta\varphi = 0\,, \tag{6}$$

together with the equation for the normal velocity of the interface. The latter can be derived from Eq. (1) for the slightly curved interface. The small curvature χ renormalizes the normal velocity c_n of the interface according to the Gibbs-Thomson condition $c_n = c_0 - \chi$, where c_0 is the velocity of the flat interface.

For the flat NS interface the velocity $c(j)$ is a function of the transport current. The one-dimensional situation had been considered in Ref. [15], where the existence of a "stall" current j^*, at which the interface velocity becomes equal to zero has been established. For $u = 5.79$ the stall current was found to be $j^* = 0.335$, and $c(j) \approx c_0(j_n) = \alpha(j^* - j_n)$, where $\alpha = 0.6$ is a numerical factor. In two dimensions the topological charge of the droplet induces a circular current j_τ, tangential to the interface which modifies its velocity. To account for the effect of the tangential current, we take the order parameter close to the nearly flat interface in the form (the interface is parallel to y axis, and we use a frame moving together with the interface along the x axis with velocity c) $\Psi = F(x - ct)\exp[ik_y y + \phi(x - ct)]$, where $k_x = \lim_{x\longrightarrow -\infty}\phi_x$ and $j_\tau = (1 - k_y^2 - k_x^2)k_y$, $j_n = (1 - k_x^2 - k_y^2)k_x$. A simple scaling analysis shows that the current renormalizes the interface velocity as

$$c(j_n, j_\tau) = c_0(\tilde{j})\sqrt{1 - k_y^2}\,, \tag{7}$$

where $\tilde{j} = j_n/(\sqrt{1 - k_y^2})^3$. If the curvature of the interface is small (i.e., $\chi \simeq 1/R \ll 1$), the interface itself is defined by the additional condition that at the (flat) interface $\mu = \mu_0 = k_x c_0(j_n, j_\tau)$. After that the problem is completely defined.

In the superconducting phase we have Eq. (6) completed by the boundary conditions for φ on the strip edges. Thus the problem under study is a generalization of the well-known problem of the Laplacian growth (see, e.g., Ref. [19,20]). A new feature is that the function φ is a multivalued one and has branch cuts. This multivaluedness means that the obtained equations implicitly contain vortex solutions: Vortices can appear and/or vanish via the formation of a singularity at the interface. The detailed consideration of these equations we leave for the future; for now we would like to mention that linear stability analysis shows that the flat interface with the current flowing through is stable with respect to small transversal perturbations. The above discussion and the results of our simulations lead us to conclude that the passage of the current suppresses the NS interface instabilities in thin superconducting films.

To study the effects of pinning we carried out simulations of TDGLE with randomly distributed pinning centers. In the presence of weak pinning the newly formed droplets assume the "fractal" configuration since the normal phase tries to settle at the pinning sites where the order parameter is already suppressed (see Fig. 2). The moving droplets percolate along easy paths connecting the pinning sites, but the pinning centers impede the interface motion. As a result, the current that penetrates the normal

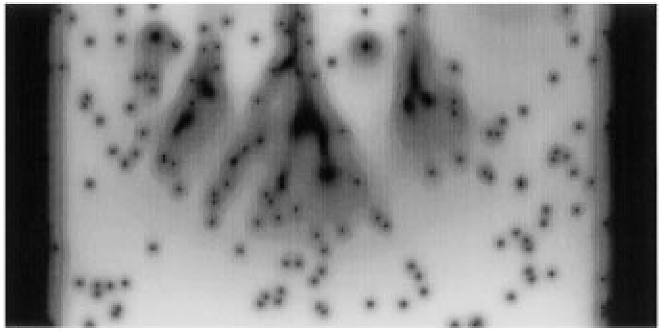

FIG. 2. Normal phase penetration at $t = 40$, $j = 0.25$, and $B = 0.018$ in the presence of 180 randomly distributed pinning centers. Other parameters are the same as in Fig. 1.

area gets smaller and can no longer support the existence of the droplet, and the droplets break up. For stronger pinning the droplets do not form at all, and single vortices penetrate the strip via jumps resembling the vortex motion through an array of linear defects [21].

Finally we discuss briefly the time scale of the observed effects. The characteristic time in dirty superconductors is $t_0 \simeq \hbar/T_c(1 - T/T_c) \approx 10^{-14}-10^{-11}$ sec, depending on the temperature interval. This means that the considered phenomena develop on the nanosecond scale. However the process of flux penetration can be considerably slowed down by pinning. In this case the characteristic time (for "dendritic" formations, for example) should include macroscopic characteristics, such as the size of the sample and the average pinning strength [22], and can increase considerably.

In conclusion, we have shown that, under a sufficient transport current, the normal phase nucleates in the superconducting strips in the form of macroscopic droplets which tear off at the edges and further propagate across the sample. These droplets possess a multiple topological charge related to the magnetic flux they carry. Pinning suppresses the droplet formation converting the normal area into multiconnected fractal formations which then split into separate vortices. We believe that the observed phenomena are not specific to thin strips, and that the same mechanism governs the normal phase formation in large samples as well.

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[1] H. Frahm, S. Ullah, and A. Dorsey, Phys. Rev. Lett. **66**, 3067 (1991); F. Liu, M. Mondello, and N. Goldenfeld, Phys. Rev. Lett. **66**, 3071 (1991); R. E. Goldstein, D. Jackson, and A. T. Dorsey, Report No. cond-mat/9411007, 1994 (to be published).
[2] R. P. Huebener, *Magnetic Flux Structures in Superconductors* (Springer-Verlag, New York, 1979).
[3] V. V. Vlasko-Vlasov *et al.*, Physica (Amsterdam) **222C**, 361 (1994).
[4] C. A. Duran, P. L. Gammel, R. E. Miller, and D. J. Bishop, Phys. Rev. B **52**, 75 (1995).
[5] M. V. Indenbom *et al.*, Phys. Rev. B **51**, 15 484 (1995).
[6] L. Burlachkov *et al.*, Phys. Rev. B **50**, 16 770 (1994).
[7] I. Aranson, M. Gitterman, and B. Ya. Shapiro, Phys. Rev. B **51**, 3092 (1995).
[8] H. Kaper *et al.* (to be published).
[9] P. W. Anderson, Rev. Mod. Phys. **38**, 298 (1966).
[10] A. A. Abrikosov, *Fundamentals of the Theory of Metals* (Elsevier, New York, 1988).
[11] I. O. Kulik, Zh. Eksp. Teor. Fiz. **59**, 584 (1970) [Sov. Phys. JETP **32**, 318 (1971)].
[12] B. I. Ivlev and N. B. Kopnin, Adv. Phys. **33**, 47 (1984).
[13] L. P. Gor'kov and N. B. Kopnin, Usp. Fiz. Nauk **116**, 413 (1975).
[14] R. J. Watts-Tobin, Y. Krähenbühl, and L. Kramer, J. Low Temp. Phys. **42**, 459 (1981).
[15] K. K. Likharev, Pisma Zh. Eksp. Teor. Fiz. **20**, 730 (1974) [JETP Lett. **20**, 338 (1974)].
[16] J. Pearl, Appl. Phys. Lett. **5**, 65 (1966).
[17] I. Aranson, L. Kramer, and A. Weber, J. Low Temp. Phys. **89**,859 (1992).
[18] A. T. Dorsey, Phys. Rev. B **46**, 8376 (1992); A. G. Aronov *et al.*, Phys. Rev. B **51**, 3880 (1995).
[19] D. Kessler, J. Koplik, and H. Levine, Adv. Phys. **37**, 225 (1988).
[20] M. Mineev-Weinstein, Phys. Rev. E **47**, R2241 (1993).
[21] D. R. Nelson and V. M. Vinokur, Phys. Rev. Lett. **68**, 2398 (1992).
[22] V. M. Vinokur, M. V. Feigelman, and V. B. Geshkenbein, Phys. Rev. Lett. **67**, 915 (1991).